Chapter 6: The Wave Theory

PART 1

Christian Huygens (1629-1695) was born at The Hague in Holland. His father Constantine, a man of wealth, position, and learning, quickly recognized the boy's unusual capabilities. Christian's father taught him both mathematics and mechanics, and long before his thirtieth birthday, Huygens had published important papers on mathematics, built and improved telescopes, discovered a satellite of Saturn, and invented the pendulum clock. In 1665, King Louis XIV of France invited Huygens to join the brilliant galaxy of intellects that the "Sun King" had clustered about him at Versailles. After 15 years in Paris, Huygens returned to spend his last years in Holland. These last years, however, proved to be as remarkable as his early years. In 1690, Huygens published the Treatise on Light*, his historic statement of the wave theory of light.*

Waves on a water surface are such a familiar and expected occurrence that a completely still, glassy pool excites surprise and admiration (Fig. 6.1). You can also observe waves on flags being blown by a strong wind. In this chapter you will be concerned with how waves propagate, what properties are used to describe them, and how waves combine with one another when several pass through the same point in space at the same time. In the wave theory, which was formulated by Christian Huygens during the seventeenth century, the space and time distribution of waves is derived from two assumptions, the superposition principle and Huygens' Principle. The wave theory is very "economical" in the sense that far-reaching consequences follow from only these two assumptions.

Waves are important in physics because they have been used in the construction of very successful working models for radiation of all kinds. You can easily imagine that dropping a pebble into a pond and watching the ripples spread out to the bank suggests interaction-at-a-distance between the pebble and the bank. The waves are the intermediary in this interaction, just as radiation was the intermediary in some of the experiments described in Sections 3.4 and 3.5. In Chapter 7, we will describe wave models for sound and light and how these models can explain the phenomena surveyed in Chapter 5. The success of these models confirms Huygens' insight into the value of wave theory. However, Huygen's contributions and wave theory were not fully appreciated and exploited until the nineteenth century.

Waves were originally introduced as oscillatory disturbances of a material (called the *medium*) from its equilibrium state. Water waves and waves on a stretched string, the end of which is moved rapidly up and down, are examples of such disturbances. The waves are emitted by a source (the pebble thrown into the pond), they propagate through the medium, and they are absorbed by a receiver (the bank). Even though waves are visualized as disturbances in a medium, their use in certain theories nowadays has done away with the material medium. The waves in these applications are fluctuations of electric, magnetic, or

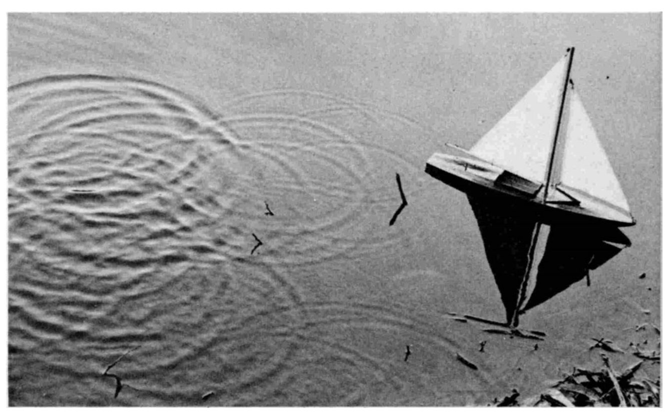


Figure 6.1 The reflected image gives information about the smoothness of the water surface. Why are the reflections of the sails dark and not white?

gravitational fields, rather than oscillations of a medium. The use of such waves to represent radiation has unified the radiation model and the field model for interaction-at-a-distance (Section 3.5). Our discussion here, however, will be of waves in a medium and not of waves in a field.

6.1 The description of wave trains and pulses

Oscillator model. We will analyze the motion of the medium through which a wave travels by making a working model in which the medium is composed of many interacting systems in a row. Each system is capable of moving back and forth like an oscillator, such as the inertial balance shown below and described in Section 3.4. You may think of the oscillators in a solid material as being the particles in an MIP model for the material.

Amplitude and frequency. Each oscillator making up the medium has an equilibrium position, which it occupies in the absence of a wave. When an oscillator is set into motion, it swings back and forth about the equilibrium position. The motion is described by an *amplitude* and a *frequency* (Fig. 6.2). The amplitude is the maximum distance of the oscillator from its equilibrium position. The frequency is the number of complete oscillations carried out by the oscillator in 1 second.

Interaction among oscillators. When waves propagate through the medium, oscillators are displaced from the equilibrium positions and are set in motion. The wave propagates because the oscillators interact with one another, so that the displacement of one influences the motion of the neighboring ones, and so on. Each oscillator moves with a frequency and an amplitude. It is therefore customary in this model to identify the frequency and amplitude of the oscillators with the frequency and amplitude of the wave. In addition, as you will see, there are properties of the wave that are not possessed by a single oscillator but that are associated with the whole pattern of displacements of the oscillators.

Conditions for wave motion. The oscillator model described above has two general properties that enable waves to propagate. One is that the individual oscillator systems interact with one another, so that a displacement of one influences the motion of its neighbors. The second is that each individual oscillator has inertia. That is, once it has been set in motion it continues to move until interaction with a neighbor slows it down and reverses its motion. These two conditions, interaction and inertia, are necessary for wave motion.

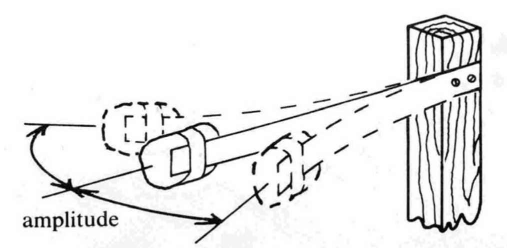


Figure 6·2 An oscillator in motion.

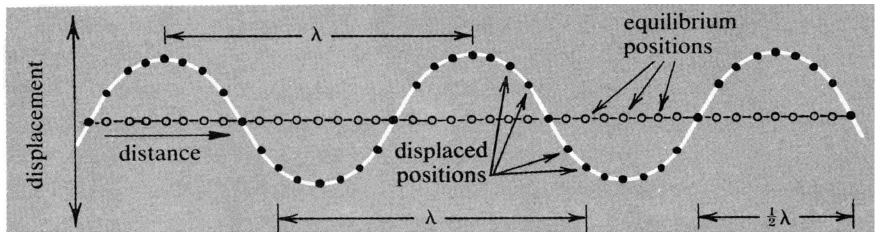


Figure 6.3 Row of oscillators in a medium, showing equilibrium positions and displaced positions in a wave. The wavelength is the distance after which the wave pattern repeats itself.

Equation 6.1

wavelength (meters) = λ

wave number (per meter) $= \mathrm{k}$

$$\lambda \times k = 1,\ k = \frac{1}{\lambda},\ \lambda = \frac{1}{k}$$

EXAMPLES

$\lambda = 0.25$ m

$$k = \frac{1}{\lambda} = \frac{1}{0.25\,m} = 4/m$$

This is 4 wavelengths/m.

$\lambda = 5.0$ m

$$k = \frac{1}{\lambda} = \frac{1}{5.0\,m} = 0.2/m$$

This is 0.2 wavelengths/m.

$\lambda = 0.0001$ m $= 10^{-4}$ m

$$k = \frac{1}{\lambda} = \frac{1}{10^{-4}\,m} = 10^{4}/m$$

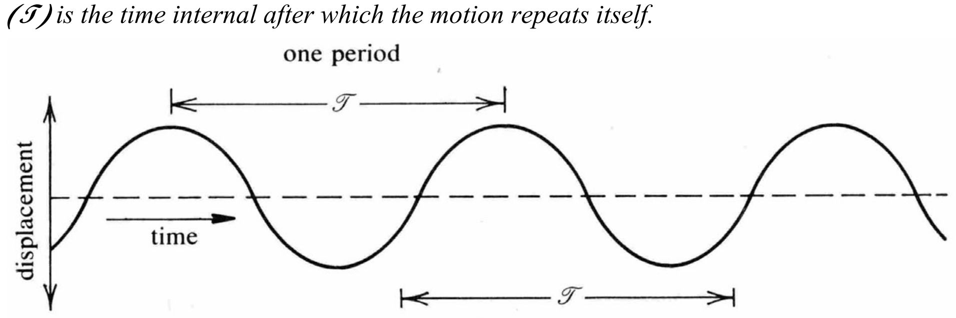
This is 10^4 or $10{,}000$ wavelengths/m.

Wave trains. Look more closely now at the pattern of the oscillators in the medium shown in Fig. 6.3. As a wave travels through the medium, the various oscillators have different displacements at any one instant of time. The wave is represented graphically by drawing a curved line through the displaced positions of all the oscillators (shown above in Fig. 6.3). This curved line, of course, changes as time goes on because the oscillators move. Note, however, that the individual oscillators in the model move only up and down.

Wavelength and wave number. You can see from Fig. 6.3 that the wave repeats itself in the medium. This pattern of oscillators is called a wave train, because it consists of a long train of waves in succession. A complete repetition of the pattern occupies a certain distance, after which the pattern repeats. This distance is called the *wavelength*; it is measured in units of length and is denoted by the Greek letter lambda, **λ**. Sometimes it is more convenient to refer to the number of waves in one unit of length; this quantity is called the *wave number* and it is denoted by the letter **k**. Wavelength and wave number are reciprocals of one another (Eq. 6.1).

Period and frequency. We have just described the appearance of the medium at a particular instant of time. What happens to one oscillator as time passes? It moves back and forth through the equilibrium position as described by a graph of displacement vs. time (Fig. 6.4) that is very similar to Fig. 6.3. The motion is repeated; each complete cycle requires a time interval called the *period* of the motion, denoted by a script "tee," $\mathcal{T}$. The number of repetitions per second is the

Figure 6.4 Graph of the motion (displacement) of one oscillator over time. The period ($\mathcal{T}$) is the time internal after which the motion repeats itself.



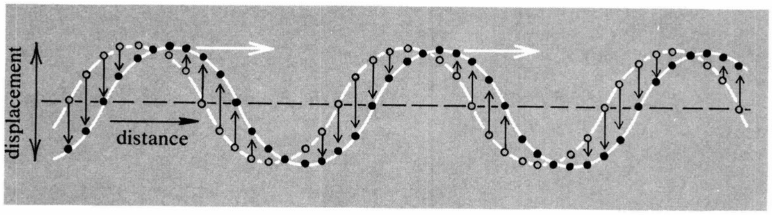
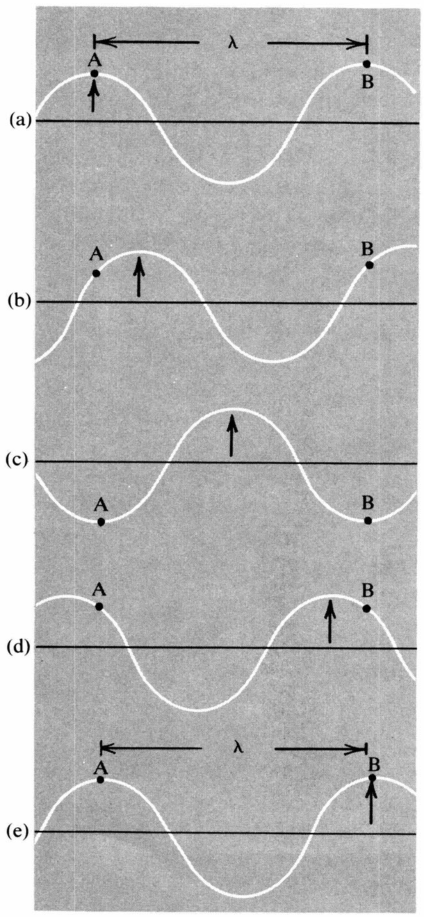


Figure 6·5 The wave moves to the right as the oscillators move up and down. The black circles and black dots represent the displacements of the oscillators at two different times.

Figure 6·6 In one period, oscillators A and B carry out a full cycle of motion from crest to trough and to crest again. The crest initially at A moves to B in this time interval.



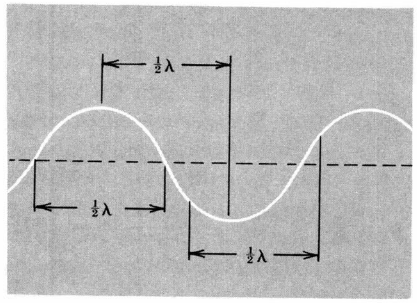
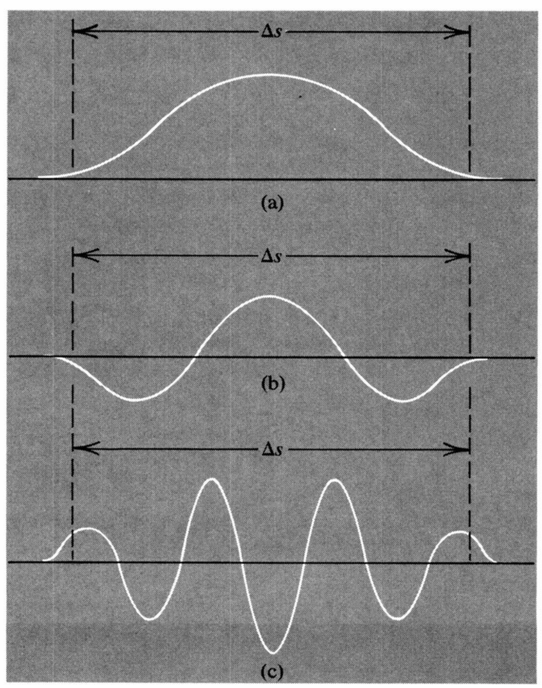


Figure 6·7 Oscillators in a wave train have opposite displacements if their separation is ½ wavelength.

Figure 6·8 Pulse patterns of disturbance in a medium. The approximate length of the pulse is denoted by Δs.



Equation 6.2 (period and frequency of a wave)

period (time for one complete repetition, in seconds) $= \mathcal{T}$
frequency (number of complete repetitions in one second, per second) $= f$

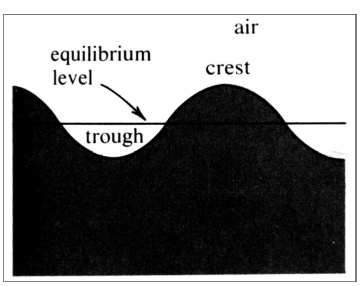
$$\mathcal{T} \times f = 1,\ f = \frac{1}{\mathcal{T}},\ \mathcal{T} = \frac{1}{f}$$

EXAMPLES
If $\mathcal{T} = 0.05$ sec,
$$f = \frac{1}{\mathcal{T}} = \frac{1}{0.05} = 20/\text{sec}.$$
If $\mathcal{T} = 3.0$ sec,
$$f = \frac{1}{\mathcal{T}} = \frac{1}{3.0} = 0.33/\text{sec}.$$
If $\mathcal{T} = 10^{-6}$ sec,
$$f = \frac{1}{\mathcal{T}} = \frac{1}{10^{-6}} = 10^{6}/\text{sec}.$$

Equation 6.3 (wave speed)

wave speed $= v$

$$v = \frac{\Delta s}{\Delta t} = \frac{\lambda}{\mathcal{T}} \qquad (a)$$
$$v = \lambda f \qquad (b)$$



frequency (symbol f). The period and frequency are reciprocals of one another (Eq. 6.2), just as are the wavelength and wave number. The period and frequency describe the time variation of the oscillator displacements, while the wavelength and wave number describe the spatial variation.

Wave speed. One of the most striking properties of waves is that they give the appearance of motion along the medium. If you look at the pattern of displacements at two successive instants of time (Fig. 6.5), you see that the wave pattern appears to have moved to the right (along the medium), although the individual oscillators have only moved up and down. Since the pattern actually moves, you can measure its speed of propagation through the medium. The wave speed is usually represented by the symbol v (Section 2.2).

You can conduct a thought experiment with the oscillator model for the medium to find a relationship among period, wavelength, and wave speed. Imagine the oscillator at a wave crest carrying out a full cycle of its motion (Fig. 6.6). While this goes on, all the other oscillators also carry out a full cycle, and the wave pattern returns to its original shape. The wave crest that was identified with oscillator A in Fig. 6.6, however, is now identified with oscillator B. Hence the wave pattern has been displaced to the right by 1 wavelength. The wave speed is the ratio of the displacement divided by the time interval (Eq. 2.2), in this instance the ratio of the wavelength divided by the period ($\lambda/\mathcal{T}$, Eq. 6.3a). By using Eq. 6.2, $f = 1/\mathcal{T}$, you can obtain the most useful form of the relationship: $v = \lambda f$, or wave speed is equal to wavelength times frequency (Eq. 6.3b).

Positive and negative displacement. Waves are patterns of disturbances of oscillators from their equilibrium positions. The displacement is sometimes positive and sometimes negative. In Fig. 6.3, the open circles and the horizontal line drawn along the middle of the wave show the equilibrium state of the medium. Displacement upward may be considered positive, displacement downward negative. In water waves, for example, the crests are somewhat above the average or equilibrium level of the water and the troughs are somewhat below the average or equilibrium level of the water. In fact, the water that forms the crests has been displaced from the positions where troughs appear.

By definition, the pattern in a wave train repeats itself after a distance of 1 wavelength. It therefore also repeats after 2, 3, ... wavelengths. Consequently, the oscillator displacements at pairs of points separated by a whole number of wavelengths are equal. If you only look at a distance of 1/2 wavelength from an oscillator, however, you find an oscillator with a displacement equal in magnitude but opposite in direction (Fig. 6.7).

Wave pulses. In the *wave trains* we have been discussing, a long series of waves follow one another, and each one looks just like the preceding one. On the other hand, a ***wave pulse*** is also a disturbance in the medium but it is restricted to only a part of the medium at any one time (Fig. 6.8). It is not possible to define frequency or wavelength for a pulse since it does not repeat itself. The concept of wave speed, however, is applicable to pulses since the pulse takes a certain amount of time to travel from one place to another. In Section 6.2 we will describe how wave trains and wave pulses can be related to one another.

Examples of wave phenomena. The oscillator model for a medium can be applied to systems in which small deviations from a uniform equilibrium arrangement can occur. One such system is a normally motionless water surface that has been disturbed so that water waves have been produced. Another example is air at atmospheric pressure in which deviations from equilibrium occur in the form of pressure variations: alternating higher or lower pressure. Such pressure variations are called sound waves. A third example is an elastic solid such as Jell-O, which can jiggle all over when tapped with a fork. In the oscillator model, movement results from oscillating displacements within the Jell-O after the fork displaced the oscillators at the surface.

Oscillator model for sound waves. Since sound in air is of special interest, we will describe an oscillator model for air in more detail. Visualize air as being made up of little cubes of gas (perhaps each one in an imaginary plastic bag). When acted upon by a sound source, the first cube is squeezed a little and the air inside attains a higher pressure (Fig. 6.9). The first cube then interacts with the next cube by pushing against it. After a while the second cube becomes compressed and the first one has expanded back to and beyond its original volume. The second cube then pushes on the third, and so on. In this way the sound propagates through the air.

The initial pressure increase above the equilibrium pressure may be

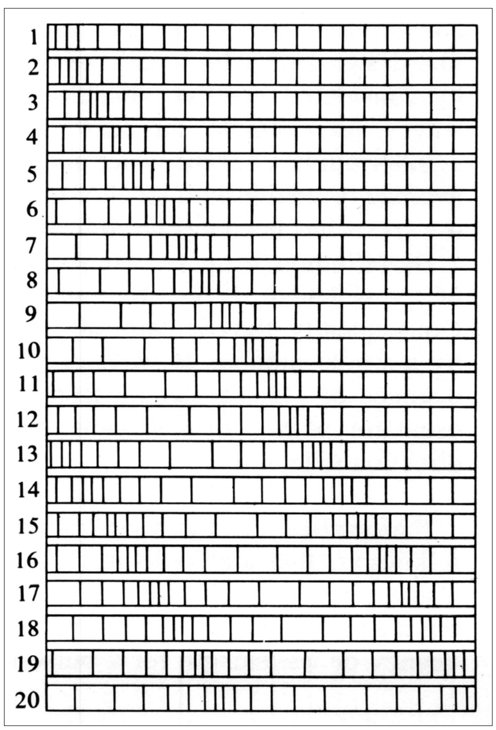


Figure 6.9 A gas bag model for air is used to represent the propagation of a sound wave. An individual bag of gas is alternately compressed and expanded. Its interaction with adjacent bags of gas leads to propagation of the compression and expansion waves.

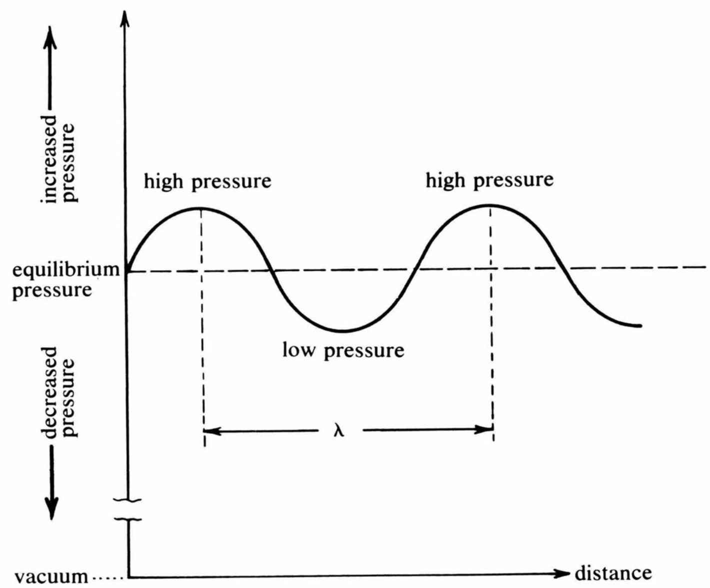


Figure 6.10 Pressure profile in a sound wave. The graph shows deviations from the equilibrium pressure.

created by a vibrating piano string or a vibrating drumhead. In addition to regions of increased pressure, the sound wave also has regions of deficient pressure where the air has expanded relative to its equilibrium state.

Thus the sound wave consists of alternating high-pressure (above equilibrium) and low-pressure (below equilibrium) regions. A pressure profile (pressure versus distance) for a pure tone has the typical wave pattern shown in Fig. 6.10.

6.2 Superposition and interference of waves

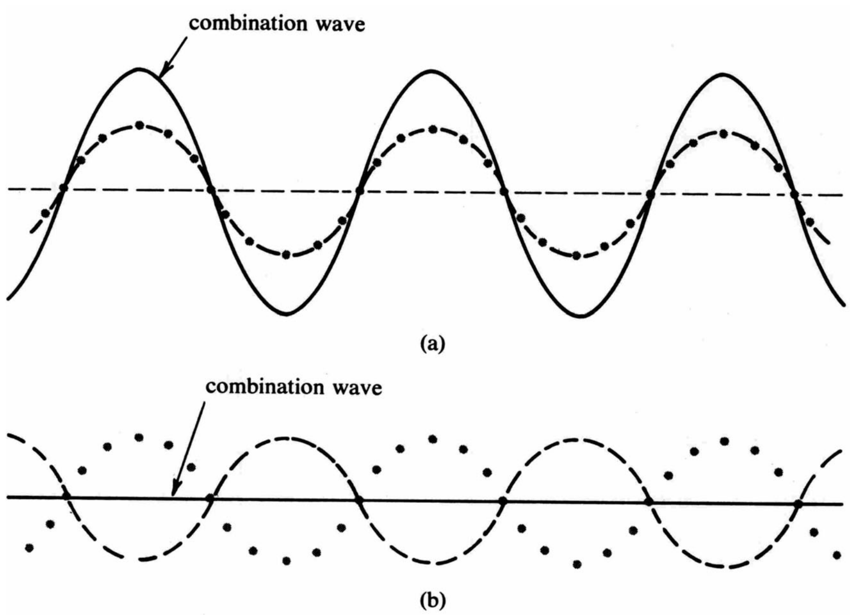
The superposition principle. Can you visualize what happens when two waves overlap? In the oscillator model, it is easy to describe the medium at a place where there are two or more waves at the same time. Each oscillator is displaced from its equilibrium position by an amount equal to the sum of the displacements associated with the waves separately (Fig. 6.11). In other words, you visualize the oscillator displacements associated with each of the wave patterns and add them together. This procedure takes for granted that the waves do not interact with one another, but that each propagates as though the others were not present.

The property of non-interaction we have just described is called the *superposition principle*. It makes the combination of waves simple to carry out in thought experiments, and it has been exceedingly valuable for this reason. Fortunately, a wave model that incorporates the superposition principle describes quite accurately many wave phenomena in nature.

Figure 6.11 Superposition of two waves leads to interference. One wave is represented by black dashes, the other by dots. The combination wave is the sum of both waves and is represented by the solid line.

(a) Constructive interference occurs when dotted and dashed waves reinforce each other.

(b) Destructive interference occurs when dotted and dashed waves cancel each other.



Interference of waves. Consider now what may happen to the oscillator motion as a result of the superposition of two waves. The two waves may combine in various ways. Perhaps each of two wave patterns has an upward displacement of an oscillator at a certain time and at a certain place. In such a case, the upward displacement in the presence of the combined wave will be twice as big as that from one wave alone (Fig. 6.11(a)). If there are simultaneous downward displacements in the two waves separately, the combined displacement will be twice as far down. Suppose you consider a point in space where one wave has an upward displacement and the other wave has an equal downward displacement at the same time. Now, the upward (positive) displacement and the downward (negative) displacement add to give zero combined displacement (zero amplitude of oscillation). In fact, it is possible for

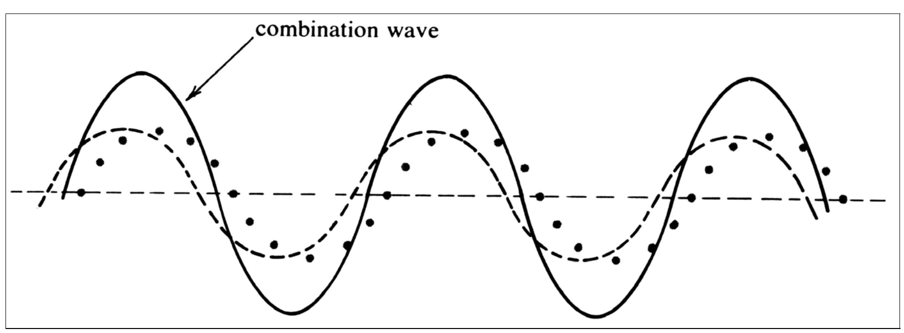


Figure 6.12 Superposition of two waves leading to partially destructive interference. The displacements of the dashed and dotted waves are added together at each point to yield the displacement of the combination wave (represented by the solid line). Note that displacement below the line is negative.

two waves to combine in such a way that they completely cancel one another, as in Fig. 6.11b.

This characteristic of waves makes their behavior different from what we expect of material objects, particularly when we think of them as single particles (Section 2.1) or as made up of particles. If one particle and another particle are combined, you have two particles, and you cannot end up with zero particles. Two or more waves, however, may combine to form a wave with larger amplitude, a wave with zero amplitude, or a wave with an intermediate amplitude (Fig. 6.12).

This result of the superposition of waves is a phenomenon called *interference*. If waves combine to give a larger wave than either one alone, you have *constructive interference*. If waves tend to cancel each other, you have *destructive interference*. There is a continuum of possibilities between the extremes of complete constructive interference shown in Fig. 6.11(a) and complete destructive interference shown in Fig. 6.11(b). With particles, the concept of destructive interference is meaningless in that the presence of one particle can never "cancel" the presence of another.

Standing waves. When two equal-amplitude wave trains of the same frequency and wavelength travel through a medium in opposite directions, their interference creates an oscillating pattern that does not move through the medium (Fig. 6.13). Such an oscillating pattern is called a *standing wave*. The points in a standing wave pattern where there are no oscillations at all are called *nodes*. At a node, there is always complete destructive interference of the two wave trains; the displacements associated with the two waves at the nodes are always equal and opposite. Because the waves move in opposite directions at the same speed, each node remains at one point in space and does not move; this is the reason behind the choice of name: a *standing* wave does not move.

You can see in Fig. 6.13 that the distance between two nodes must be exactly ½ wavelength. This holds true not only for the illustration but also for *all* standing wave patterns. The reasoning is as follows. At any node, the two wave displacements must always be equal and opposite to produce complete destructive interference. At a distance of ½ wavelength, the displacement associated with each wave has exactly reversed (as illustrated in Fig. 6.7). Thus, the two displacements must again be equal and opposite and again produce a node.

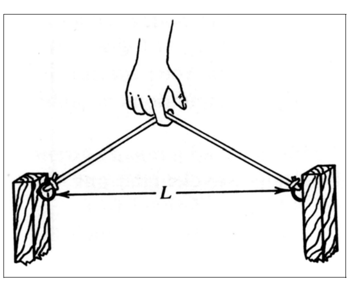
An easy way to set up standing waves is to place a reflecting barrier in the path of a wave. The reflected wave interferes with the incident wave to produce standing waves. The nodes are easy to find because the oscillators remain stationary at a node. This offers a convenient way to determine the wavelength: measure the distance between nodes and multiply by 2.

Tuned systems. It is very fruitful to pursue the standing wave idea one step further. Suppose an elastic rope is tied to a fixed support at each end and the middle is set into motion by being pulled to the side and released (see drawing to left). How will the rope oscillate? To solve this problem, think of the pattern as being made up of wave trains in

The "one-particle model" for a real object is a "very small object that is located at the center... of the region occupied by the real object." (Section 2.1) This is a way to think about an object so as to focus on the object's position, motion, and inertia without considering its shape and orientation. Complex objects can be thought of as two or more particles that interact in defined ways, or with the many-interacting-particles (MIP) model. In such models, each particle is thought of as a single, tiny bit of matter. The matter itself is thought of as indestructible, or "conserved." Such particles cannot "cancel" one another to cause destructive interference.

A wave, on the other hand, is quite different. A wave, as explained in this chapter, is thought of as a disturbance or oscillation that passes through matter. The displacement of the particles can be positive or negative and, as with (+1) + (–1) = 0, two waves can cancel one another.

In the 20th century, physicists found that matter in the micro-domain behaves in ways that conform with neither the particle nor the wave model. This led to the "wave-particle duality" and quantum mechanics. (Chapter 8).



Equation 6.4 (Possible number of half wavelengths that fit within L)

$$L=\frac{1}{2}\lambda$$

or

$$L=2\times\left(\frac{1}{2}\lambda\right)=\frac{2}{2}\lambda$$

or

$$L=3\times\left(\frac{1}{2}\lambda\right)=\frac{3}{2}\lambda$$

or

$$L=4\times\left(\frac{1}{2}\lambda\right)=\frac{4}{2}\lambda$$

or

$$L=5\times\left(\frac{1}{2}\lambda\right)=\frac{5}{2}\lambda$$

. . . and so on

Equation 6.5 (wavelengths permitted on a tuned system, from above)

$$\lambda=\frac{2}{1}L, \text{ or}$$

$$\lambda=\frac{2}{2}L = L, \text{ or}$$

$$\lambda=\frac{2}{3}L, \text{ or}$$

$$\lambda=\frac{2}{4}L = \frac{1}{2}L, \text{or}$$

$$\lambda=\frac{2}{5}L$$

and so on . . .

Equation 6.6 (finding frequency for a given speed and wavelength)

$$f=\frac{v}{\lambda}$$

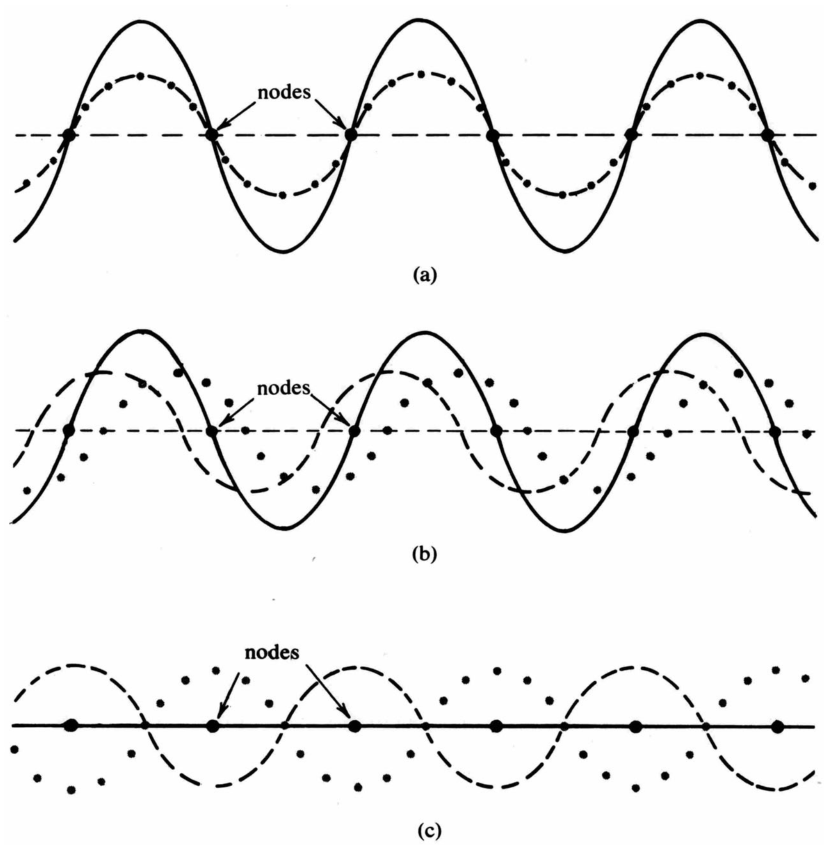


Figure 6.13 The formation of standing waves by the superposition of two wave trains propagating in opposite directions (dotted wave towards right and dashed wave toward left). The combination wave is the solid line. Note the stationary position of the nodes, marked by the large dots.
(a) Constructive interference of the two wave trains.
(b) Partially destructive interference after 1/8th of a period.
(c) Destructive interference after 2/8ths (1/4th) of a period.
Can you draw the pattern after 3/8ths of a period? After 4/8ths (1/2) of a period?

combinations, some moving to the right, others to the left. Because the ends are fixed, the wave pattern must be such that the ends of the rope are its nodes. The length of the rope is the distance between the nodes, which must be an integral multiple of ½ wavelength (Eq. 6.4). It follows that the wavelengths of the waves that can exist on this rope are related to the length of the rope by Eq. 6.5 to satisfy the conditions of nodes at the ends.

A system such as the rope with fixed ends is called a *tuned system*, because it can support only waves of certain wavelengths (Eq. 6.5) and the frequencies related to them by Eq. 6.6 (derived from Eq. 6.3b). The wave speed is a property of the medium from which the tuned system is constructed.

Musical instruments. Musical instruments employ one or more tuned systems whose frequencies are in a suitable relation to one another. For stringed instruments, such as the violin and guitar, the tuned system is a wire or elastic cord; for wind instruments, it is an air column in a pipe closed at one end; for drums, it is an elastic membrane whose edge is fixed; and so on.

The tone of the instrument is determined by the oscillation frequency of the tuned system. It is possible to change the frequency either through changing the length of the tuned system (and therefore changing the wavelength of the allowed standing waves) or through changing the wave velocity by modifying the medium in the tuned system.

Sound waves of a single frequency can be produced in closed pipes of a certain length. Longer pipes produce lower tones. A pressure wave starts at one end of the pipe and travels down the pipe, confined by the walls. When the wave reaches the other end of the pipe, it is reflected back and interferes with waves coming down the pipe. The interference forms a standing wave. This standing wave is of the characteristic wavelength determined by the length of the pipe and has the frequency that we hear.

Beats. Standing waves are created by the interference of waves with the same frequency. What will be the combined effect of two waves of differing frequencies? To answer this question, apply the superposition principle in a thought experiment in which two such waves are combined. Suppose the two waves are in constructive interference at one instant of time. Since one wave has shorter cycles than the other before repeating, they will soon get out of step. After a while, the two waves will be in destructive interference, and a little later in constructive interference again. So the net effect is an alternation from constructive interference (loud) to destructive interference (soft) and back again. These alternations in volume are called beats.

It is easily possible to calculate the time interval between two beats from the difference in frequency of the two interfering wave trains. During this time interval the two waves must go from constructive interference to destructive interference and back to constructive interference. Therefore, the higher-frequency wave must vibrate exactly once more than the lower frequency wave. The additional oscillation restores the constructive interference of the two waves, since waves repeat exactly after a whole oscillation. Hence the wave amplitude after the interval is equal to its value before, meaning that the next beat is ready to begin.

The number of oscillations made by either of the two waves is equal to its frequency (oscillations per second) times the time interval ($N_1 = f_1\Delta t$ and $N_2 = f_2\Delta t$, Eq. 6.7). The two numbers, according to the condition, must differ by one ($N_1 - N_2 = (f_1 - f_2)\Delta t$, Eq. 6.8). The conclusion is that the frequency difference times the time interval is equal to one ($\Delta f\ \Delta t = 1$, Eq. 6.9). The frequency of the individual waves determines the overall pitch of the sound, not the beat frequency; in fact, the beat frequency is $f_1 - f_2$.

Wave packets. Standing waves and beats are wave phenomena that are observable when two wave trains are combined. You may, of

Equation 6·7

frequencies of the two wave trains (per second) f_1, f_2
time interval (seconds) Δt
number of oscillations N_1, N_2

$$N_1 = f_1\Delta t, \qquad N_2 = f_2\Delta t$$

Equation 6·8

$$\begin{aligned} 1 &= N_1 - N_2 \\ &= f_1\Delta t - f_2\Delta t \\ &= (f_1 - f_2)\Delta t \end{aligned}$$

Equation 6·9

frequency difference Δf

$$\Delta f = f_1 - f_2 \quad \text{(a)}$$

$$\Delta f \Delta t = 1 \quad \text{(b)}$$

EXAMPLE

Frequencies of 255/sec and 257/sec

$$\Delta f = 2/sec$$

$$\Delta t = \frac{1}{\Delta f} = \frac{1}{2/sec} = 0.5\ sec$$

course, use the superposition principle and the rules for constructive and destructive interference to combine as many different wave patterns as you wish. In the early nineteenth century, it was discovered by Joseph Fourier (1768-1830) that any wave pattern could be formed by a superposition of one or more wave trains, as illustrated below. All wave phenomena can thereby be related to the frequencies, amplitudes, wavelengths, and velocities of the component wave trains in a wave pattern.

To illustrate Fourier's discovery, we will construct a wave pulse close to the one shown in Fig. 6.8a by combining the four wave trains drawn in Fig. 6.14. You are invited to read off the wave amplitudes from the graphs, to add the wave amplitudes of the four waves, and to verify that the combined wave drawn in Fig. 6.14 really is obtained by superposition of the four wave trains. By combining more and more wave trains of other wavelengths and successively smaller and smaller amplitudes, you can achieve further constructive and destructive interference at various locations in the pulse. In this way you could obtain a closer and closer approximation to the wave pulse shown in Fig. 6.8a and Fig. 6.14 (see Fig. 6.15).

The representation of wave pulses by a superposition of wave trains has led to the introduction of the suggestive phrase *wave packet* (instead of wave pulse), which we will also adopt. The superposition procedure can be quite tedious to work out in detail if many wave trains must be combined to achieve success. The essence of the procedure, however, is to select wave trains that interfere destructively in one wing of the wave packet, constructively at the center, and destructively again in the other wing. This can be achieved if one wave train has one more

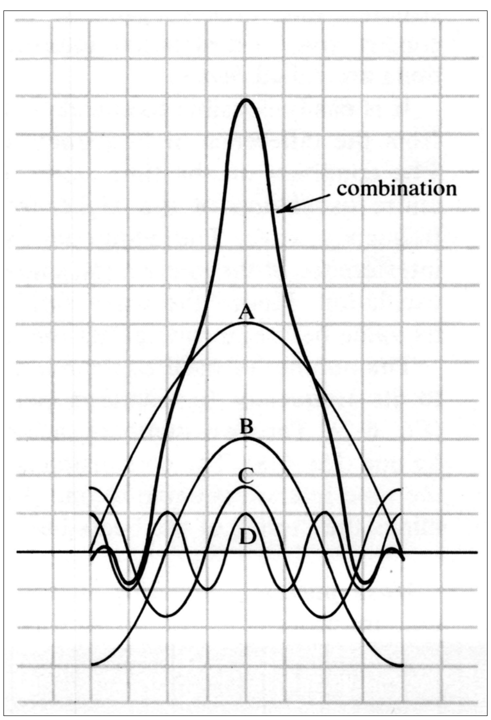
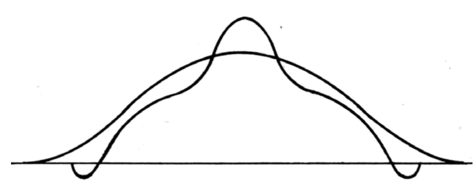
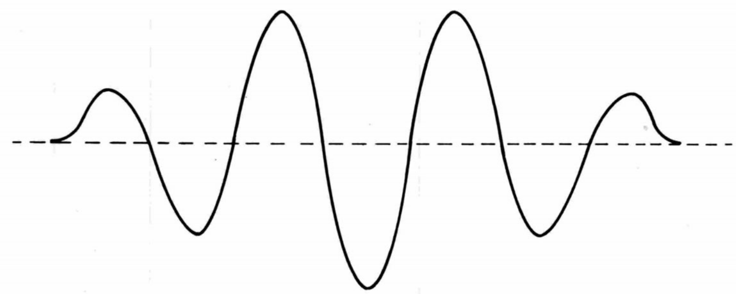


Figure 6.14 (left) The superposition of four wave trains to produce a wave pulse.

Figure 6.15 (below) The wave packet in Fig. 6.14 and the pulse in Fig. 6.8a have been drawn to the same scale for easier comparison.





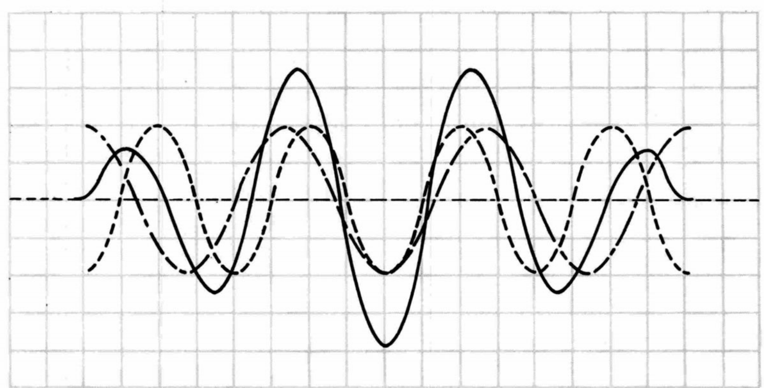


Figure 6.16 The superposition of wave trains to produce a wave packet.
(a) The wave packet pictured in Fig. 68c, enlarged.
(b) A very similar wave packet constructed by the superposition of two wave trains.

full wave over the length of the packet than does the other one. Look, for example, at the wave packet with about four ripples shown in Fig. 6.8c, and reproduced here (Fig. 6.16a). We can combine two wave trains (one with four full waves over the length of the packet and one with three) to find a first approximation to the desired wave packet (Fig. 6.l6b).

Uncertainty principle. We will now formulate a general principle governing the superposition of wave trains to form wave packets. It is called the *uncertainty principle*, and it has played a very important role in the application of the wave model to atomic phenomena, which we will describe in Chapter 8.

Physical significance. The content of the uncertainty principle is that a wave packet that extends over a large distance in space (large Δs) is obtainable by superposition of wave trains covering a narrow range in wave numbers, but that a wave packet that extends over only a short distance in space (small Δs) must be represented by the superposition