Chapter 6: The Wave Theory

PART 2

of wave trains covering a wide range in wave number. It is consequently impossible to construct a wave packet localized in space (small Δs) out of wave trains covering a narrow range in wave number. This idea is known as the "uncertainty principle" because it means that there is an inherent uncertainty in our ability to measure the exact position of a wave packet; Δs represents the "length" of the wave packet and thus the range of uncertainty in our measurement of the packet's position. The size of Δs is closely related to the range of wave numbers included in the packet. We cannot specify the range of wave numbers (Δk) precisely, but we can relate it to the size (Δs) of the wave packet. We will now derive a mathematical model that expresses this relationship.

Equation 6·10

wave number of the two wave trains (per meter) k_1, k_2
wave packet length (meters) Δs
number of waves N_1, N_2

$$N_1 = k_2 \Delta s, \qquad N_2 = k_2 \Delta s$$

Equation 6·11

$$\begin{aligned} 1 &= N_1 - N_2 \\ &= k_1 \Delta s - k_2 \Delta s \\ &= (k_1 - k_2)\Delta s \end{aligned}$$

Equation 6·12

wave number difference Δk

$$\Delta k = k_1 - k_2 \quad \text{(a)}$$

$$\Delta k \Delta s = 1 \quad \text{(b)}$$

Mathematical model. The calculation proceeds in the same way as the calculation for the time interval between beats in Eqs. 6.7, 6.8, and 6.9. First, we select two wave trains with different wave numbers k_1 and k_2, one a little larger and one a little smaller than the average wave number of all the waves needed. Each wave train has a certain number of waves ($N_1 = k_1\Delta s$ and $N_2 = k_2\Delta s$) within the length (Δs) of the wave packet (Eq. 6.10). By how much do these two numbers have to differ? They have to differ sufficiently so that the two wave trains are in destructive interference in the regions to the left and to the right of the wave packet's center, where they are in constructive interference. The distance between the two regions is approximately the spatial length Δs of the wave packet. Now, to achieve the desired destructive interference in both regions, the wave train with the shorter wavelength has to contain at least one more whole wave than the other in the distance Δs, that is: $1 + N_2 = N_1$, or $1 = N_1 - N_2$. This condition is applied in Eq. 6.11 to yield an important result: the range of wave numbers (Δk) times the width of the wave packet (Δs) is equal to one (Eq. 6.12b).

Comparison of beats and wave packets. It is clear that Eqs. 6.9 and 6.12b are closely similar. You may consider both of them as statements of an uncertainty principle for wave packets if you are willing to think of one beat pulsation as a wave packet. Equation 6.12 refers to the size of the wave packet in space. Equation 6.9 refers to the duration of the wave packet in time. The wave trains included in a wave packet have a certain average wave number or frequency, and extend above and below these average values by an amount equal to about one half of the wave number difference Δk or frequency difference Δf. The wave packet includes wave trains of substantial amplitude within this range of wave number or frequency, and wave trains of progressively smaller and smaller amplitude outside this range. The exact amplitude distribution of the included wave trains is determined by the shape of the wave packet and can be calculated by more complicated mathematical procedures developed by Fourier and later workers. We apply the uncertainty principle to wave packets below in Examples 6.1 and 6.2

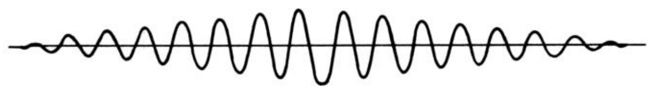
EXAMPLE 6.1. A telegraph buzzer operates at a pitch of 400 vibrations per second. A sound wave packet is formed by depressing the key for 0.1 second. What is the frequency range in the wave packet?

Solution:

$\Delta f\, \Delta t = 1$, $\Delta t = 0.1$ sec., hence $\Delta f = (1/\Delta t) = (1/0.1 \text{ sec}) = 10/\text{sec}$.

The frequency range is about 395 per second to 405 per second.

EXAMPLE 6.2. The wave packet pictured here is 0.08 meter long and contains approximately 16 ripples. What is the wave number range in this wave packet?



Solution : Average wave number $k = \dfrac{16}{0.008} = 200/m$

$$\Delta k\,\Delta s = 1,\ \Delta s = 0.08\ m,\ \Delta k = \frac{1}{\Delta s} = \frac{1}{0.08m} = 12/m$$

The wave number range is 194/m to 206 /m

6.3 Huygens' Principle

Ripple tank. Let us now return to study the propagation of waves by experimenting with water waves. A ripple tank is a useful device for observing water waves. It is a shallow tank with a glass bottom through which a strong light shines onto a screen (Fig. 6.17). Dipping a wire or paddle into the water, creates waves on the water surface; the crests of the waves create bright areas on the screen and troughs create shadows. The patterns of disturbance of the water surface may be observed (Fig. 6.18). A wide paddle generates straight waves (Fig. 6.18a), while the point of a wire generates expanding circular waves (Fig. 6.18b).

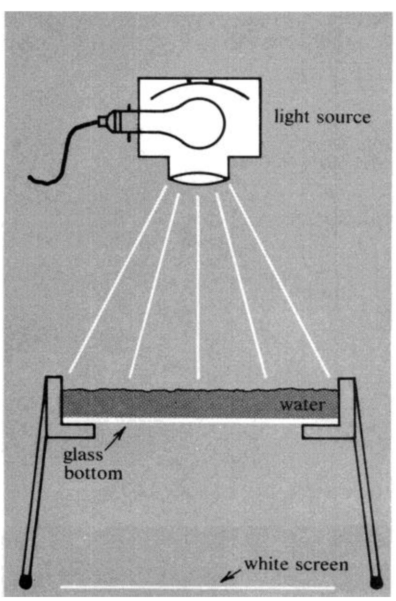


Figure 6.17 (left) Diagram of a ripple tank used for the production and observation of water waves. The wave crests and troughs create bright areas and shadows on the screen.

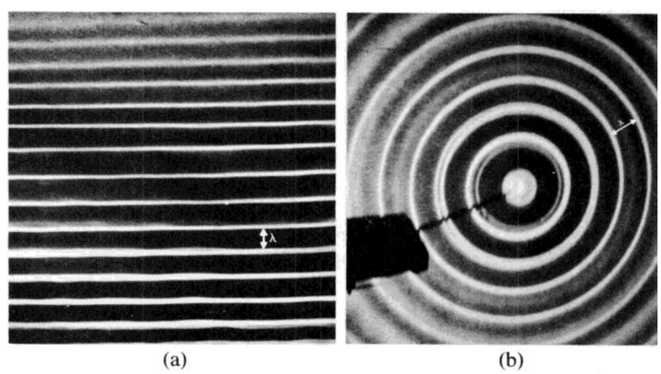


Figure 6.18 (above) Water waves in a ripple tank. (a) Waves generated by a wide paddle. (b) Waves generated by the point of a wire.

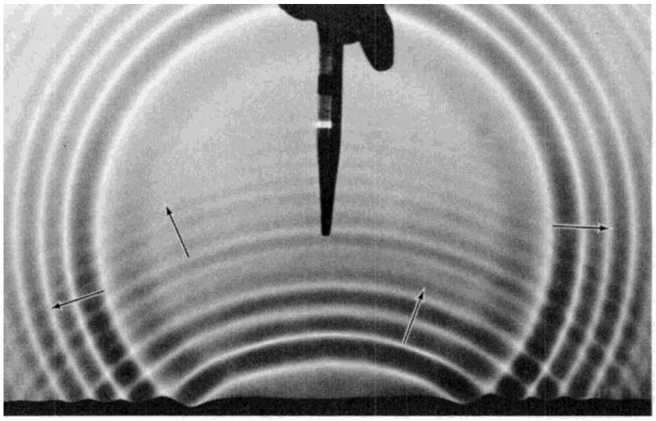


Figure 6.19 The bright lines of the wave crests indicate the wave fronts. The arrows at right angles to the wave fronts indicate the direction of propagation. The waves were originally produced by the tip of the pointer at the center of the photo. The wave fronts form circles centered on the point where they were created until they reflect from the barrier at the bottom of the photo. Where does the wave appear to be diverging from after it is reflected? Can you relate this to what you see in a plane (flat) mirror, as in Fig. 5.17?

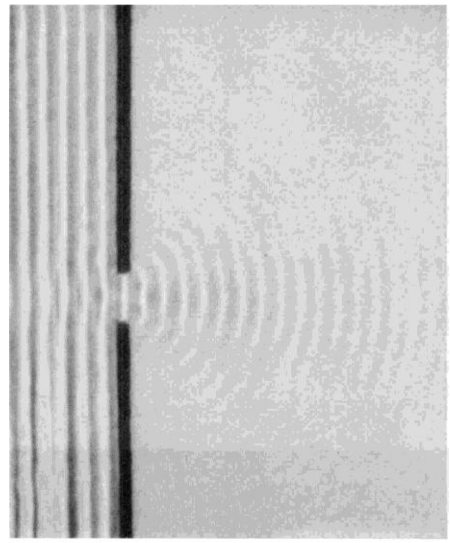


Figure 6.20 Straight waves from the left impinge on a barrier with a hole. Note the curved, circular shape of the wave front to the right of the barrier

The point of a wire may be considered as a point source of waves. The reflection of a circular wave pulse created by a pencil point touched to the water surface is shown in Fig. 6.19.

Wave patterns. To describe the pattern, we identify the *wave front*, which is the line made by each wave crest or trough, and the *propagation direction* in which the wave is traveling. The wave always travels in the direction at right angles to the wave front. Therefore, the wave travels in different directions at different parts of a curved wave front such as the one shown in Fig. 6.19.

You can make an interesting discovery if you use a barrier to block off all but one small section of the water. The waves passing through the hole from one side of the barrier to the other spread out in ever increasing circles (Fig. 6.20). This shows a very important result: the small section of the wave front acts as if it were itself a point source of waves.

Huygens' wavelets. In the oscillator model, the oscillator in the small section moves in rhythm with the waves impinging from the source side of the barrier; it also interacts with the oscillators on the other side and

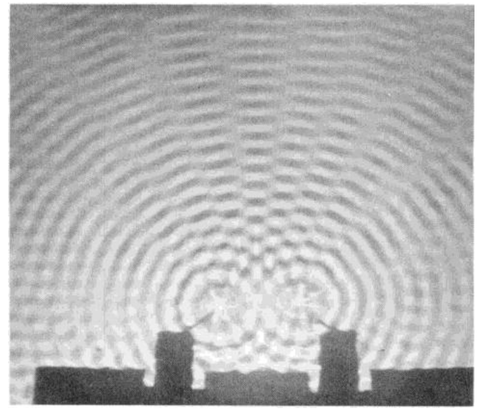


Figure 6.21 Two point sources produce an interference pattern. Note the lines of "nodes" fanning out from the sources.

sets them in motion as though it were a point source. In fact, you can think of every point of a wave front as the source of *wavelets* (numerous mini-waves generated by another wave) that radiate out in circles. That is, each oscillator interacts equally with the other oscillators in all directions from it. This principle is called *Huygens' Principle*. The wavelets have the same frequency of oscillation as their source points in the old wave front. When a wave front encounters a barrier, then most parts of the wave front are prevented from acting as wave sources. What remains is the circular wavelet originating from that part of the wave front that passes through the hole in the barrier.

Two-hole interference. When the barrier has two holes, the waves not only pass through both holes and spread out, but there also is interference between the waves coming from these two "sources." The observable result is very similar to the interference produced by waves from two adjacent point sources (Fig. 6.21). Note the lines of "nodes" fanning out at various angles from the sources, forming what is known as a "two-hole (or double-slit) interference pattern." This pattern demonstrates the existence of interference and can be observed in all waves (including light and sound), not just those in a ripple tank.

Construction of wave fronts. The position of the wave front at successive times may be found by seeking the region of constructive interference of the wavelets emanating from all the source points in a wave front. When there is no barrier, the complete circular wavelets originating from each point in the wave front are not seen because of destructive interference among them.

Schematic diagrams for the procedure of locating the constructive interference are drawn in Fig. 6.22. These diagrams show a wave crest at three successive instants. Huygens' Principle is applied to source points a in the initial wave crest AB to obtain the circles b, c, d. The destructive and constructive interference of all these wavelets results in a new wave crest at the position of the common tangent line CD of all the circles. After a second equal time interval, all the circles

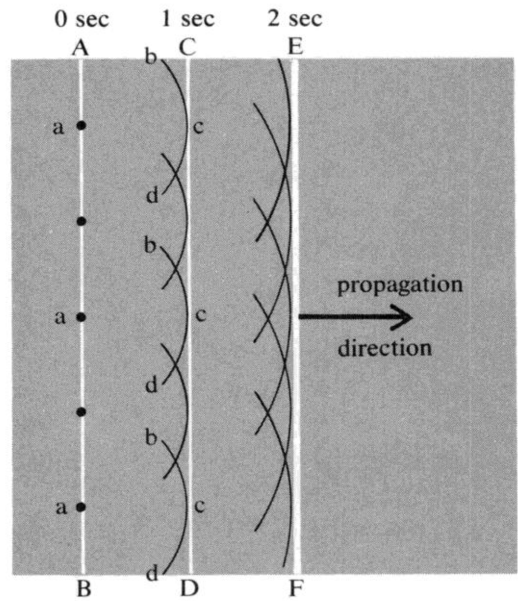


Figure 6.22 The wave front AB (thick white line) advances to CD and then EF, which are the common tangent lines of all the circular wavelets (thin black lines) from Huygens' sources (black dots) in the wave front at AB.

are twice as large, but again the interference effects result in a wave crest EF at the position of the common tangent line of all the larger circles. In this way the straight wave crest advances.

6.4 Diffraction of waves

It is clear from Fig. 6.20 that waves do not necessarily travel in straight lines. Even though the incident wave is headed to the right, the wave transmitted through the hole has parts that travel radially outward

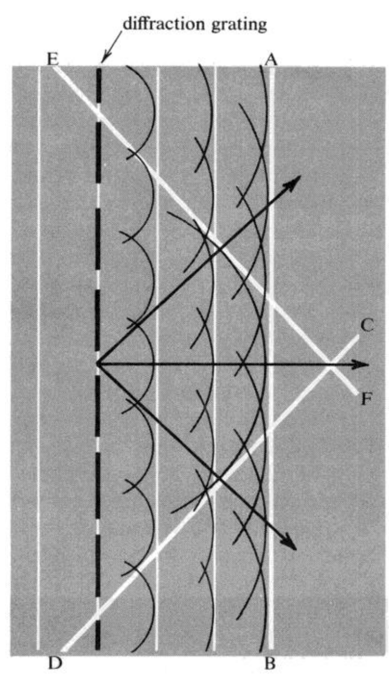


Figure 6.23 Huygens' Principle is used to find the waves transmitted by a diffraction grating. Note the wavelets (thin, curved black lines) centered on the slits. The white lines indicate the undiffracted wave crests (along common tangent line AB) and the diffracted wave crests (common tangent lines CD and EF). The black arrows show the directions of propagation of the observable waves.

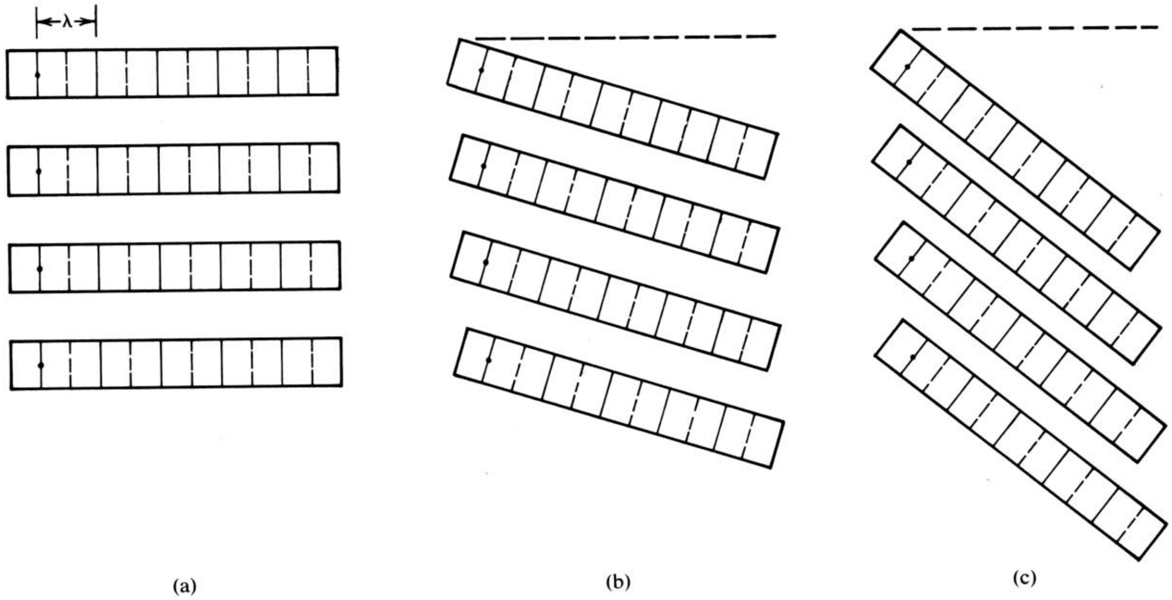


Figure 6.24 Paper strip analogue model for diffraction of waves by a grating. Four paper strips are marked at equal intervals to represent wave troughs and crests. The four strips are then pinned in a row to represent four wave trains passing through equidistant slits in a grating. The strips may be rotated, but are always kept parallel so that the strip direction represents the propagation direction. Interference is determined by the superposition of crests and troughs on the strips.
(a) Constructive interference in the un-diffracted direction is indicated by the alignment of crests with crests, troughs with troughs.
(b) Destructive interference is indicated by the alignment of the crests of one "wave" and the troughs of the adjacent "wave."
(c) Constructive interference in the diffracted direction is indicated by the alignment of crests with crests, troughs with troughs.

from the hole. In other words, the wave was deflected (or bent) by the barrier. This process of deflection of waves passing beside barriers is called *diffraction*. Diffraction makes it possible for waves to bend around a barrier.

Diffraction grating. Let us now apply Huygens' Principle to a device called a *diffraction grating*. A diffraction grating has many evenly spaced slits through which waves can travel. Between the slits, waves are absorbed or reflected. A wave coming through the slits radiates out from each slit in circular wavelets according to Huygens' Principle (Fig. 6.23). You do not observe simple circular waves, however, because the many waves interfere, sometimes constructively and sometimes destructively. A convenient analogue model for diffraction that can be constructed from four strips of paper is described in Fig. 6.24.

(a)

(b) [En-large-ment of (a)]

(c) [Enlarge-ment of (b)]

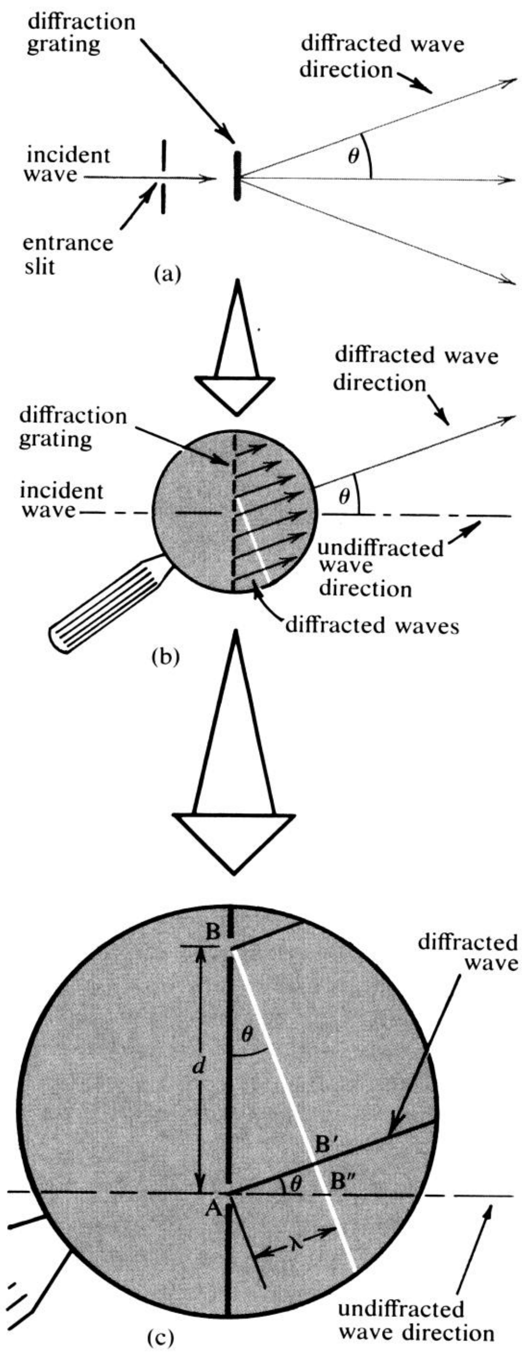


Figure 6·25 Construction of a mathematical model for diffraction by a large diffraction grating.
(a) Waves impinge on the grating from the left. Part of the wave pattern is diffracted at an angle θ, part continues in the undiffracted direction.
(b) Enlarged view of the grating shows that waves passing through adjacent slits travel different distances to contribute to the same wave front (white line).
(c) Constructive interference of diffracted waves occurs if the wave trains from adjacent slits are exactly 1 wavelength out of step as they contribute to one wave front (white line).

distance between slits	d
wavelength	λ
diffraction angle for constructive interference	θ
additional path length for waves passing slit A compared to waves passing slit B	$\overline{AB}'$

Diffraction condition: $\overline{AB}' = \lambda$

Step I: By definition,
sine $\angle ABB' = \overline{AB}'/\overline{AB} = \lambda/d$.
Step II: Prove $\angle ABB' = \theta$.
(i) Extend line BB′ to the undiffracted wave direction at B″.
(ii) θ ***is complementary to*** $\angle AB''B'$ ***in right triangle*** $AB''B'$.
(iii) $\angle ABB'$ ***is complementary to*** $\angle AB''B'$ ***in right triangle*** $AB''B$.
(iv) Hence $\angle ABB' = \theta$.
Step III: It follows from I and II that sine $\theta = \lambda/d$.

With a large diffraction grating of many slits (perhaps 10,000 slits or more), constructive interference of the waves from all the slits occurs only when the adjacent strips are exactly one, two, or three waves out of step. For all other directions, you can find pairs of close or distant slits that give complete destructive interference and thereby cancel one another's wavelets. Waves are therefore diffracted by the grating only

Equation 6.13
(diffraction grating)

distance between slits (meters) $= d$
diffraction angle $= \theta$

$$\text{sine } \theta = \frac{\lambda}{d}$$

into certain special directions. The diffraction angle can be calculated from the condition for constructive interference (Fig. 6.25).

The diffraction grating formula states that the sine of the angle of diffraction is equal to the ratio of the wavelength to the distance between slits. (See Eq. 6.13 and Example 6.3.) The most important practical application of the diffraction grating has been to the study of light, which will be described in the next chapter.

EXAMPLE 6·3. **Use of the diffraction formula.**

(a) $\lambda = 0.2$ m, $d = 0.3$ m, $\theta = ?$

$$\text{sine } \theta = \frac{\lambda}{d} = \frac{0.2\text{m}}{0.3\text{m}} = 0.67$$
$$\theta = 42°$$

(b) $d = 10^{-6}$ m, $\theta = 25°$, $\lambda = ?$

$$\text{sine } 25° = 0.42$$
$$\lambda = d \text{ sine } \theta = 10^{-6} \text{ m} \times 0.42 = 0.42 \times 10^{-6} \text{ m}$$

(c) $\lambda = 10^3$ m, $\theta = 15°$, $d = ?$

$$\text{sine } \theta = 0.26$$
$$d = \frac{\lambda}{\text{sine } \theta} = \frac{10^3\text{m}}{0.26} = 3.9 \times 10^2 \text{ m}$$

(d) $\lambda = 10^{-4}$ m, $d = 10^{-2}$ m, $\theta = ?$

$$\text{sine } \theta = \frac{\lambda}{d} = \frac{10^{-4}}{10^{-2}} = 10^{-2}$$
$$\theta = 0.6°$$

(e) $\lambda = 0.3$ m, $d = 0.2$ m, $\theta = ?$

$$\text{sine } \theta = \frac{\lambda}{d} = \frac{0.3}{0.2} = 1.5$$

θ does not exist.

Diffraction by single slits and small obstacles. Huygens' Principle can also be applied to diffraction by a single slit opening (Fig. 6.20) and to diffraction by a short barrier. The result of the theory suggests that the ratio of the wavelength to a geometrical dimension of the diffracting barrier is of decisive importance for diffraction. In fact, if this ratio is very small (short wavelength, large slit, or large obstacle), the angles of diffraction are very small, so that diffraction is hardly noticeable. If the ratio is large (long wavelength, small slit, or small obstacle),

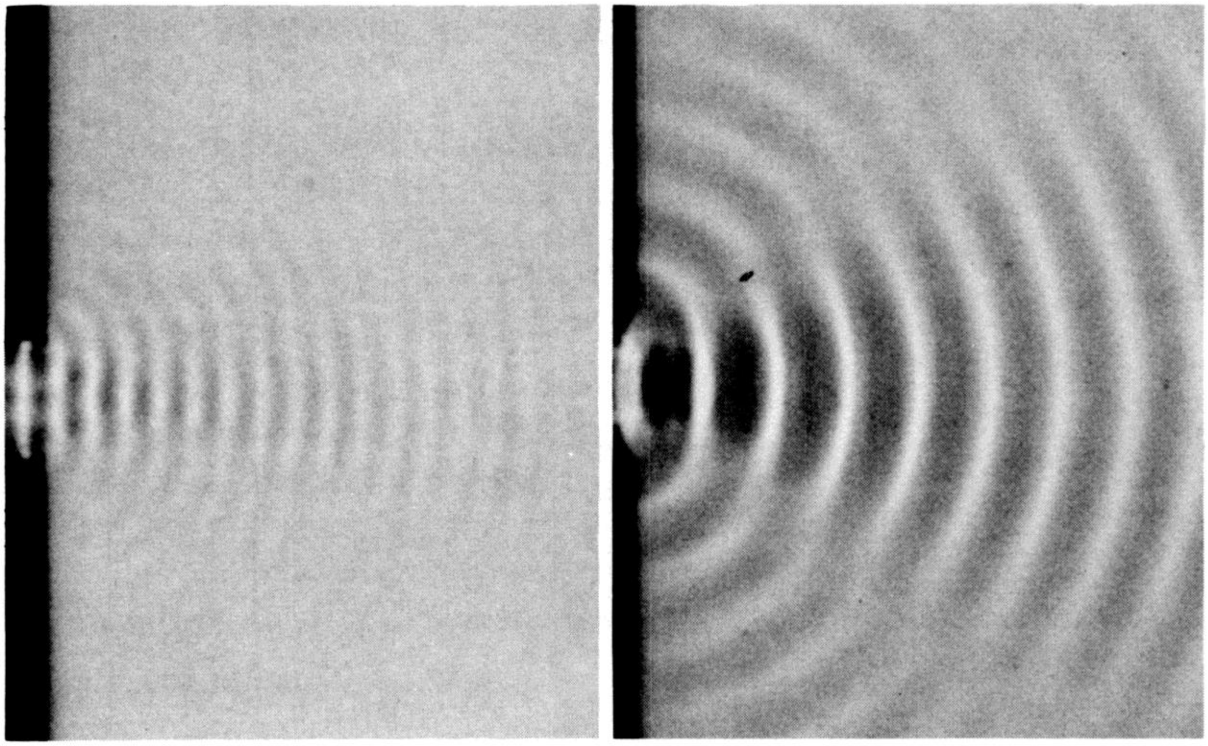
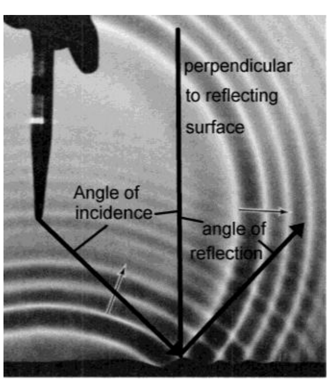


Figure 6.26 Diffraction of waves by an opening. In both photos, the waves are moving from left to right. In the left photo, the wavelength is relatively short (1/3 the width of the opening), so there is little diffraction, and only very weak waves are diffracted away from the original direction of propagation. In the right photo, the wavelength is longer (2/3 the width of the opening), and the waves experience substantial diffraction, spreading out in all directions. After passing through the opening, the wave fronts are essentially semicircles, showing that the waves are now moving in various directions away from the opening. This is a practical demonstration of Huygen's Principle: the waves passing through the opening act as point sources of new waves, which then travel in all directions away from the sources.

then diffraction covers all angles, but the amplitudes of the diffracted waves are very small because the slit or obstacles are small. For intermediate values of the ratio (wavelength comparable to the slit or obstacle in size), diffraction is an important and easily noticeable phenomenon. Two photographs of waves in a ripple tank (Fig. 6.26) show long and short wavelength waves passing through an opening and being diffracted when they pass through an opening. The greater diffraction of the longer wavelength waves is obvious.



6.5 Reflection of waves

The ripple tank photograph to the left (from Fig. 6.19) shows reflection of an expanding circular wave packet. We picked one point on the barrier and drew arrows showing the approximate direction of propagation before and after reflection from that point. The angles of incidence and reflection (as defined in Fig. 5.11) are shown. You can measure the angles to test whether they are equal; we measured one to be 43.5° and the other to be 45°; this is satisfactory agreement given the accuracy of our measurements.

We can also use Huygens' Principle to investigate the relation of these angles in a more general way. According to this principle, each point in a wave front acts like a source of wavelets propagating outward. The wavelets have the same frequency and wavelength as the original waves. The common tangent line of the wavelets is the wave front they produce by constructive interference.

The reflection process is illustrated in Fig. 6.27. A straight wave is incident on the reflecting barrier obliquely from the left. Between the

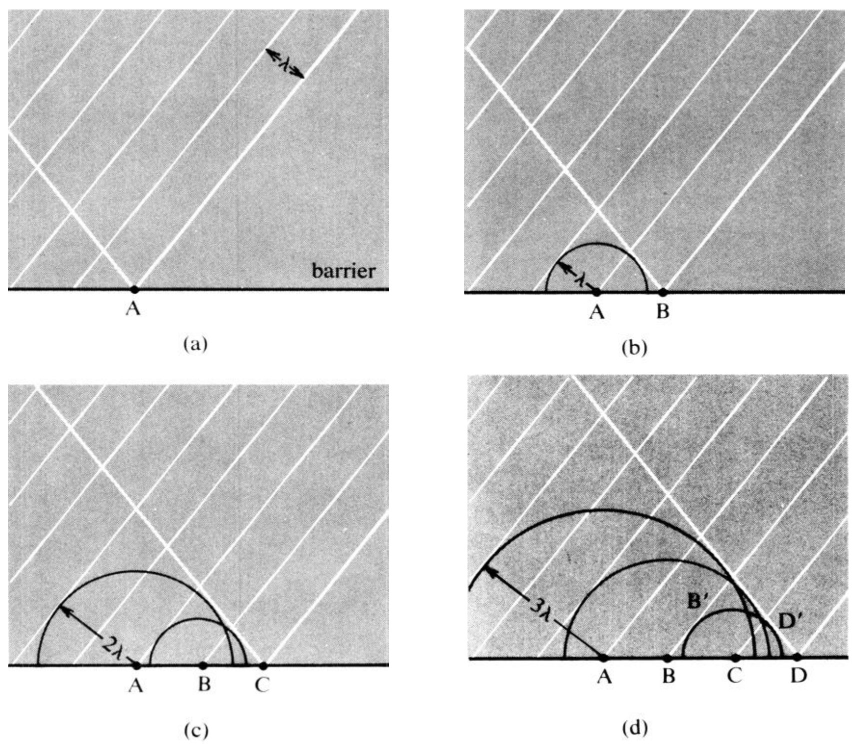


Figure 6.27 Reflection of waves by a barrier. The incident wave crests (white lines) are advancing toward the lower right. Only one reflected wave, moving toward the upper right, is shown. To avoid clutter in the diagram, we have not drawn the other reflected waves.

(a) Three wave crests are striking the barrier, which has reflected a section of the first crest. Point A on the first crest acts as a source of Huygens' wavelets.

(b) The wave crests advance by a distance of one wavelength (λ), and the wavelet from Point A has expanded into a semicircle of radius λ. Point B becomes a source of wavelets.

(c) The wave crests advance by another wavelength; the wavelet from Point A now has a radius of 2λ; the wavelet from B has a radius λ. and the Point C becomes a source of wavelets.

(d) The wavelet from A has radius 3λ; the wavelet from B has radius 2λ, the wavelet from C has radius λ, and Point D becomes a source of wavelets.

The wavelets constructively interfere all along the common tangent line DD', which defines the location and direction of the reflected wave fronts.

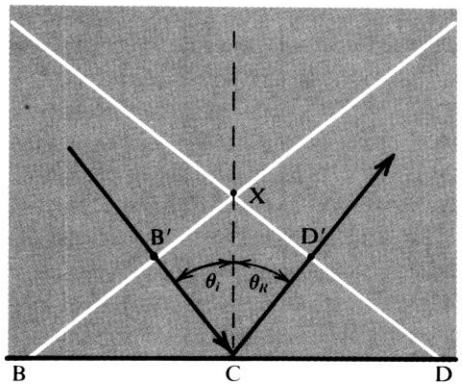


Figure 6·28 Construction of a mathematical model for wave reflection based on Fig. 6·27. Arrows B′C and CD′ represent, respectively, the incident and reflected propagation directions. They are at right angles to the corresponding wave fronts BB′ and DD′ (white lines). Consider the two right triangles XCB′ and XCD′. They share the common hypotenuse XC and have two sides equal, $CB' = CD' = \lambda$. Hence the two triangles are congruent. It follows that corresponding angles are equal, $\theta_i = \theta_R$.

four successive instants shown in Fig. 6.27, the wave advances between each drawing by 1 wavelength. The Huygens wavelets formed by the first wave crest passing through points A, B, C, and intermediate points on the barrier have a common tangent, which is the reflected wave front. The crests of the Huygens' wavelets all fall on the common tangent, where they interfere constructively; at all other points, the wavelets interfere destructively and cancel one other.

To relate the angles of incidence and reflection, the directions of propagation have to be taken into account. This is done in Fig. 6.28, where only one incident and one reflected wave crest from Fig. 6.27d are included. The application of Huygens' Principle in Fig. 6.28 results in a familiar conclusion: the angle of incidence is equal to the angle of reflection (Eq. 6.14). This statement may be called the *law of wave reflection*.

Equation 6.14 (Law of Reflection)

angle of incidence = θ_i

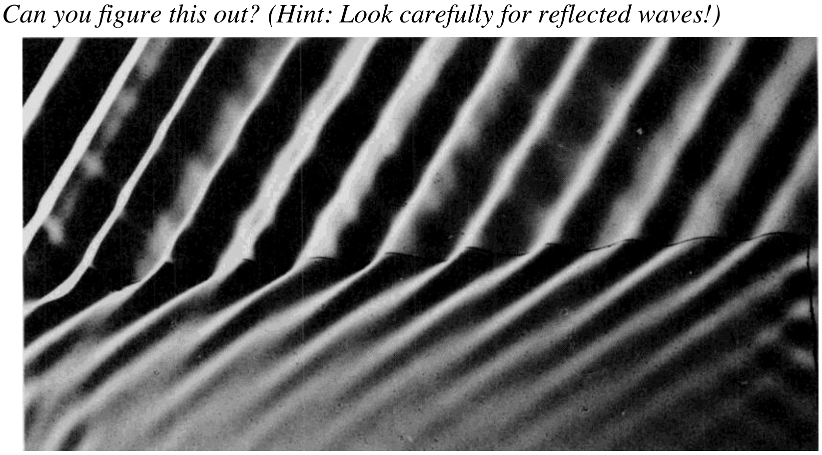
angle of refraction = θ_R

$$\theta_i = \theta_R$$

6.6 Refraction of waves

When a wave propagates from one medium into another, its direction of propagation may be changed. An example of this happening with water waves is shown in Fig. 6.29. The boundary here is between deep water above and shallow water below. Even though water is the

Fig 6.29 Water waves passing from a deeper region to a shallower region are refracted and travel in a different direction at the boundary. Huygens' Principle does not reveal which direction the waves are traveling. Can you figure this out? (Hint: Look carefully for reflected waves!)



Equation 6.3b

$v = f\lambda$

material on both sides of the boundary, it acts as a different medium for wave propagation when it has different depths. You can see that the wavelength is shorter in the shallow water and can infer from this that the wave speed is slower there (Eq. 6.3b).

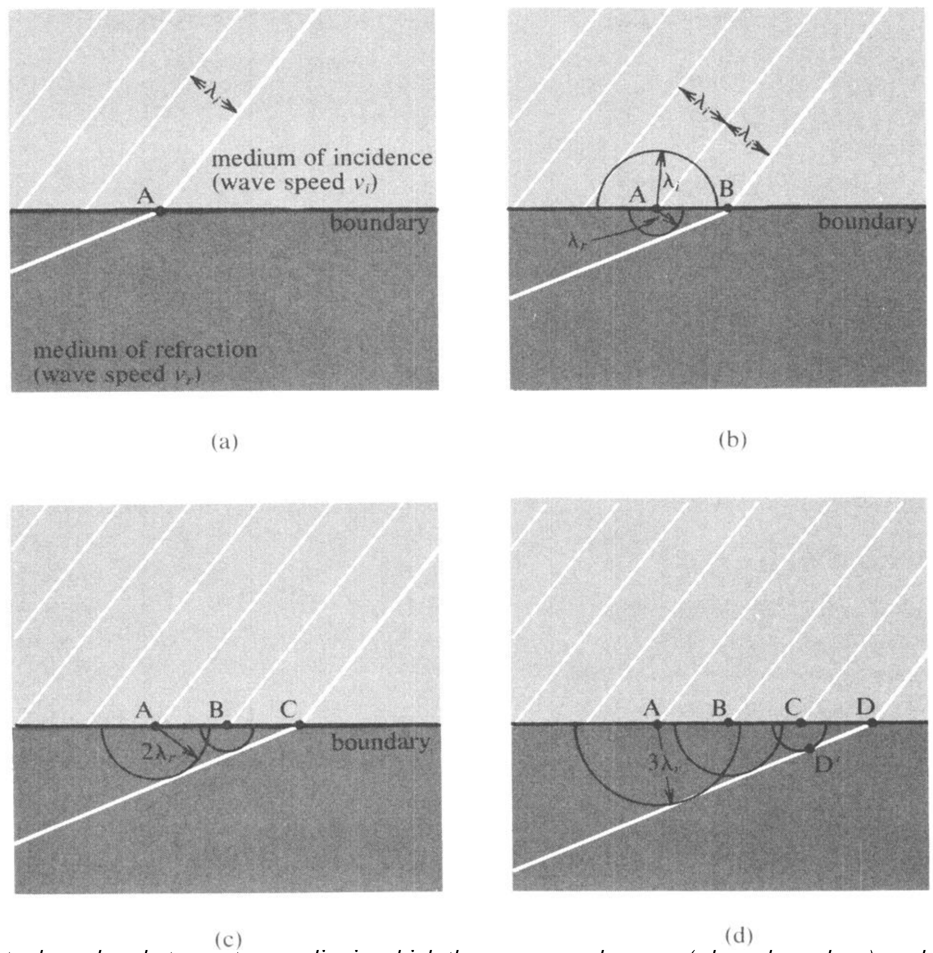


Figure 6.30 Refraction at a boundary between two media, in which the wave speeds are v_i *(above boundary) and* v_r *(below boundary). The wave speed is assumed to be* ***less*** *below the boundary than above it.* (v_r *is* ***less*** *than* v_i). *The waves are moving downward to the right.*

(a) Three wave crests (white lines) are incident on the boundary, which has refracted a section of the waves. Point A on the first crest acts as a source of Huygens' wavelets.

(b) The wavelet from point A has expanded into a semicircle of radius λ_r *below the boundary and a semicircle of radius* λ_i *above the boundary. The latter gives rise to a reflected wave (see Fig. 6.27) and will not be described further. The wave crests advance by the distance* λ_i*. Point B becomes a source of wavelets.*

(c) The wavelet from A has reached radius $2\lambda_r$*, the wavelet from B has radius* λ_r*. Point C becomes a source of wavelets.*

(d) The wavelet from A has radius $3\lambda_r$*, the wavelet from B has radius* $2\lambda_r$*, and the wavelet from C has radius* λ_r*. The common tangent line DD' coincides with the refracted wave front. Note the change in the direction of propagation (which is perpendicular to the wave front): we can see from the diagram that the refracted wave (below the boundary) is traveling* ***slower*** *and in a direction* ***farther*** *from the boundary surface than the incident wave (that is,* ***closer*** *to the perpendicular to the boundary).*

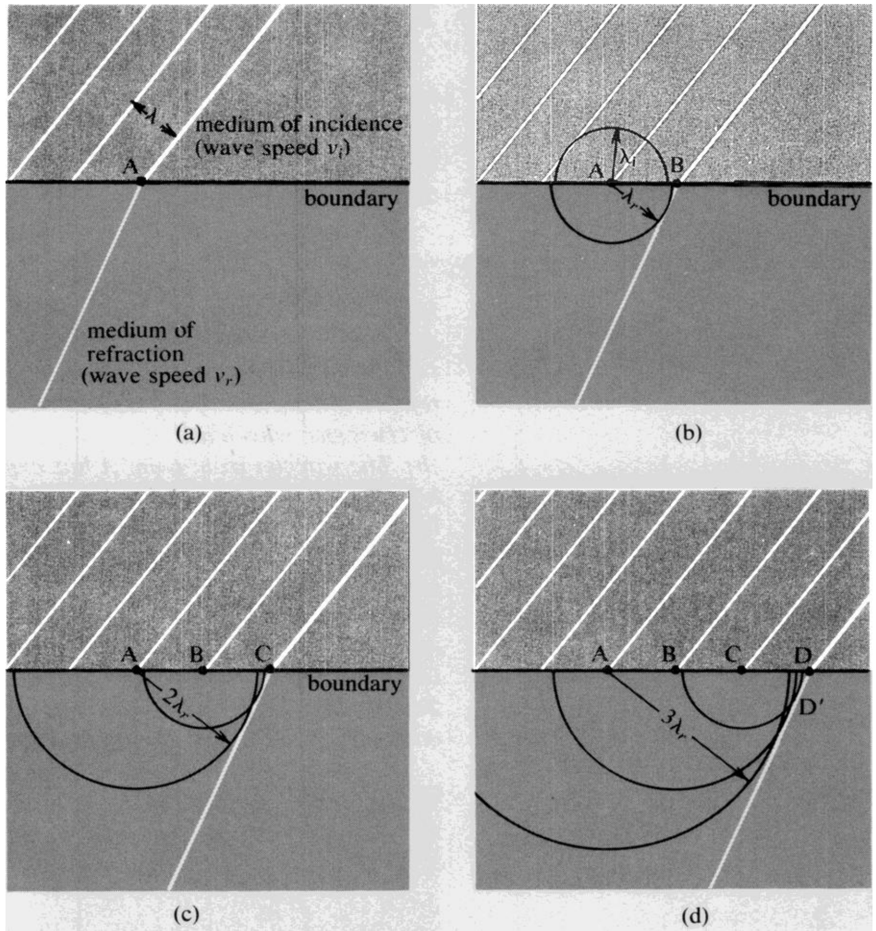


*Figure 6.31 Refraction at a boundary between two media, in which the wave speeds are v_i (above boundary) and v_r (below boundary). The wave speed is assumed to be **greater** below the boundary (v_r is **greater** than v_i). (This is similar to Figure 6.30 but the speeds are reversed.) The waves are assumed to be moving downward to the right.*

(a) Four wave crests (white lines) are incident on the boundary, which has refracted a section of the waves. Point A on the first crest acts as a source of Huygens' wavelets.

(b) The wavelet from point A has expanded into a semicircle of radius λ_r below the boundary and a semicircle of radius λ_i above the boundary. The latter gives rise to a reflected wave (see Fig. 6.27) and will not be described further. The wave crests advance by the distance λ_i. Point B becomes a source of wavelets.

(c) The wavelet from A has radius $2\lambda_r$; the wavelet from B has radius λ_r. Point C becomes a source of wavelets.

*(d) The wavelet from A has radius $3\lambda_r$, the wavelet from B has radius $2\lambda_r$, and the wavelet from C has radius λ_r. The common tangent line DD' coincides with the refracted wave front. Note the change in the direction of propagation (which is perpendicular to the wave front): we can see from the diagram that the refracted wave (below the boundary) is traveling **faster** and in a direction that is **closer** to the boundary surface than the incident wave (that is, **further** from the perpendicular to the boundary).*

Note: Although we have assumed above that the waves are traveling toward the right, this demonstration can also be carried out using the same diagram with the waves traveling in the opposite direction. Thus wave theory based on Huygens' Principle predicts that refracted waves will follow the same path in either direction. Does this seem reasonable to you? Can you suggest any observations or experiments that would confirm or refute this?

The refracting boundary. The change in the direction of propagation is called refraction, the same term that was introduced in Section 5.2. We will now find the law of refraction of waves by applying Huygens' Principle to the propagation of the wave across the boundary between two media with different wave velocities. Each point in the wave front that touches the medium of refraction acts like a source of wavelets that propagate into that medium. These waves have the same

Equation 6·15

wave speed in medium of incidence v_i
wave speed in medium of refraction v_r
wavelength in medium of incidence λ_i
wavelength in medium of refraction λ_r

$$v_i = \lambda_i f$$

$$v_r = \lambda_r f$$

Equation 6.17

$$\frac{\text{sine}\,\theta_\text{i}}{\text{sine}\,\theta_\text{r}} = \frac{\lambda_i}{\lambda_r} \quad (a)$$

$$\frac{\text{sine}\,\theta_\text{i}}{\text{sine}\,\theta_\text{r}} = \frac{v_i}{v_r} \quad (b)$$

frequency as their source, and therefore the same frequency as the wave in the medium of incidence. The wave in the medium of refraction, however, where the speed is different, has an altered wavelength, because wavelength, frequency, and speed are related by $v = \lambda f$ (Eq. 6.15). The ratio of the wavelengths in the two media is equal to the ratio of the wave speeds, since these two properties of the wave are directly proportional as long as the frequency remains the same (Eq. 6.16). Thus, the change in medium results in a changed wavelength.

Construction of the refracted wave front. The procedure for finding the law of refraction is very similar to that used in the preceding section to find the law of reflection. A straight wave is incident on the refracting boundary obliquely from the left. Between each of the four successive instants shown in Figs. 6.30 and 6.31, the wave advances by 1 wavelength. The Huygens' sources on the boundary generate wavelets that propagate into the second medium with the wave speed and therefore the wavelength appropriate to that medium. The case of reduced wave speed and wavelength is illustrated in Fig. 6.30, while the case of increased wave speed and wavelength is illustrated in Fig. 6.31. In both cases the wavelets originating in points A, B, and C (and intermediate points on the boundary) have a common tangent that is the refracted wave front.

Law of refraction. To relate the angles of incidence and refraction, the directions of propagation have to be taken into account. This is done for both cases above in Fig. 6.32, where only one incident and one refracted wave crest from the previous figures are included. The conclusion from the application of Huygens' Principle is that the sines of the angles of incidence and refraction have the same ratio as the wavelengths (Eq. 6.17a) and, therefore, the same ratio as the wave speeds in the two media (Eq. 6.17b). This result is similar in form to Snell's Law of Refraction: (n_i sine $\theta_i = n_r$ sine θ_r, Eq 5.2, Section 5.2), a key assumption in Newton's ray model of light. We shall study this further below in Section 7.2, where we will compare and evaluate the ray and wave models in some detail.

If you look at the propagation direction of the refracted waves in Figs. 6.30 and 6.31, you will recognize that the effect of crossing the boundary can be described as follows. In the medium with the slower wave, the propagation direction is farther away from the boundary surface; in the medium with the faster wave, the propagation direction is closer to the boundary surface. You may use the tables of the sine functions (Appendix, Table A.7) to solve problems on the refraction of waves.

Reflection at the boundary. The application of Huygens' Principle to the boundary between the two media leads to reflected wavelets as well as refracted ones. One reflected wavelet is indicated in Fig. 6.30b and one is indicated in Fig. 6.31b. Since these wavelets are in the medium of incidence, their speed and wavelength are appropriate to that medium. By pursuing their formation further, we could have obtained the same sequence of diagrams as are shown in Fig. 6.27. The wavelets would interfere constructively to form a reflected wave according to the law of reflection (Eq. 6.14). Thus wave theory suggests that we should also look for reflected waves, and, in fact, by looking carefully, you can indeed identify reflected waves in the deeper water of Fig. 6.29! In other

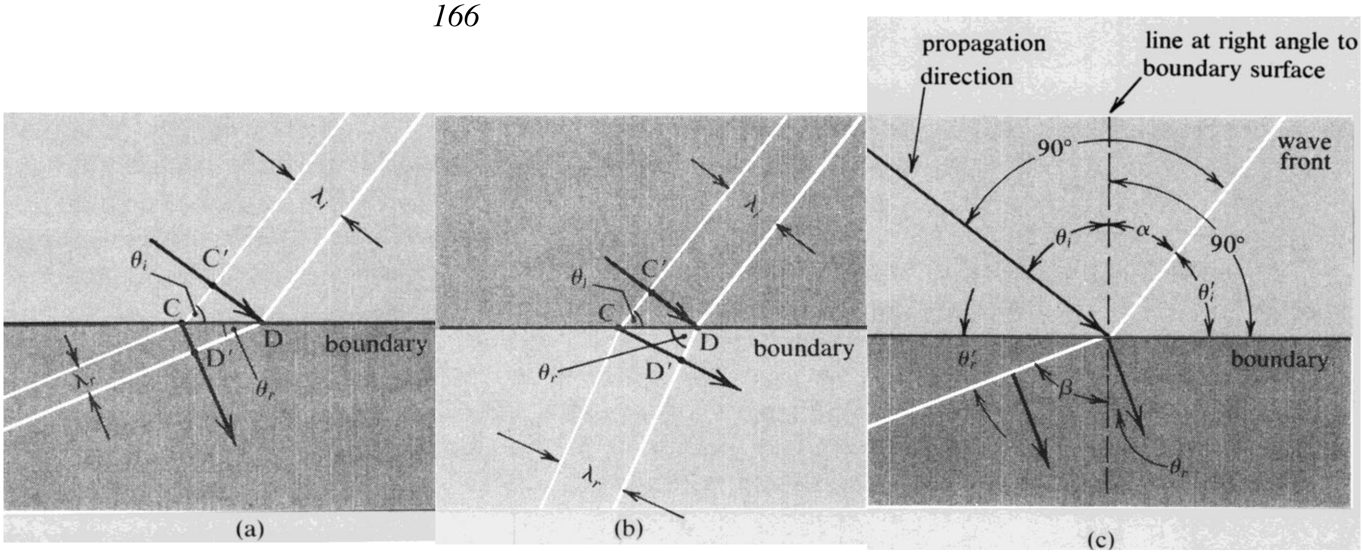


Figure 6.32 Construction of a mathematical model for wave refraction, based on Figs. 6.30 and 6.31. Note that in (a) the medium with the longer wavelength (faster speed) is at top in lighter shading. However, in (b) the medium with the longer wavelength (faster speed) is at bottom, also in lighter shading.
In both (a) and (b), Arrows C'D and CD' represent, respectively, the incident and refracted propagation directions. They are at right angles to the corresponding wave fronts (white lines) CC' and DD'. ASSUMPTION: Angle C'CD is equal to the angle of incidence θ_i and D'DC is equal to the angle of refraction θ_r ; we will prove this assumption in (c) below. The definition of the sine functions can be applied to right triangles CDC' and CDD' with the following results:

$$\text{sine } \theta_\text{i} = \frac{C'D}{CD} = \frac{\lambda_\text{i}}{CD} \quad (1)$$

$$\text{sine } \theta_\text{r} = \frac{D'C}{CD} = \frac{\lambda_\text{r}}{CD} \quad (2)$$

Divide Eq. (1) by Eq. (2)

$$\frac{\text{sine } \theta_\text{i}}{\text{sine } \theta_r} = \frac{\lambda_\text{i}}{\lambda_\text{r}} \qquad (3)$$

(c) Proof of ASSUMPTION asserted above: the angle of incidence θ_i equals the angle between the boundary and the wave front θ_i'. Two overlapping right angles in the medium of incidence are indicated in the figure (c) above. Both angles θ_i and θ_i' are complementary to the angle α. Consequently the two angles are equal, $\theta_i = \theta_i'$. The same construction with respect to the angle β in the second medium leads to the conclusion that $\theta_r = \theta_r'$.

words, the incident wave appears to be split by the boundary into a reflected wave and a refracted wave. Huygens' Principle has indeed served us well; however, it does not reveal how the energy carried by a wave is divided between reflection and refraction.

The occurrence of partial reflection gives an important clue about the sense of the direction of propagation of waves. By examining only the incident and refracted waves, such as in Fig. 6.29, you would not be able to determine whether the waves were incident as described at the beginning of this section (from upper left), or whether the waves were incident from the lower right and passed from the shallow water to the deeper water. The observation of reflected waves in the upper part of the photograph is evidence that the waves were incident from above.

Summary

Equation 6.3b (wave speed)

$$v = \lambda f$$

The concept of waves has its roots in water waves. More generally, waves are oscillatory displacements of a medium from its equilibrium state. Two important forms that such disturbances can take are the wave train, in which the displacement pattern repeats over and over, and the wave pulse, in which the displacements are localized in space and time. The wavelength, wave number, period, frequency, amplitude, and speed of the waves can be defined for a wave train, but only the last two of these can be defined for a pulse. The frequency, wavelength, and speed of a wave train are related by $v = \lambda f$ (Eq. 6.3b).

The wave theory is built upon the above ideas and applies to a wide variety of types of waves. The goal of wave theory is the construction of mathematical models to describe the behavior and propagation of waves. Wave theory explains and clarifies a large variety of phenomena. Such phenomena include sound, music, water waves, radio, light, constitution of the atom, traffic flow, and earthquakes.

The wave theory rests on two key assumptions about waves: 1) the superposition principle and 2) Huygen's Principle. The theoretical deductions from these assumptions can be compared with observation to identify the successes and the limitations of the wave theory.

According to the superposition principle, the displacement of the combination of two or more waves passing through the same point in space at the same time is the sum of the displacements of the separate waves. The result is constructive or destructive interference, depending on whether the separate waves reinforce or oppose one another.

Huygens' principle is used to investigate the propagation of waves. Each point in a wave front is considered as a source of circular outgoing wavelets. The amplitude and frequency of the wavelets are determined by the amplitude and frequency of the wave at the source point. The wavelets interfere constructively along their common tangent line, which is therefore the front of the propagating wave. Elsewhere, the wavelets interfere destructively and are not separately observable.

Huygens' Principle allows us to conduct thought experiments on the propagation of waves and furnishes a procedure for determining

the results. We have used Huygens' Principle to understand diffraction, reflection, and refraction of waves.

The wave theory does not attempt to relate the wave speed, amplitude, and energy to properties of the medium, the wave source, and the wave absorber. These matters require more detailed working models for the three systems; their treatment is beyond the scope of this text.

List of new terms

medium
  (for wave propagation)
wave train
wave pulse
amplitude
frequency (f)
wavelength (λ)
wave number (k)
period ($\mathcal{T}$)
wave speed (v)

superposition
interference
constructive
  interference
destructive
  interference
node
tuned system
beats
wave packet
uncertainty
  principle

Huygens' Principle
wave front
propagation direction
Huygens' wavelets
diffraction
diffraction grating
reflection of waves
refraction of waves
standing waves

List of symbols

k	wave number	Δf	frequency range $(f_1 - f_2)$
λ	wavelength	Δk	wave number range $(k_1 - k_2)$
f	frequency	v	wave speed
$\mathcal{T}$	period	θ	diffraction angle
N	number of waves	θ_i	angle of incidence
Δs	pulse width	θ_R	angle of reflection
Δt	time for one beat	θ_r	angle of refraction

Problems

Here are some suggestions for problems that have to do with water waves. Observations on a natural body of water are made most effectively from a bridge or pier overhanging the water. You may observe wind-generated wave trains or pulses generated by a stone. By dipping your toe rhythmically into the water, you may be able to generate a circular wave train.

Experiments can be conducted in a bathtub or sink if natural bodies of water are not available. A pencil or comb dipped horizontally into the tub near one end can generate straight wave pulses. Dipping your finger, a pencil or a comb vertically will generate circular waves. To observe the waves, place a lamp with one shaded bulb over the bathtub so as to direct the light at the water surface and not into your eyes. You should also avoid looking at the reflected image of the bulb. Under these conditions, waves cast easily visible shadows on the bottom of the tub or on the ceiling. **Caution: You must be careful when using electricity near the bath or sink; an electrical shock from household**

current can be dangerous. Keep water away from the lamp and do not under any circumstances touch the lamp with wet hands nor while any other part of your body is wet or touching something wet.

1. Measure the speed of water waves by measuring how long they take to traverse a given distance. Describe the conditions of your observations, especially the depth of the water and the amplitude of the waves. If you observe wave trains, determine their frequency and wavelength and test Eq. 6.3b.

2. Identify the interaction(s) that are involved in the propagation of waves on a water surface.

3. Observe waves at the seashore and report qualitatively about as many of the following as you can observe.
   (a) Differences in speed of various waves.
   (b) Differences in direction of propagation.
   (c) Applicability of the superposition principle.
   (d) Effect of the depth of the water on the wave motion.
   (e) Reflection of wave fronts.
   (f) Refraction of wave fronts.
   (g) Diffraction of waves.
   (h) Transfer of energy from the waves to other systems.

4. Sand ripples are frequently observed on the ocean or lake bottom in shallow water. They are formed by the interaction of sand and water just as water waves are formed by the interaction of water and wind. Measure the wavelength of sand ripples that you observe. Comment on their propagation speed.

5. Observe reflection of water waves in your sink or bathtub. Estimate the angles of incidence and reflection as well as you can and compare your results with the law of reflection for waves.

6. Observe single-slit diffraction of water waves in your sink or bathtub. Report the slit width you found most suitable and other conditions that helped you to make the observations.

7. Several different diffraction gratings diffract water waves with a wavelength of 0.03 meter. Find the diffraction angle for a diffraction grating with a slit spacing of (a) 0.30 meter; (b) 0.10 meter; (c) 0.05 meter; (d) 0.025 meter.

8. Water waves are diffracted by a grating with a slit spacing of 0.30 meter. Find the wavelengths for the waves when the diffraction angle is (a) 10°; (b) 25°; (c) 60°.

9. Find the result of superposing the following three waves:
   wave A -- wavelength (λ) = 6 centimeters (cm), amplitude = 3 cm;
   wave B -- λ = 3 cm, amplitude = 2 cm;
   wave C -- λ = 2 cm, amplitude = 1 cm.
   Start from a point where all three waves interfere constructively; keep plotting until all three waves again interfere constructively.

10. Sound waves in air have a wave speed of 340 meters per second. Find the wavelength and wave number of the following sound waves: (a) middle C, frequency (f) = 256 per second; (b) middle A, f = 440 per second (c) high C, f = 1024 per second.

11. Use the paper strip analogue (Fig. 6.24) to study diffraction of waves. Report the wavelength, "slit" separation, and diffraction angle(s) for three different "gratings." Choose λ/d small (0.5), medium (2.0), and close to one for the three cases. (Note: one grating may give several diffraction angles, according to whether the waves from adjacent slits are 1, 2, 3, ... wavelengths out of step.) Make as many paper strips as you feel necessary to help you.

12. Use the paper strip analogue (Fig. 6.24) to study diffraction from only two slits. Measure and/or use geometrical reasoning to find the angles of diffraction amplitude maxima (constructive interference) and diffraction amplitude minima (destructive interference). Compare your results with those obtained for a diffraction grating and describe qualitatively the reasons for similarities and differences.

13. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

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