The word position has several meanings, two of which, "posture or attitude" and "site or location," are easily confused. We will use the word position always with the latter meaning.

In physics, the terms "relative position" and "relative motion" refer to the fact that position and motion must be defined "relative to" or "in relation to" something other than the object itself.

In common speech, we often simply use ourselves or the earth as the reference object without saying so. For example, we say, "The moon is far away," or "The train is moving." It usually seems unnecessary to say "The moon is far away from me." or "The train is moving with respect to the station." However, in physics we often must describe motion from various reference frames, including especially those in which we are not at the origin and/or in which we are not at rest. Therefore, whenever there is any possibility of confusion, we will explicitly name the reference frame, and you should do the same.

You may associate the word "relativity" with mathematical mystery and scientific complexity, yet the basic concept, which we will try to explain in these pages, is simple. The matters of concern in relativity are the position (location) and motion of objects. The basic concept is that position and motion of an object can only be perceived, described, and recognized with reference to (that is, "relative" to) other objects. When you say, "The physics books are at the left rear of the book store," you refer the position of the books to the entrance and outline of the store. Objects such as the store entrance, to which position or motion are related, are called *reference objects*. Several reference objects used in combination to describe position are said to form a *reference frame (*or *frame of reference)*, and we speak of the position or motion of the original object relative to the reference frame.

If we know the position and motion of an object relative to one reference frame, we might ask about the position and motion relative to a second reference frame. This is the root of the theory of relativity: development of specific mathematical models for relating position and motion as observed relative to one reference frame to position and motion as observed relative to another reference frame. Einstein's theory of relativity is the most complete theory of these relationships. We will describe some aspects of Einstein's work in Section 7.3, but we will not go into the mathematical details in this text.

2.1 Relative position

Look at the girl in the field of daisies (Fig. 2.1). How would you tell someone where she is? Most directly, you could go to the edge of the field and point at her, saying, "The girl is there." By this action, you indicate the position of the girl relative to your outstretched arm and finger.

If you had to describe the girl's position to someone who was not watching, you could say, "She is a little way in from the south edge of the field, near the southeast corner." This statement indicates her position relative to the edges and corners of the field. In other words, it is impossible to describe the position of the girl (or of anything else) without referring to one or more other objects. Even if you were to draw a map of the girl's position, you would have to include on it some objects that could be used to align it with the actual field.

Reference objects and reference frames. For practical purposes, the reference objects must be easy to locate and identify, or they cannot be used as guides in finding the object whose position is being described. It would be hopeless, for example, to try to find the girl in the daisy field if her position were described by saying, "The girl is between two daisy blossoms." Something more distinctive is needed: the edges and comers of the field, as used earlier, or possibly a scarecrow at the center of the field.

The use of reference objects in everyday life is highly varied and adapted to many special circumstances. A piece of furniture, corners of

Figure 2.1 Can you find 3 children hiding among the daises? Describe their relative positions.

a room, street intersections, or a tall building can be used as a reference frame for the complete description of the location of a residence, restaurant, or mailbox.

Examples. An imaginary conversation is recounted in Fig. 2.2. What happened in this conversation? What finally allowed Percy to communicate the location of the hawk without confusion, ambiguity, or absurdity? First, he established a reference frame by selecting a large, easily identifiable branch on the tree and pointing in the direction of the tree. Clyde could grasp this reference frame. Next, Percy specified the direction ("above it") and distance ("the second branch") from the branch to the hawk.

The reference frame first used by Percy consisted of a reference *point* (Percy's body) and a reference *direction* (along Percy's pointing finger). These two components are necessary parts of a reference frame and are defined through more or less easily identified reference objects or earth-based directions, such as north and up. Percy's initial attempt to use the tree as reference point failed because there were several trees.

An overdose of relativity.

Mr. Jones was going to a doctor's office and had never been there before. He called the doctor's office to ask for directions. After the receptionist told him how to get there, he asked whether it was on the north or south side of the street.

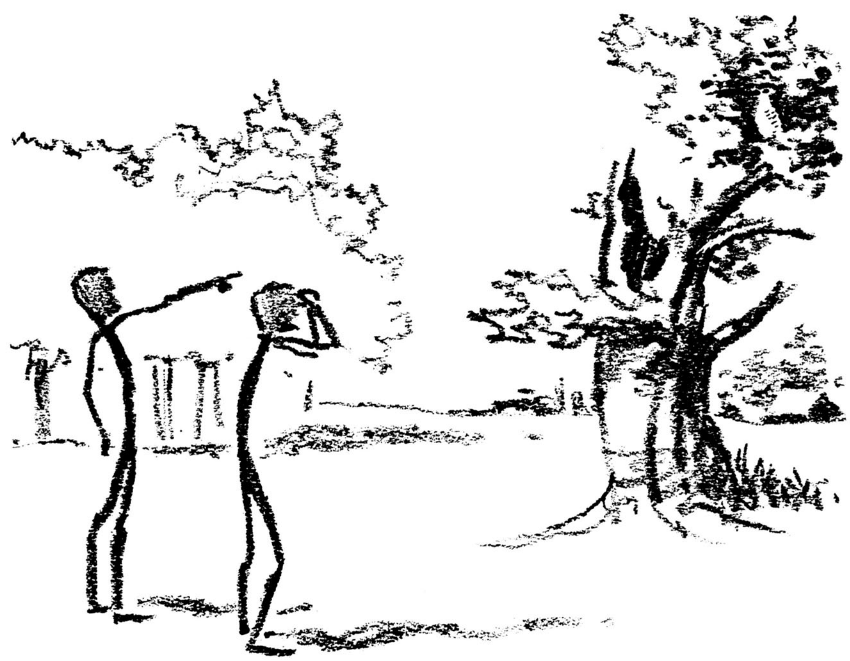
The response: "It depends which way you're walking."

One of the most difficult communications problems is to give instructions for locating a book to a person who is not acquainted with the room in which the book is kept. In such a case it is most helpful to use the person's body as the reference frame by telling him to stand in the door to the room, look for the bookcase on his left, and then scan the middle of the second shelf of that bookcase. This example illustrates how you might use large elements of the environment (the room) to locate smaller ones (the person, and directions defined by the body), and then still smaller ones (the bookcase, ultimately the book) by a narrowing down process.

Describing position is more difficult when you do not have any reference objects to use for a narrowing down process. For example, a passenger on a ship who observes something in the ocean faces this problem. In these circumstances you would have to start with the ship you are on and work outward. You may sight a flying fish "500 yards off the starboard bow" (ahead and to the right), using the ship as reference frame. You could say instead that the fish is "500 yards northwest," using the ship as reference point and compass directions to complete the reference frame.

One-particle model. So far we have been content with describing the position of a very small object that is located at a certain point in space. Real objects, of course, actually occupy an entire region of space, which may be small or large, round or thin, upright or slanted. For a complete description of an object, you therefore should take into account its shape and orientation as well as its location. A useful approach that avoids much unnecessary detail is to make a *one-particle model* for each object of interest. A particle is a very small object

Figure 2.2 Percy and Clyde took a long walk through the county of McDougall.
Percy: Clyde, do you see the falcon sitting in that tree over there?
Clyde: What falcon? In which tree? Where?
Percy: In that big, broken tree over there (pointing his finger).
Clyde: Oh, that tree in front of us! I see it, but I don't see the falcon.
Percy: It's on the branch.
Clyde: There are too many branches. I give up. Let's forget it.
Percy: No, let's start over again. Do you see that broken branch about halfway up the trunk on the right side?
Clyde: Yes, I do.
Percy: Fine. Now look at the second branch above it, on the same side of the tree. Now move to the right and you just have to see the falcon.
Clyde: Oh sure, but that's a hawk.



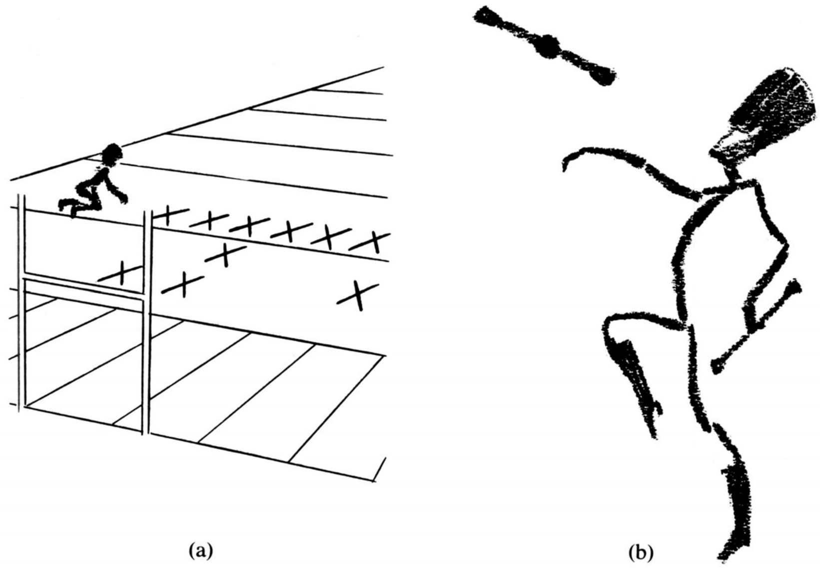


Figure 2.3 Two applications of a one-particle model.
(a) An X represents each football player. Eleven " particles" represent the team.
(b) A working model for the baton is the particle at its center.

that is located at the center or midpoint of the region occupied by the real object (Fig. 2.3). This working model greatly oversimplifies most objects, but is nevertheless accurate enough for most purposes in this text.

Coordinate frames. The laboratory scientist, who tries to describe natural phenomena in a very general way, avoids using incidental objects such as the laboratory walls or table surfaces as reference objects. Instead, a scientist frequently uses a completely artificial reference frame consisting of an arbitrarily chosen reference point and reference direction. The only requirement is to be able to describe the position of objects by using numbers. The numerical measures are called *coordinates;* the reference frame is called a *coordinate frame.* Two coordinate frames in common use are degrees of latitude and longitude, to define position on the earth relative to the equator and the Greenwich meridian, respectively, and distances measured in yards from the end zones and the sidelines on a football gridiron.

Polar coordinates. The procedure of giving the distance from the reference point and the direction relative to the reference direction gives rise to two numbers called *polar coordinates.* The distance may be measured in any unit, most commonly in meters, centimeters, or millimeters. The relative direction is usually measured in angular degrees. How this works is shown in Fig. 2.4. The necessary tools are a ruler to measure distance and a protractor to measure angles. Polar coordinates provide an operational description of the relative position of a point.

A polar coordinate grid, from which you may read the polar

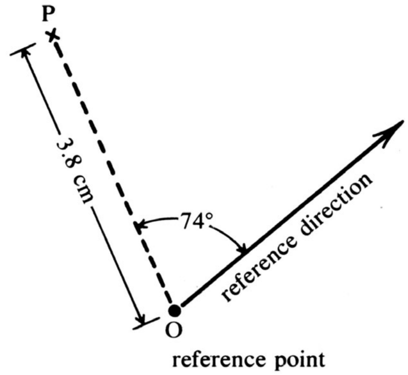


Figure 2.4 Polar coordinates of the point P relative to the reference point 0 and the reference direction indicated by the arrow.

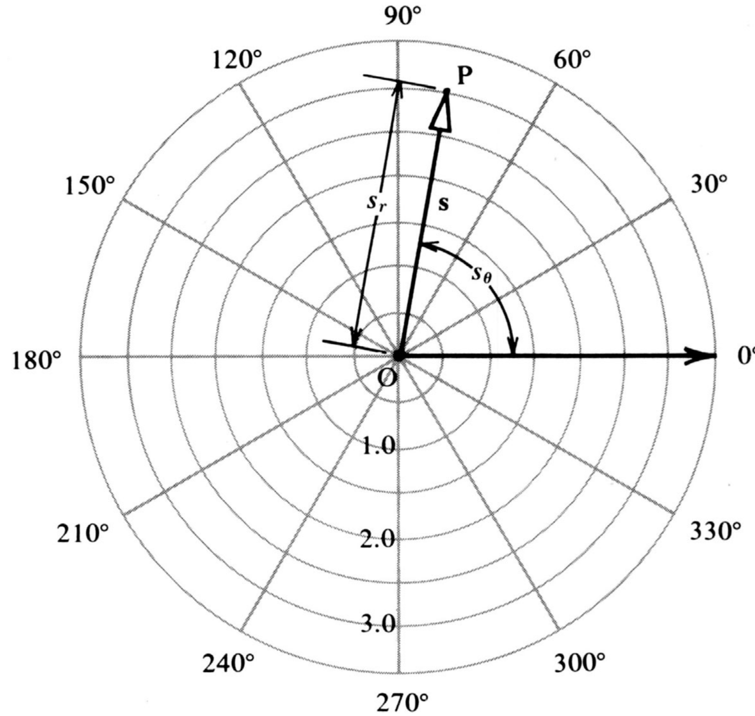


Figure 2.5 Polar coordinates of the point P which has relative position **s** = (s_r, s_θ) = (3 cm, 80°).

s_r and s_θ are examples of variables with subscripts. The subscripts (r and θ) indicate that the main variable (s) has two (or more) distinct values. The subscripts can also be numbers, for example s_1, s_2, s_3 and so on.

You should also be aware of the difference between the meaning of subscripts (described above) and superscripts, for example s^3, which means $s \times s \times s$.

The reference point in polar coordinates is called a pole because the grid lines converge on it, as do the meridians at the poles of the earth.

coordinates of a point directly, is shown in Fig. 2.5. We shall indicate the relative position of the point P by an arrow in the diagram and by the boldface symbol **s** in the text. The polar coordinates will be indicated by the length of the arrow s_r (*r* for radius) and the direction of the arrow s_θ (Greek θ, theta, for angle), as indicated in the figure. Angles are customarily measured counterclockwise from the reference direction.

Examples. An example of the application of polar coordinates is shown in Fig. 2.6. The surveyor is using the direction of the road as the reference direction and the location of his tripod as the reference point. Some of the measurements he has made are given in the figure. These measurements may be used to make a map by locating the objects on a polar coordinate grid, as in Fig. 2.7. Polar coordinates have the intuitive advantage that they represent relative position in the way a person perceives them, namely, with the objects at various distances in various directions from the observer at the center.

Polar coordinates are used to direct aircraft to airports, with the control tower as reference point and north as the reference direction, as well as in other situations where a unique central point exists (e.g., in radar surveillance with the transmitter acting as reference point).

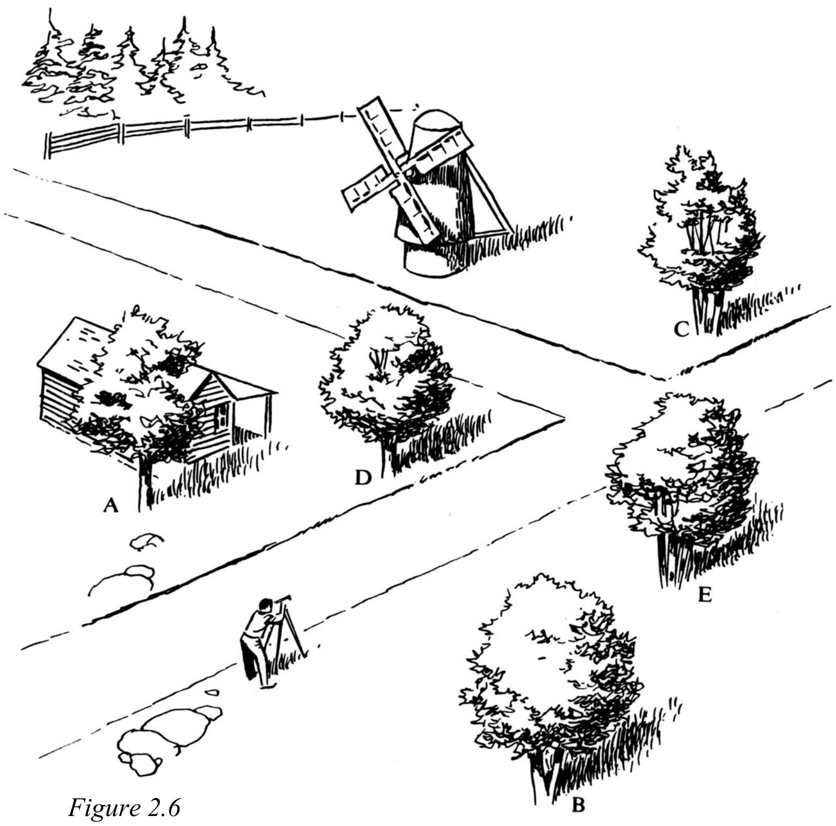


Figure 2.6
Polar coordinates.

	direction	distance
tree A	80º	10 m
tree B	270º	6 m
tree C	10º	31 m
windmill	40º	38 m

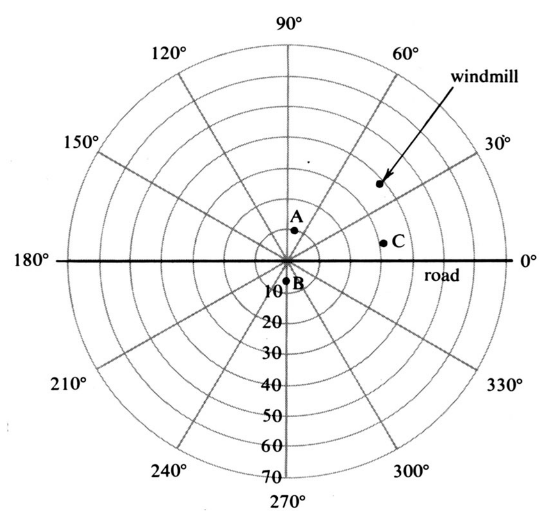


Figure 2.7 The position of the objects in Fig. 2.6 is mapped on a polar coordinate grid.

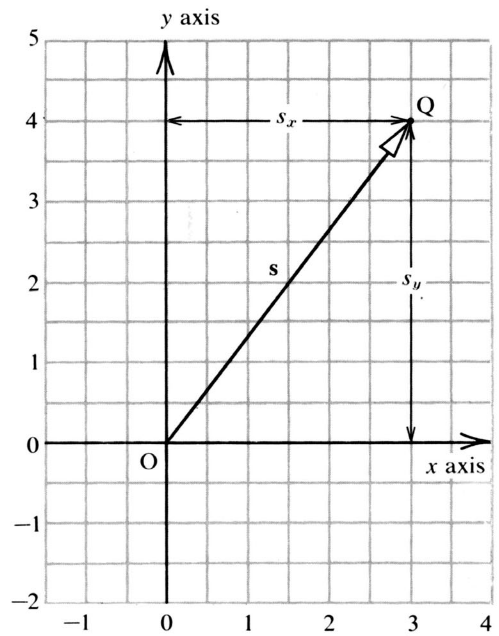


Figure 2.8 Rectangular coordinates s_x and s_y of the point Q, whose relative position is **s** *= [s_x, s_y] = [3.0, 4.0]. The two rectangular axes are indicated by arrows, and the origin of the coordinates by O.*

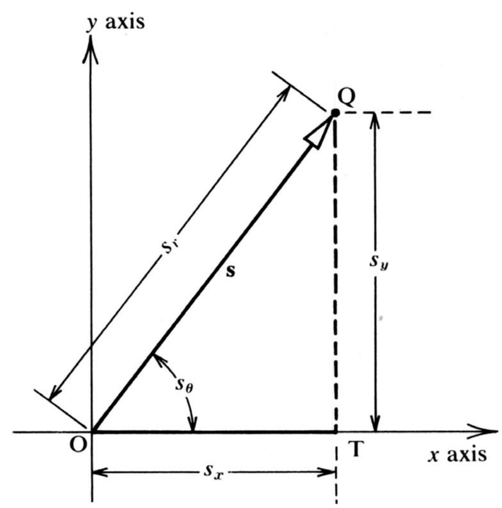


Figure 2.9 Here Fig. 2.8 is redrawn to show the rectangular and polar coordinates of the point Q. These are related by means of the right triangle QTO.

Rectangular coordinates. A particularly useful technique for describing relative position that we will employ extensively later in the course makes use of two lines at right angles to each other called *rectangular coordinate axes* (Fig. 2.8). They are usually labeled the x-axis and y-axis. Their point of intersection is called the *origin of coordinates*. Distances are measured to the desired point Q along lines perpendicular to each axis, and the two measurements obtained are called the *rectangular coordinates* of the point Q relative to the two axes. As before, we introduce the boldface symbol **s** for the relative position of a point and use ordinary letter symbols with subscripts for the rectangular coordinates, this time s_x and s_y. The relative position of point Q is indicated by an arrow from O to Q in Fig. 2.8, just as it was in Fig. 2.5. Sometimes, for the sake of brevity, we will write the rectangular position coordinates in square brackets, with s_x first and s_y second: [s_x , s_y] (Fig. 2.8).

Rectangular coordinates are sometimes called Cartesian coordinates, after René Descartes. Descartes, a French philosopher, first described this coordinate frame in the Discourse on Method*, published in l637.*

You can see that the rectangular coordinates, unlike polar coordinates, do not give directly the distance s_r of a point from the origin. You can find the distance, however, by applying the Pythagorean theorem (Appendix, Eq. A.5) to right triangle QTO in Fig. 2.9, where the distance s_r *is* the length of the hypotenuse OQ:

$$s_r = \sqrt{s_x^2 + s_y^2}\text{, see Ex. 2.1, below.}$$

EXAMPLE 2.1. Relate the polar coordinates of the point Q to its rectangular coordinates (Fig. 2.9) $s_x = 3.0$, $s_y = 4.0$.

Solution:

(a) To find s_r use the Pythagorean theorem (Appendix, Eq. A.5).

$$s_r = OQ = \sqrt{{s_x}^2 + {s_y}^2} = \sqrt{(3.0)^2 + (4.0)^2} = \sqrt{25.0} = 5.0$$

(b) To find s_θ, use the definition of the trigonometric ratios (Appendix, Eq. A.6):

$$tangent(s_\theta) = \frac{s_y}{s_x} = \frac{4.0}{3.0} = 1.33$$

$$s_\theta \approx 53^\circ \text{ (from Table A.7)}$$

Rectangular coordinate frames, unlike polar coordinate frames, do not have a single center, as you may observe easily when you compare the polar and rectangular coordinate grids in Fig. 2.5 and 2.8. Whereas there is a unique reference point, the "pole," in the polar grid, it is possible to use any point in a rectangular grid as reference point by selecting the horizontal and vertical lines passing through this point as *x* axis and *y* axis. With R as reference point in Fig. 2.10, for instance, the point Q has the rectangular coordinates [*5, 1*], as you may verify in the figure. Rectangular coordinates, therefore, are advantageous when you are interested in the position of two points or objects relative to one another, rather than only relative to the origin of the coordinate frame. We will use this feature when we calculate changes in the position of a moving object.

2.2 Relative motion

So far we have considered ways of describing the relative position of objects that are stationary. Now, consider objects that change position. When an object changes position, you commonly say that it "moves," or that it is "in motion." But what do you mean by "motion?" Since position is defined relative to a reference frame, it is plausible to expect that motion would also be defined relative to a reference frame. It is therefore customary to use the phrase "relative motion."

Examples of relative motion. Imagine that a truck moving on a roadway is being described relative to two different reference frames. Reference frame A is attached to the roadway, reference frame B to the truck. To make the description simpler and more concrete, we will introduce two observers, one representing each reference frame (Fig. 2.11).

As the truck moves down the road, Observer A reports its position first on his left, then in front of him, then on his right. The position of

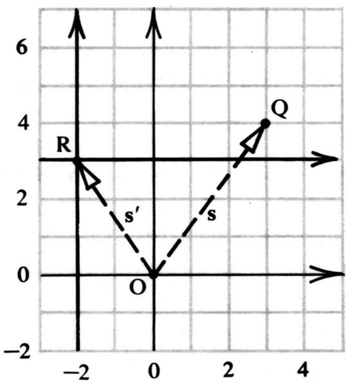


Figure 2.10 (above) The point R is chosen as reference point for describing the relative position of Q. The coordinates of Q relative to R are [5, 1]. (Note the reduced scale of the diagram compared to Figs. 2.8 and 2.9.)

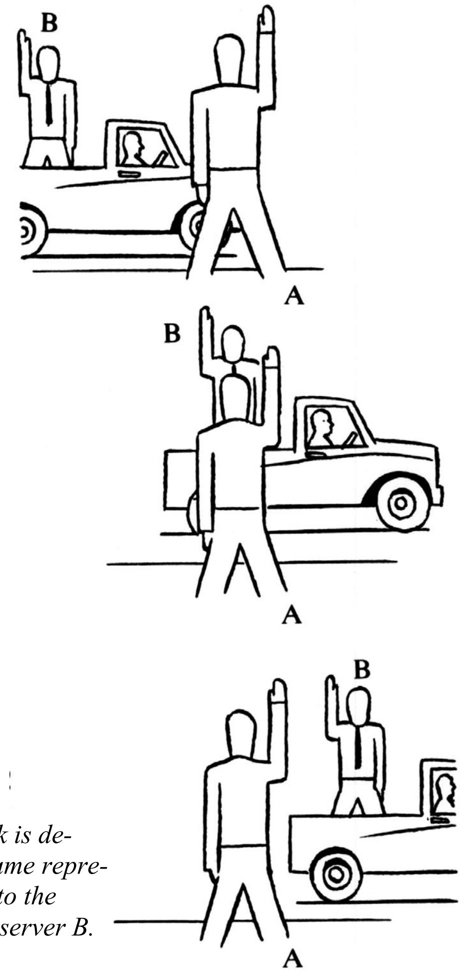


Figure 2.11 The motion of the truck is described relative to the reference frame represented by Observer A and relative to the reference frame represented by Observer B.

the truck relative to Observer A has changed. But Observer B always reports the truck as being in the same position, with the platform under his feet and the driver's cab on his left. The position of the truck relative to Observer B, therefore, has not changed from the beginning to the end of the experiment. Thus, you find that you have two different sets of data, one from each reference frame. In one reference frame you would conclude that the position had changed, and in the other that it had not. If we define relative motion as the change of position relative to a reference frame, then the two observers disagree (but each is correct) not only with regard to the relative position of the truck at various times in the experiment, but also about the truck's relative motion.

We can extend the discussion to motion of other objects. For instance, does the earth move? This depends on the reference frame used to define the earth's position. Relative to a reference frame attached to the earth, to which we are all accustomed, the earth is stationary. Relative to a sun-fixed reference frame, which was introduced by Copernicus and about which you probably studied in school, the earth

"For every apparent change in place occurs on account of the movement either of the thing seen or of the spectator, or on account of the necessarily unequal movement of both. For no movement is perceptible relatively to things moved equally in the same directions — I mean relatively to the thing seen and the spectator. Now it is from the Earth that the celestial circuit is beheld and presented to our sight. Therefore, if some movement should belong to the Earth . . . it will appear, in the parts of the universe which are outside, as the same movement but in the opposite direction, as though the things outside were passing over. And the daily revolution . . . is such a movement."

Copernicus
De Revolutionibus
Orbium Coelestium, 1543

moves in its orbit. In Chapter 15 we shall describe the resistance which Copernicus and Galileo encountered when they took the sun-fixed reference frame seriously.

Look at some of the consequences of this concept of relative motion. (You may find these consequences fascinating or merely strange, depending on how willing you are to break out of habitual modes of thinking.) In order to determine the position and motion of objects in an experiment, it is important to decide upon a reference frame. You have already noted that observers may disagree about the relative motion of an object. You must also recognize that an observer will always report an object to be stationary in his own reference frame so long as he is attached to that object. Such an observer's report about the motion of the remainder of the world will seem unusual indeed, if the observer's reference frame is attached to a merry-go-round, a satellite in orbit, or even a sewing machine needle.

Speed and relative motion. Another consequence of the relative motion concept is that different observers might disagree about the direction and the speed of a moving object they both observe. Think about the following example, in which an object is reported to travel at different speeds relative to different reference frames.

The speed of a riverboat going upstream as reported by its passengers looking at the shoreline is a snail-like 1 mile per hour, but the speed as reported by the captain is a respectable 10 miles per hour. Who is right? All steering and propulsion take place in the reference frame of the water. The motion of a riverboat relative to the shore is different from its motion relative to the water unless the water is still. Since the water is flowing downstream at a speed of 9 miles per hour (relative to the shore), and the riverboat is traveling upstream at a speed of 10 miles per hour (relative to the water), the speed of the riverboat (relative to the shore) is 1 mile per hour. Thus, the conflicting reports of the two observers are understandable and correct, for they are observing from different reference frames. The question, "Who is right?" can only be answered, "Each is right from his own point of view."

Here is a second example, in which we would like you to imagine you are each of the observers in turn:

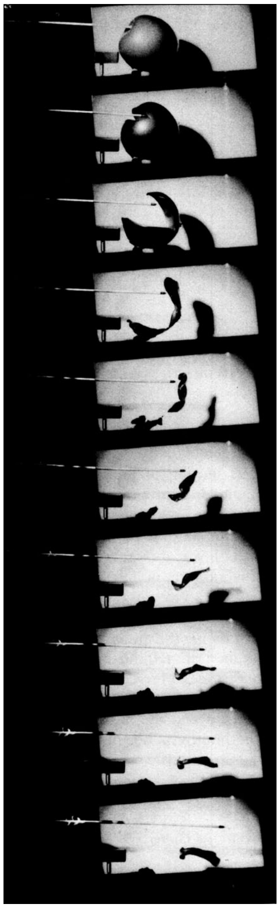
Three cars, A, B, and C, are traveling north on a highway at speeds of 55, 65, and 75 miles per hour, respectively. Observers attached to each car make the following reports.

What does the observer in car B report about the speed of the roadside? What does the observer in car C report about the speed of car A and the roadside?

Observer in car A: Relative to me, car B is traveling north at 10 miles per hour, car C is traveling north at 20 miles per hour, car A (my car) is stationary, and the roadside is traveling south at 55 miles per hour.

Observer in car B: Relative to me, car A is traveling south at 10 miles per hour and car C is traveling north at 10 miles per hour.

Observer in car C: Relative to me, car B is traveling south at 10 miles per hour, and car C is stationary.



Recording and reproducing relative motion. Since relative motion is a transitory phenomenon, it cannot be recorded on a diagram or map with the ease with which relative position can be recorded. The motion-picture film is the most familiar way of recording and recreating relative motion.

Motion pictures. Motion pictures consist of a strip of photographs (Fig. 2.12) that show a scene at very short intervals (approximately 1/24 second). Of course, the scene changes, but does not change much in this short time. When the pictures are rapidly projected in the correct order, the viewer's eye and mind perceive smooth motion. If the pictures are taken with too great a time interval, so that the scene changes significantly between, then the smooth motion becomes jerky.

Flip books. Another way to represent relative motion is through flip books. Flip books create the illusion of motion through the same device as motion pictures, a series of pictures that show the same scene with slight changes in appearance. The pictures are bound in a book and are viewed when the pages of the book are flipped. With a flip book, you can examine each individual scene more easily than with a motion picture, you can control the speed, and you can view the sequence both "forward" and "backward" by starting at the front or the back of the book.

Multiple photographs. Still another technique for representing but not recreating motion is the multiple photograph. This is produced by taking many pictures at equal short time intervals on the same piece of film as in the example shown in Fig. 2.13. The multiple photograph gives a record of the path of the racquet and of the ball during a serve.

Blurred photographs. Even a single photograph can give evidence of motion when the camera shutter remains open long enough for the image projected onto the photographic film to change appreciably.

Figure 2.12 A section of motion-picture film taken at high speed, showing an arrow bursting a balloon. Harold Edgerton, the master of high speed photography, took the photograph by means of a rotating prism synchronized with a strobe [regularly flashing] light. In 1940, Edgerton won an Academy Award for movies made with this type of camera. For other photos by Edgerton see Stopping Time *by G. Kayafas, listed in the bibliography at the end of this chapter.*

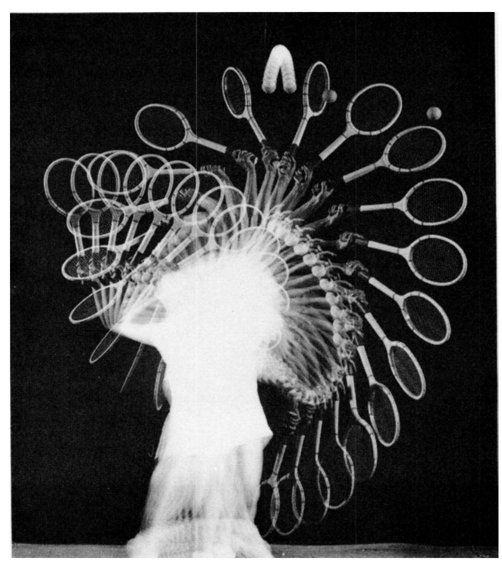


Figure 2.13 Multiple exposure photograph of a tennis serve, taken by Harold Edgerton. Can you estimate the time interval between exposures?

Figure 2.14 How many people are shown in this picture?

This can happen either because the photographic subject moves (Fig. 2.14), or, as every photographer knows, because the camera moves. In other words, the significant fact is motion of the subject relative to the camera. Indeed, an experienced photographer can "stop" the motion of a racing automobile by purposely sweeping his camera along with the automobile (Fig. 2.15). The automobile appears sharply in this picture, but the background, which was moving relative to the automobile and therefore relative to the camera, is blurred.

Example. An illustration of how significant relative motion can be was discovered by Berkeley physicist Luis W. Alvarez in a magazine reproduction of part of a motion-picture film showing the assassination of President John F. Kennedy. The President was riding in a motorcade, and Alvarez noticed something exciting in photograph 227: the motorcade in the photo was blurred, but the background and foreground were sharp. This was in contrast to most other photos, where the background was blurred and the motorcade was sharp. Apparently, Alvarez reasoned, the photographer had been sweeping his camera sideways to keep it lined up with the moving car but had suddenly stopped the

Figure 2.15 The racing car is stationary relative to the camera. Were its wheels stationary?

motion for a fraction of a second. The film, which showed the relative motion of camera and photographed objects by a steady change in picture from frame to frame, showed a change in this relative motion, a change that Alvarez ascribed to the photographer's neuromuscular reaction to the sound of a rifle shot. Further investigation of the original film in the National Archives, and of human flinching reactions to sudden sounds, confirmed Alvarez's interpretation of the blurred motorcade as evidence that a rifle was fired at that instant.

FORMAL DEFINITION
The average speed is equal to the ratio of the distance traversed divided by the time interval required to traverse the distance.

Equation 2.1

distance traversed $= \Delta s$
(pronounced "delta ess")
time interval $= \Delta t$
(prounced "delta tee")
average speed $= v_{av}$

$$v_{av} = \frac{\Delta s}{\Delta t}$$

Note: The "Δs" and "Δt" symbols stand for single quantities and do not indicate multiplication of Δ by "s" or by "t". The meaning of the Δ symbol will be explained further below (Section 2.3).

Units of speed:
meters per second (m/sec)
miles per hour (mph)
feet per second (ft/sec)

1 m/sec ≈ 2.2 mph ≈ 3.3 ft/sec

1 mph ≈ 0.45 m/sec ≈ 1.5 ft/sec

(≈ indicates approximately equal)

Definition of speed. Up to this point we have used the word "speed" without defining it. Automobile speed in miles per hour is usually read directly off the dial of a speedometer, the speed of a runner is expressed in his or her times for a particular distance, wind speed is indicated by a device called an anemometer, and so on. To be comparable with one another, these speeds must all be derived from the same definition. As the unit of speed (miles per hour) suggests, the generally accepted definition (at left) is the rate at which distance is traversed (Eq. 2.1).

This definition can be applied whenever a distance and a time measurement have been made. You are probably familiar with the highway "speedometer checks," roadside markers that identify a one-mile stretch; by driving at a steady speed and observing the time required to traverse the distance, you can calculate the speed (Example 2.2).

EXAMPLE 2.2. Find the speed if the distance traversed and the time interval are given.

(a) $\Delta s = 2.5\, m \qquad \Delta t = 4.5\,\text{sec}$

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{2.5\ m}{4.5\ sec} \approx 0.56\ m/sec$$

(b) $\Delta s = 140\, m \qquad \Delta t = 0.4\,\text{sec}$

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{140\ m}{0.4\ sec} \approx 350\ m/sec$$

(C) $\Delta s = 1\, mile \qquad \Delta t = 4\,\text{min}$

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{1\ mile}{4\ min} \approx 0.25\ mile/min$$

$$= 15\ mph \approx (15 \times 0.45)\ m/sec = 6.8\ m/sec$$

The definition can be applied in many other circumstances, too, where the distance and time interval may be measured in any convenient unit. The speed of a taxicab in a large city, for instance, may be described as six blocks per minute, the speed of a bus may be only three blocks per minute, the speed of an elevator may be one floor in three seconds (one-third floor per second or 20 floors per minute), and so on.

You can convert from one to another of these units of speed if you know how the units of distance and time compare (number of feet per floor, number of seconds per minute).

"If two particles are carried at a uniform rate, the ratio of their speeds will be the product of the ratio of the distances traversed by the inverse ratio of the time intervals occupied."
Galileo Galilei
Dialogues Concerning Two New Sciences, 1638

Curiously, the work of Galileo, who was the first to investigate moving bodies systematically and quantitatively, contains no reference to this idea of speed as a numerical quantity (v) equal to the ratio of distance divided by time. Instead, he always compared two or more speeds with one another (often by comparing the times to go equal distances, or by comparing the distances traveled in equal times), and he was able to derive and state his results by using ratios of distances (or times) to one another. One of Galileo's major contributions was a clear understanding of what we now call "average speed" and "instantaneous speed." We now explain these two key concepts.

Average speed. When you think of a bus making its way in city traffic, you immediately realize that the speedometer reading has little direct connection with a measured speed of, say, three blocks per minute. After all, the bus is stopped a good fraction of the available time. The speedometer needle may swing from 0 miles per hour (the bus is stopped) up to 20 or even 30 miles per hour while the bus is moving, and then back to 0 miles per hour again at the next stop. If you count how many blocks the bus travels in a minute, you include the stops and the motion. The speed determined in this way is called the *average speed*, because it is an average value intermediate between the maximum and minimum values. The average speed is always referred to a certain distance or time interval, such as the average speed over a mile of highway (speedometer check) or in a minute of city driving (bus and taxi examples). A car that required a minute to drive 1 mile on the highway was traveling at the average speed of 1 mile per minute, 60 miles per hour, or about 90 feet per second.

FORMAL DEFINITION
The instantaneous speed is equal to the average speed measured during an "instant." An instant is a time interval short enough that the speed does not change to a significant extent.

Instantaneous speed. After this explanation, you may wonder what the car's speedometer indicates. The speedometer indicates the "actual speed" of the car. The actual speed is equal to the average speed if the car is driven steadily without speeding up or slowing down. In this way the speedometer check can be used as intended by the highway builders. In other words, the average speed, which can be measured in the standard units of distance and time, is used to calibrate the speedometer dial.

OPERATIONAL DEFINITION
The instantaneous (or actual) speed is equal to the number shown by a speedometer.

There is a second relation between average speed and actual speed – a relation that has led to the term *instantaneous speed* for the latter. Imagine the average speed measured during a very short time interval, such as 1 second or less. During such a short time interval, the car has barely any possibility of speeding up or slowing down. Hence the average speed in this short time interval is practically equal to the actual speed. Since a very short time interval is called an instant, the name instantaneous speed is generally used (Eq. 2.2).

Equation 2.2

instantaneous speed = v

$$v = \frac{\Delta s}{\Delta t}$$

(Δt *is an instant, an interval of time chosen short enough so that the speed does not change to a significant extent.)*

How short is an "instant"? The instant is defined to be so short that the speed of the moving object does not change appreciably. Just how short it must be depends on the motion that is being studied. For a car that accelerates from a standing start to 60 miles per hour in 10 seconds, the instant must be considerably shorter than 1 second. For a bullet being fired, an instant must be very much shorter yet, for the entire time interval during which the bullet accelerates inside the gun barrel is much,

much shorter than 1 second. At the other extreme, consider the ice in a glacier slowly gliding down a mountain valley. For this motion, even a day may be a brief instant because years elapse before the speed changes.

Applications. Since the average speed is defined by means of a mathematical formula ($\Delta s/\Delta t$), you can use mathematical reasoning (Section A.2) to solve a variety of problems. For instance, you can compute the distance traversed by a moving object if you know its average speed and the travel time (Section 1.3, Eq. 1.2, and Fig. 1.6). Or you can compute the time required for a trip if you know the average speed and the distance to be covered. These ideas are illustrated in Example 2.3.

EXAMPLE 2.3

(a) Find the distance if the average speed and time interval are given. How far does a pedestrian walk in 1.6 hours?

Solution: We wish to use Equation 2.1 to find Δs; thus we multiply both sides of Equation 2.1 by Δt to get: $\Delta s = v_{av}\,\Delta t$

For a pedestrian we can estimate $v_{av} \approx 3$ mph and $\Delta t = 1.6$ hours.
Thus $\Delta s = v_{av}\,\Delta t = 3$ mph x 1.6 hours ≈ 5 miles.

(b) Find the time interval required if the average speed and distance are given. If a bullet's average speed is 700 m/sec, how long does a bullet take to travel 2000 meters?

Solution: We wish to find Δt; thus we multiply both sides of Equation 2.1 by Δt and divide by v_{av} to get: $\Delta t = \dfrac{\Delta s}{v_{av}}$

For the bullet,

$$v_{av} = 700\,m/sec = 7\times 10^2\ m/sec$$
$$\Delta s = 2000\,m = 2\times 10^3\ m$$
$$\Delta t = \frac{\Delta s}{v_{av}} = \frac{2\times 10^3\ m}{7\times 10^2\ m/sec} \approx 0.28\times 10 = 2.8\,sec.$$

2.3 Displacement

The concept connecting relative position with relative motion is the change of relative position, which enables you to apply coordinate techniques to motion. For example, when a particle moves from one point R to another point Q (Fig. 2.16), then its position relative to any reference point fixed on the page changes. The change in position of a moving object is called the *displacement* because you can think of the moving object being displaced from one point to the other, from R to Q. By marking the successive displacements of a moving object, you can trace its path in space (Fig. 2.17).

The symbol for displacement is the boldface **Δs** with the Greek **Δ**

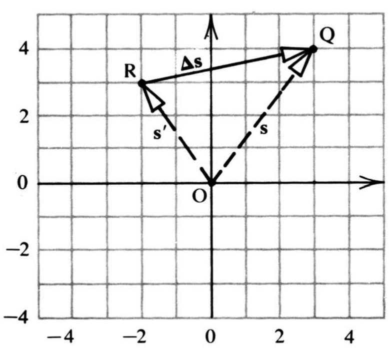


Figure 2.16 *The dashed arrows represent the positions of points Q and R relative to the coordinate axes. The solid arrow represents the displacement* **Δs** from *R to Q.*

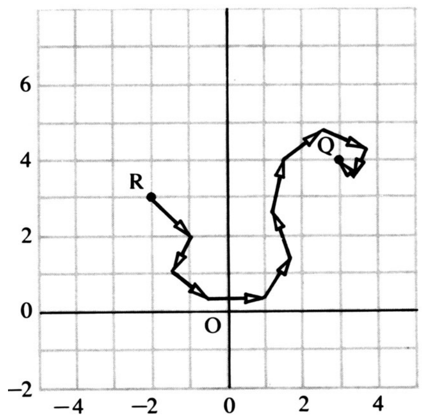


Figure 2.17 Arrows represent the successive small displacements of a particle whose overall (or net) displacement is from R to Q.

Equation 2.3

Relative position of Q

$$s = [3,4]\begin{cases} s_x = 3 \\ s_y = 4 \end{cases}$$

Relative position of R

$$s' = [-2,3]\begin{cases} s_x' = -2 \\ s_y' = 3 \end{cases}$$

Equation 2.4

Change of relative position from R *to* Q *(displacement* **Δs**)

$$\begin{aligned} \Delta s_x &= s_x - s_x' \\ &= 3-(-2) = 5 \\ \Delta s_y &= s_y - s_y' \\ &= 4-3 = 1 \\ \mathbf{\Delta s} &= \mathbf{s - s'} \\ &= [s_x - s_x', s_y - s_y'] \\ &= [5, 1] \end{aligned}$$

Note that the x- and y- components of **Δs** *are independent of one another and are calculated separately.*

(delta) standing for "difference" and **s** standing for relative position. The boldface symbol **Δs** is a single entity and does not signify multiplication of two factors. It should be distinguished from the symbol Δs, the distance traveled, which was used in the definition of speed (Eq. 2.1). The displacement (**Δs)** is a more complex quantity than simply the distance traveled (Δs). The displacement **Δs** includes the distance traveled (Δs) **and** the direction of that movement in space.

With the help of the coordinate grid in Fig. 2.16, you can find the coordinates of Q and R relative to the origin O (Eq. 2.3). The coordinates of the change in relative position, which are called the components of the displacement from R to Q, are written Δs_x and Δs_y. They are equal to the differences of the coordinates of Q and of R relative to O (Eq. 2.4).

In a diagram, a position or displacement will be indicated by an arrow (with an open arrowhead, Figs. 2.16 and 2.17) from the reference or starting point at the tail to the actual or final point at the head. The length of the arrow represents the magnitude of the displacement, and the direction of the arrow represents the direction of the displacement. If several arrows have to be drawn, then their tails, magnitudes, and directions must be properly related. As you will discover in later chapters, the arrow description of a magnitude and a direction in space will be used for force, velocity, and other physical quantities, as well as for displacements

Placement of arrows. Consider, for instance, the surveyor (Fig. 2.6), whose measurements were represented by relative position arrows in Fig. 2.7. In this diagram the tails of all the arrows would be placed at the same point, which represents the surveyor's benchmark. If, however, you want to track a sailboat moving on a zigzag course

(Fig. 2-18), then you must represent the displacement on each straight part by an arrow whose tail is placed at the head of the arrow representing the preceding displacement. Thus, you obtain a map of the boat's path, and you can find the overall displacement from the starting point to the finish point by interpreting the map.

Addition of displacements. The process of combining the displacements to find the overall displacement by placing the tail of one arrow at the head of the previous one is called *addition of displacements*, and the overall displacement is called the *sum.* This is analogous to the

Figure 2.18 The sailboat proceeds from the starting point to the finishing point along a six-part zigzag course. The individual displacements $\mathbf{\Delta s_1}$, $\mathbf{\Delta s_2}$, $\mathbf{\Delta s_3}$, $\mathbf{\Delta s_4}$, $\mathbf{\Delta s_5}$, *and* $\mathbf{\Delta s_6}$ *combine (add together) to give the sum, or overall displacement* $\mathbf{\Delta s}$. *(Scale: length of side of small square in graph below = 1/2 mile.)*

First, we find the various displacements by counting squares on the figure below:

$\mathbf{\Delta s_1} = [+2.0,\ +3.0]$ $\mathbf{\Delta s_3} = [+5.0,\ +3.0]$ $\mathbf{\Delta s_5} = [+2.5,\ -1.5]$

$\mathbf{\Delta s_2} = [+1.0,\ -3.0]$ $\mathbf{\Delta s_4} = [-0.5,\ -2.0]$ $\mathbf{\Delta s_6} = [-0.5,\ +5.5]$

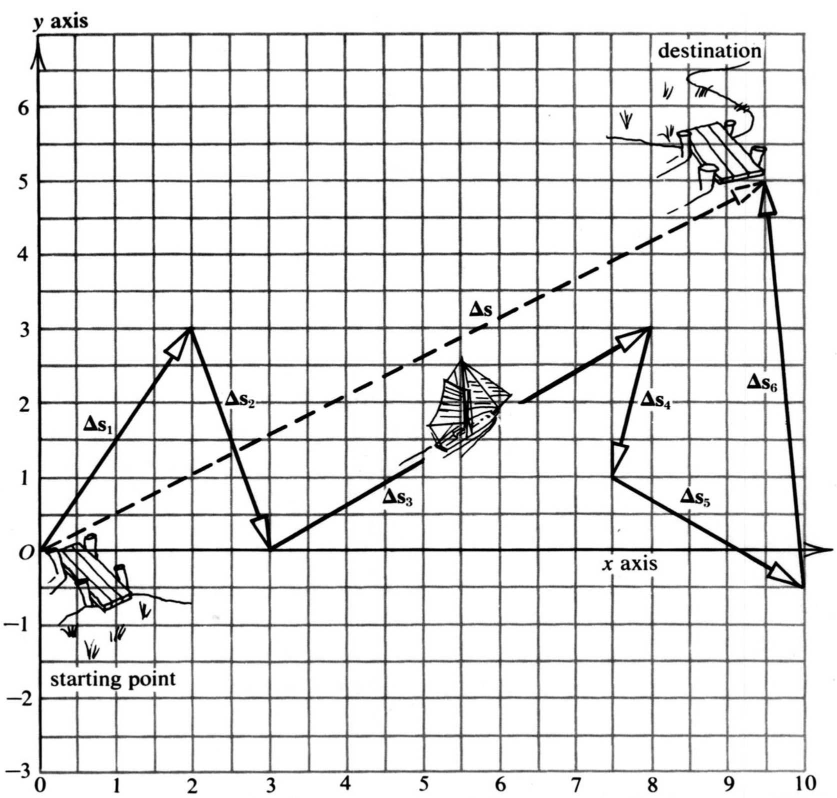
$\mathbf{\Delta s} = [\Delta s_x ,\ \Delta s_y] = [+9.5,\ +5.0]$ (counted directly from figure below)

We can check the above result by adding the individual displacements to find the sum: $\mathbf{\Delta s_1} + \mathbf{\Delta s_2} + \mathbf{\Delta s_3} + \mathbf{\Delta s_4} + \mathbf{\Delta s_5} + \mathbf{\Delta s_6} = \mathbf{\Delta s}$

Now, to actually find the x- and y-components of $\mathbf{\Delta s}$***, we must add the individual x- and y-components separately, so they do not get mixed up with one another!***

x components : $\Delta s_x = 2.0 + 1.0 + 5.0 - 0.5 + 2.5 - 0.5 = 9.5$ *(agrees with above)*

y components: $\Delta s_y = 3.0 - 3.0 + 3.0 - 2.0 - 1.5 + 5.5 = 5.0$ *(agrees with above)*



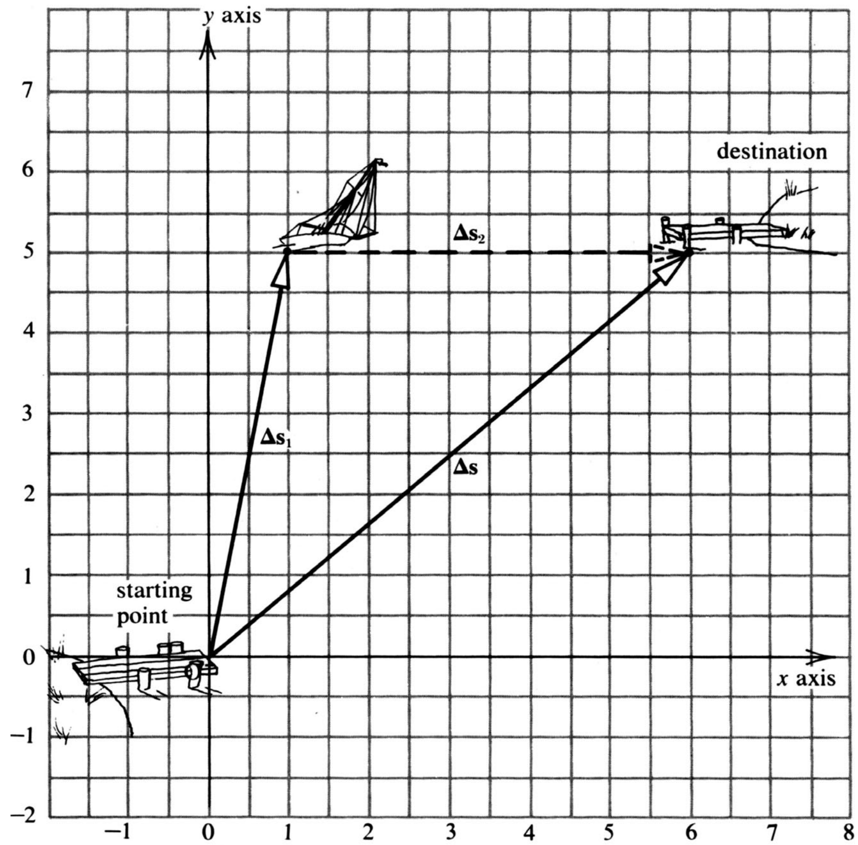


Figure 2.19 The sailboat has accomplished displacement $\mathbf{\Delta s_1}$ *and still needs to make displacement* $\mathbf{\Delta s_2}$ *to reach its destination. The displacement* $\mathbf{\Delta s_2}$ *is the difference between* $\mathbf{\Delta s}$ *and* $\mathbf{\Delta s_1}$*. (Scale: 1 square = 1/2 mile.)*
$\mathbf{\Delta s_1}$ *= [1, 5],* $\mathbf{\Delta s_2}$ *= [5, 0],* $\mathbf{\Delta s}$ *= [6, 5].*
$\mathbf{\Delta s_2} = \mathbf{\Delta s} - \mathbf{\Delta s_1}$ = [*6, 5*] - [*1, 5*] = [6 - 1, 5 - 5] = [5, 0]. (**Note that the x- and y-components are subtracted separately so they do not get mixed together!**)
As you can see in the figure, $\mathbf{\Delta s_2}$ indeed has an x-component of 5 and a y-component of 0.

addition of numbers, where $100 combined with $60 gives the sum of $160. The graphical process of adding displacements is illustrated in Fig. 2.18. You can also use normal arithmetic to do this if you know the rectangular components of each displacement. The rectangular components may be read off Fig. 2.18 and are listed in the legend to that figure. It is clear that the *x* component of the overall displacement is the sum of the *x* components of the individual components, and the same is true of the y components. It is essential to keep the x- and y-components separate.

Finding the sum (or difference) of displacements*: the x- and y-components are independent and <u>must be kept track of separately</u>; thus x-components are only added to (or subtracted from) other x-components, and y-components are only added to (or subtracted from) other y-components.*

Subtraction of displacements. The course of the sailboat in Fig. 2.18 gave a natural illustration of the sum of displacements. To find illustrations of the difference of displacements, consider first two ways of interpreting the difference of two numbers: what is left over after part is removed and what is needed to obtain a larger quantity. The first way applies when you have \$100 and spend \$60; you are left with the difference, which is \$40. The second way applies when you want \$100 and have \$60; you still need the difference, which is \$40. The second interpretation can be applied to displacements. If you are in the sailboat, are aiming for a destination at a displacement $\mathbf{\Delta s}$ from the starting point, but have only made the progress described by the displacement $\mathbf{\Delta s_1}$, the displacement $\mathbf{\Delta s_2}$ must still be traversed (Fig. 2.19). The displacement $\mathbf{\Delta s_2}$ is the difference between the goal $\mathbf{\Delta s}$ and the partial achievement $\mathbf{\Delta s_1}$, that is, $\mathbf{\Delta s_2} = \mathbf{\Delta s} - \mathbf{\Delta s_1}$. The difference may be found either graphically or arithmetically from the rectangular displacement components by subtraction as shown in Fig. 2.19.

Equation 2.5

$$\frac{\mathbf{\Delta s}}{b} = \left[\frac{\Delta s_x}{b}, \frac{\Delta s_y}{b}\right]$$

EXAMPLE
$\mathbf{\Delta s}$ *= [14, 5] and b = 4*

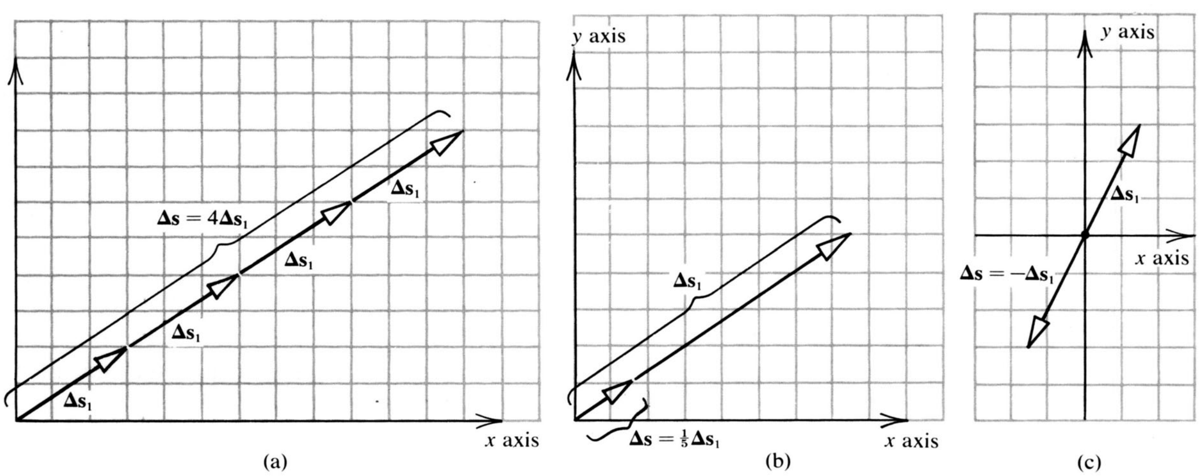
Then

$$\frac{\mathbf{\Delta s}}{b} = \frac{[14,5]}{4} = [3.5, 1.25]$$

Multiplication and division. Certain other arithmetic operations can be carried out with displacements by performing these operations on all the rectangular components of the displacements, just as you have calculated sums and differences by applying the appropriate arithmetic operation to the rectangular components. By adding a displacement to itself repeatedly (Fig. 2.20a), you obtain a multiple of the displacement. You can also divide a displacement into equal parts, such that each part is a fraction of the original displacement (Fig. 2.20b). Finally, you can find the negative of a displacement, which is just a displacement of equal magnitude and in the opposite direction from the original displacement (Fig. 2.20c).

An important algebraic concept is the division of a displacement by a number (Eq. 2.5), which will be used in the definition of velocity and

Figure 2.20 Arithmetic operations with displacements.
(a) Multiple displacements.
(b) Fractional displacement.
(c) Negative displacement.



acceleration in Chapter 13. Specifically, a displacement divided by a number is just another, smaller displacement in the same direction; to calculate this, simply divide each rectangular component by the number, as illustrated in Eq. 2.5.

Summary

Position and motion of objects can only be observed and described relative to reference frames. Position or motion of an object may differ when described relative to different reference frames. A reference frame may be centered on you or on any other point in space that is stationary or in motion relative to you. Quantitative ways of describing relative position make use of polar coordinates (s_r , s_θ) and rectangular coordinates [s_x, s_y]. A quantitative description of relative motion makes use of the average speed, which is defined as the distance traveled divided by the time interval required (Eq. 2.1).

The change of an object's relative position is called the displacement. The displacement has a magnitude and a direction in space. It is represented in diagrams by an arrow and is described quantitatively by polar coordinates or rectangular components, [Δs_x , Δs_y]. The processes of arithmetic (addition, subtraction, multiplication and division) can be carried out with displacements in rectangular coordinates by calculating the x- and y-components separately.

List of new terms

reference object	coordinate frame	displacement
reference frame	polar coordinates	component
reference point	rectangular coordinates	average speed
reference direction	coordinate axis	instantaneous speed
particle	origin of coordinates	

List of Symbols

s (bold)	relative position in space		
[s_x , s_y]	rectangular position coordinates	Δs	distance traversed
		Δt	time interval
[s_r , s_θ]	polar position coordinates	v	speed
Δs (bold)	displacement		
[Δs_x, Δs_y]	displacement components (or displacement rectangular coordinates)		

Problems

1. Lie on your bed on your back. Use your head as reference point and the directions front, back, right, left, above, below (or combinations of these) to describe the position of objects relative to this reference point. Give the approximate direction and distance of several of the following: pillow, lamp, radio, door, your feet, and so on.

2. Describe the position of your bedroom by using the building as reference frame. (Do *not* give detailed instructions as to how a person could walk to your bedroom.)

3. Describe the position of the children in Fig. 2.1 by using the picture edge as reference frame.

4. Find an alternate way of locating the hawk for Clyde (Fig. 2.2).

5. (a) Estimate the polar coordinates of the house and trees D and E in Fig. 2.6.
(b) Mark the location of the house and trees D and E on the map in Fig. 2.7.

6. Find the distance from R to Q in Fig. 2.10.

7. Find an arithmetic relationship among the coordinates of Q relative to O, R relative to O, and Q relative to R in Fig. 2.10.

8. A man on the earth travels 10 miles south, then 10 miles east, then 10 miles north. After the 30-mile trip is finished, he is back at his starting point. Identify his starting point.

9. Give two examples from everyday experience where you intuitively identify motion relative to a reference frame that is moving relative to the earth.

10. Explain how a motion-picture strip can be used to determine the speed of a moving object. Refer to average and instantaneous speeds. Apply your method to Fig. 2.12. You should estimate the approximate distances in the figure.

11. Explain how a multi-flash photograph like Fig. 2.13 can be used to determine the speed of the moving object at various points along the path. Apply your method to an object in Fig. 2.13. You should estimate the approximate distances in the figure.

12. Measure the average speeds of two or three objects in everyday life. You may use any convenient units, but you should convert to standard units (meters per second). Explain for each example how you chose to define "average."

Problems 13-16 ask you to compare or describe motion. You should describe the path(s) of the motion in words, name the reference frame(s) and make a drawing(s). You should also describe the speed of the motion.

13. A record is being played on a phonograph. Compare the motion of the needle relative to the phonograph base with the needle's motion relative to the record on the turntable.

14. Describe the motion of the moon relative to a reference frame fixed on the sun. Use a one-particle model for the moon.

15. Describe the motion of the earth relative to a reference frame fixed on the surface of the moon. Use a one-particle model for the earth.

16. Describe the motion of the sun relative to a reference frame fixed on the moon. Use a one-particle model for the sun.

17. A girl is pedaling a bicycle in a straight line.
(a) Describe the motion of the tire valve relative to the axle of the wheel.
(b) Describe the motion of the tire valve relative to the road.
(c) Describe the motion of one pedal relative to the road.
(d) Describe the motion of one pedal relative to the other pedal.

18. Using the two different methods outlined below, find the displacement from tree A to the windmill in Fig. 2.7.
(a) Use ruler and protractor to solve the problem geometrically. State the result in polar coordinates. You answer should have both a distance (s_r) and an angle (s_θ), expressed as $[s_r , s_\theta]$.
(b) Impose a rectangular coordinate frame (graph paper) on Fig. 2.7 and solve the problem arithmetically. You answer should have both an x-component (s_x) and a y-component (s_y), expressed as $[s_x , s_y]$.

19. These questions deal with Fig. 2.18.
(a) Find the combined displacement of the second and third legs of the sailboat's course.
(b) Find the combined displacement of the fourth, fifth, and sixth legs of the sailboat's course.
(c) Find the combined displacement of the first, third, and fifth legs of the course.
(d) Find the displacement still required for the sailboat to reach its destination after the first leg of the course.
(e) Find the displacement still required for the sailboat to reach its destination after the fourth leg of the course.

20. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your dissatisfaction (conclusions contradict your ideas, or steps in the reasoning have been omitted; words or phrases are meaningless; equations are hard to follow; etc.) and pinpoint your criticism as well as you can.

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