

# *Chapter 9: Operational Definitions of Energy*

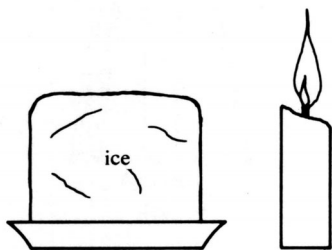
Your common-sense notion of energy is derived from certain systems (bent bows, candle-oxygen systems) that can act as energy sources. In Chapter 4, we identified several forms of energy storage, such as the elastic energy of the bow and the chemical energy of the candle-oxygen system. We also pointed out that a system stores a quantity of energy that depends on the state of the system.

In this chapter, we will introduce two operational definitions of energy. To do so, we will describe standard units of measurement and procedures for comparing the standard units with the system whose energy we wish to measure. You can apply these operational definitions to real systems in experiments; you can also use the operational definitions to estimate the energy stored in systems by making working models for such systems and carrying out thought experiments on the models. We will use these operational definitions extensively in later chapters of this text to find mathematical models for the energy stored in a wide variety of systems. In fact, energy is the quantity that connects all of physics, a "natural currency" or a "medium of exchange." We will use the concept of energy and our operational definitions as the basis for understanding the remaining topics in this book: temperature, work, force, electricity, motion, Newton's laws, periodic motion and the solar system, heat engines and refrigerators, and kinetic theory of gases.

## 9.1 Measurement of energy

Unlike length or mass, the energy stored in a system cannot be compared directly with a standard unit of energy. For instance, when you compare two bows that have been bent to a different degree, it is not possible to arrange them side by side and infer by direct comparison that the one has, say, three times the stored energy of the other one. It is not even possible to place one standard system beside the two bows and to read off their energies, as you would use a ruler to compare the heights of two children. Instead of a direct comparison, you have to make an indirect comparison by transferring the energy from each bow in turn to a third system (perhaps a third bow, a spring, or the gravitational field of a weight-earth system, see Fig. 9.1). Suitable coupling elements have to be provided, a procedure that is easy in thought experiments but difficult in real experiments. In the end, the third system permits the energy comparison if a suitable energy scale is available.

**A burning candle.** Think of a candle, and how you could determine the energy stored in the candle-air system, energy that is transferred to other systems when the candle burns. The best way to determine this energy is to burn the candle and let the energy be transferred to a system that shows the effect of the added energy in a measurable way: either cold water that is heated, or a block of ice that is partially melted will do. The result of your measurement is expressed as the temperature rise of the water or as the mass of ice melted. Now you know the



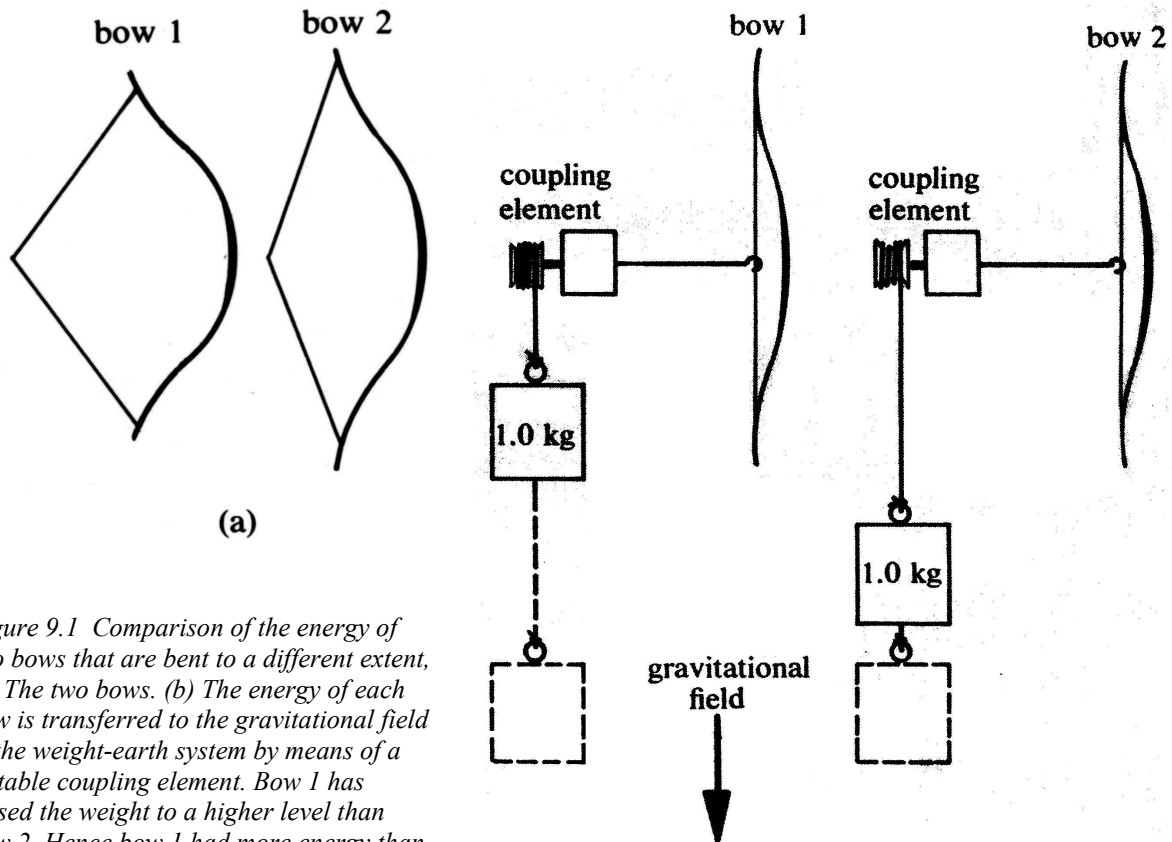


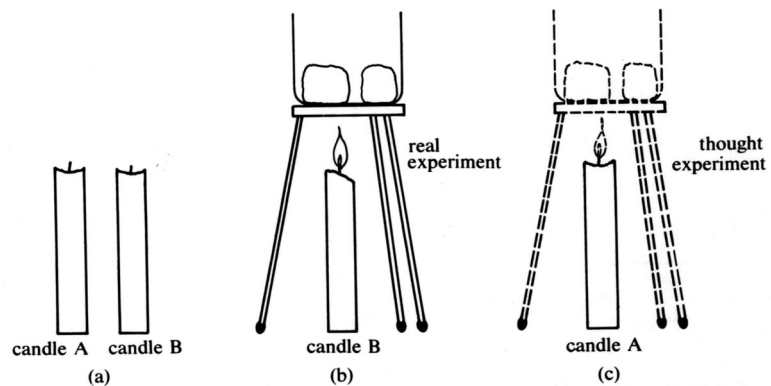
Figure 9.1 Comparison of the energy of two bows that are bent to a different extent, (a) The two bows. (b) The energy of each bow is transferred to the gravitational field of the weight-earth system by means of a suitable coupling element. Bow 1 has raised the weight to a higher level than bow 2. Hence bow 1 had more energy than did bow 2.

Figure 9.2 Measurement of the energy of an air-and-candle system.

(a) You need two identical candles, A and B.

(b) Candle B is burned and the energy is transferred to melting ice.

(c) In a thought experiment, candle A can transfer an equal quantity of energy to melting ice.



energy, but you no longer have the candle. If you could only find the energy stored without burning the candle!

This is where a thought experiment comes in. Instead of burning your candle A, take another candle B, as similar to candle A as possible,

*"... the very considerable quantity of Heat that was produced in this Experiment... would have been capable of melting 6 1/2 lb. of ice, or of causing near 5 lb. of ice-cold water to boil."*

*Benjamin  
Thompson,  
Count Rumford  
Philosophical  
Transactions,  
1798*

*Rumford was an unusual individual who made key contributions to the study of energy and heat. We will explain his work below in Section 10.5.*

and burn it to measure the energy transferred (Fig. 9.2). Now imagine that you burn candle A. Since the two candles were very much alike, candle A will heat the water or melt the ice to the same extent as did candle B. The energy of the air and candle A system, therefore, is equal to the energy transferred during the burning of candle B. You now have a measure of the energy of the air and candle A system, and you still have candle A. In fact, you can apply your knowledge to all other similar candles after you have burned the one candle and measured the energy transfer.

The foregoing description has several points that should be noted. First of all, it shows that energy can be measured directly only by being transferred, as was true also in the bow example. Second, it suggests two operational definitions of energy similar to those used over 200 years ago by Count Rumford: energy may be measured by the temperature rise of water or by the mass of melted ice. Third, it shows that a standard system (the cold water, the ice) must be chosen for the definition of the unit of energy. Fourth, it illustrates how the energy of one system, such as candle A, may be estimated through a thought experiment that makes use of the operational definition and of observations on another, but very similar, system.

## 9.2 Energy scales

Before we describe operational definitions of energy, we will define units of energy and construct scales on which energy may be measured. This task is analogous to the selection of a unit of length (the meter, defined as the distance between two scratches on a certain platinum-iridium bar, Fig. 1.9), and the construction of a ruler, by making equidistant marks using the unit of length. Just as the ruler has a scale for measuring lengths or distances, so we now need a scale for measuring energy.

**Thermal energy scales.** The energy scale consists of a set of systems that have 0 units, 1 unit, 2 units,... of energy, just as the ruler is marked for 1, 2, . . . units of length. The thermal unit of energy may be selected in any one of several ways. For instance, it may be defined as the energy required to bring 1 kilogram of water from its freezing temperature to its boiling temperature. This unit of energy is inconveniently large, and the generally accepted thermal unit of energy, called the *Calorie*, is capable of raising 0.010 kilogram of water from its freezing temperature to its boiling temperature (Fig. 9.3). Other thermal units of energy could be defined in the same way as the Calorie, but using substances such as aluminum, mercury, or alcohol instead of water. Water, however, is the material most conveniently available and generally used for energy scales.

Still another thermal unit of energy may be defined as the energy required to melt 1 kilogram of ice (solid water) at its melting temperature. A system for constructing a thermal energy scale based on this unit is shown in Fig. 9.4.

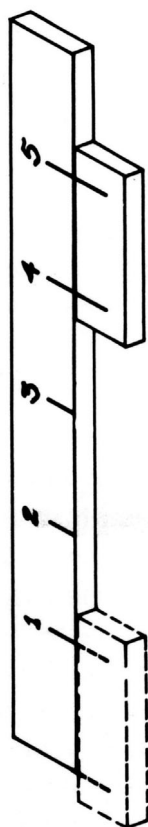


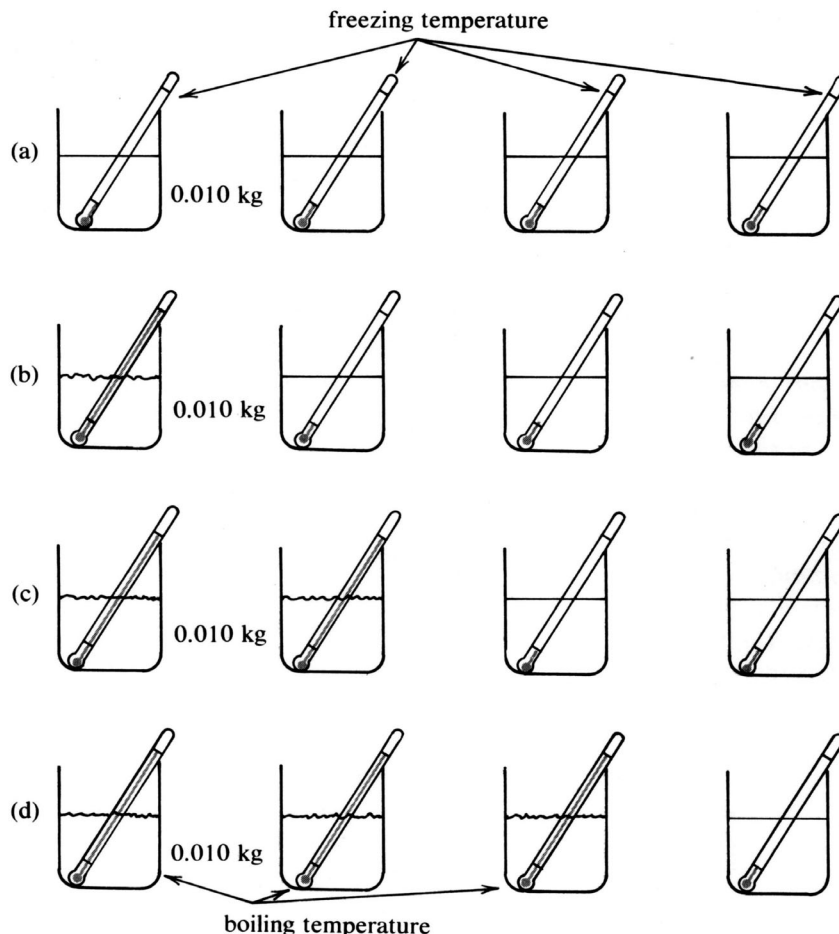
Figure 9.3 A system that provides a thermal energy scale consists of four (or more) 0.010 kilogram samples of water.

(a) All samples are at freezing temperature. The system has 0 Calories of energy,

(b) One sample has been heated to boiling temperature; the three others remain at freezing temperature. The system now has 1 Calorie of energy,

(c) Two samples have been heated to boiling temperature; two remain at freezing temperature. The system now has 2 Calories of energy,

(d) Three samples have been heated to boiling temperature; one remains at freezing temperature. The system now has 3 Calories of energy.



#### Equation 9.1 (Thermal energy)

$$\begin{aligned} \text{mass of ice melted* (kg)} &= M_G \\ \text{energy increase of} \\ \text{system (Cal)} &= E \end{aligned}$$

$$E = 80M_G$$

(\* $M_G$  is a negative number if water freezes)

#### EXAMPLES

a) mass of ice melted = 2.6 kg

$$M_G = 2.6 \text{ kg}$$

$$\begin{aligned} E &= 80 \times 2.6 \text{ Cal} \\ &= 208 \text{ Cal} \end{aligned}$$

b) mass of ice melted = 0.3 kg

$$M_G = 0.3 \text{ kg}$$

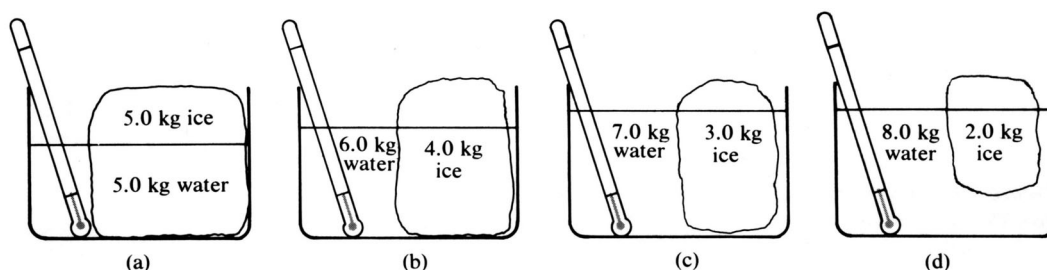
$$\begin{aligned} E &= 80 \times 0.3 \text{ Cal} \\ &= 24 \text{ Cal} \end{aligned}$$

#### Calorie, calorie and kilocalorie

We will use the Calorie (with an uppercase "C") as the unit of thermal energy. The calorie (with a lowercase "c") is a much smaller unit:  $1000 \text{ cal} = 1 \text{ Cal}$ . Many books also refer to the "kilocalorie" ( $=10^3 \text{ cal}$ ), which is simply another name for the Calorie.

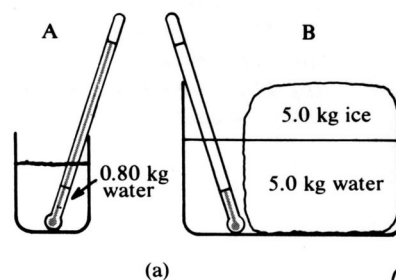
You may ask how the Calorie and the ice-melting unit of energy compare. An experiment to make the comparison is illustrated in Fig. 9.5. The result of this experiment is that 1 ice-melting unit equals 80 Calories. A second question is how the water-heating and ice-melting energy scales compare. In other words, if 1 ice-melting unit equals 80 Calories, are 2 ice-melting units equal to 160 Calories, and so on? The experimental result is that the two energy scales are equivalent, that quantities of energy can be measured on either scale and converted to the other scale, at 80 Calories per ice-melting unit (Eq. 9.1). One Calorie, therefore, is the energy required to melt  $1/80$  kilogram (0.0125 kg) of ice.

**Mechanical energy scales.** Mechanical units of energy may be defined as the elastic energy of a standard spring that is wound up a specified amount, or of a standard rubber band that is stretched a specified amount. Another conceivable mechanical unit of energy is the kinetic energy of an object with an inertial mass of 1 kilogram, moving

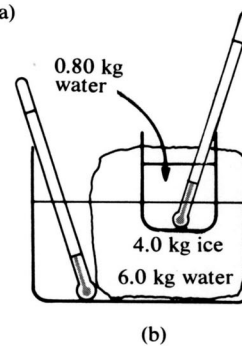


**Figure 9-4** A system that provides a thermal energy scale based on the ice-melting unit consists of 5 kilograms of liquid water and 5 kilograms of ice at equilibrium.

- (a) The system has zero energy.
- (b) The system has 1 unit of energy.
- (c) The system has 2 units of energy.
- (d) The system has 3 units of energy.



(a)



(b)

**Figure 9-5** Comparison of the Calorie and ice-melting units of energy.

- (a) The initial state of the system. Subsystem A at the boiling temperature has an energy of 80 Calories. Subsystem B at the freezing temperature has an energy of 0 ice-melting units.
- (b) The final state of the system. The two subsystems have come to equilibrium at the freezing temperature. Subsystem A has an energy of 0 Calories, subsystem B has an energy of 1.0 ice-melting units.

The 0.10 kg mass (more accurately, 0.102 kg) is chosen for reasons that will be explained in Section 14.4.

with a speed of 1 meter per second. Undoubtedly other possibilities will also occur to you. Energy scales based on two of the examples mentioned are illustrated in Figs. 9.6 and 9.7.

The mechanical energy scale we will use is derived from energy stored in the gravitational field of the earth interacting with certain standard weights. The unit is defined as the energy required to raise a weight with a 0.10 kilogram gravitational mass through a height of 1 meter. This unit of energy is called the *joule*.

When you come to make an energy scale based on the joule, you would seem to have two alternatives: a) to raise the standard weight through various heights (1 meter for 1 joule, 2 meters for 2 joules, and so on, Fig. 9.8a), or b) to use more weights and always raise them through 1 meter (0.20 kilogram for 2 joules, 0.30 kilogram for 3 joules, 0.05

Figure 9.6 A system that provides a mechanical energy scale consists of a set of similar springs.

(a) The system has 0 "spring units" of energy.

(b) The system has 1 "spring unit" of energy.

(c) The system has 2 "spring units" of energy.

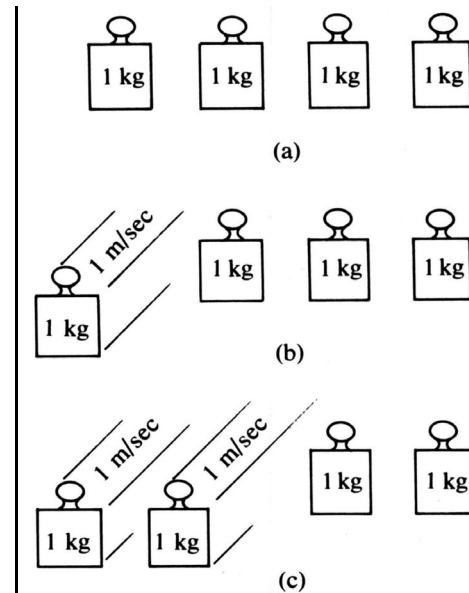
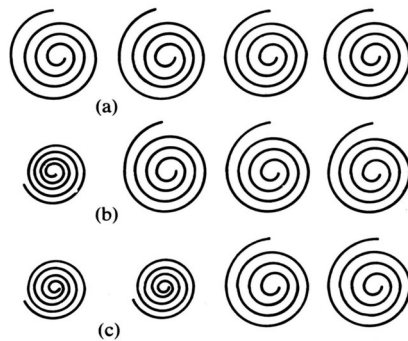


Figure 9.7 (above) A system that provides a mechanical energy scale consists of a set of 1 kilogram weights that can be put in motion at 1 meter per second.

(a) The system has 0 "kinetic units" of energy.

(b) The system has 1 "kinetic unit" of energy.

(c) The system has 2 "kinetic units" of energy.

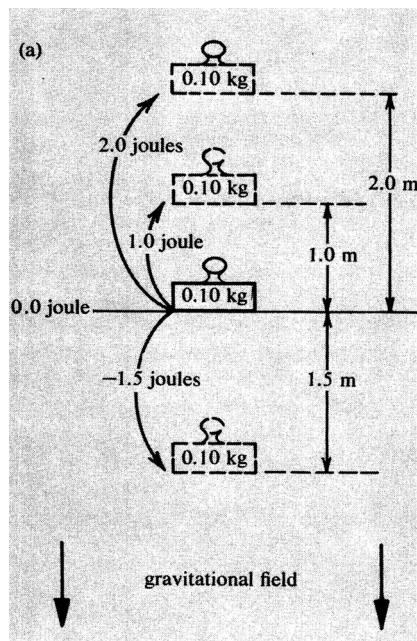
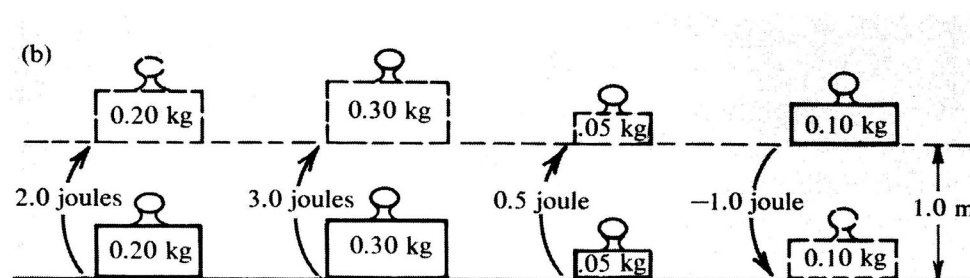


Figure 9.8 Energy scales in joules.

(a) (to left) One weight is raised or lowered by various heights.

(b) (below) Various weights are raised or lowered through 1 meter.



**Equation 9.2 (Mechanical energy)**

gravitational mass of  
the weight (kg) =  $M_G$   
height the weight is  
raised above  
starting level (m) =  $h$   
energy (joules) =  $E$

$$E = \left( \frac{M_G}{0.10} \right) h = 10 M_G h$$

( $h$  is a negative number  
if the weight is lowered  
below starting level.)

**EXAMPLES**

$$M_G = 2.0 \text{ kg}; \quad h = 10 \text{ m}$$

$$E = 10 M_G h$$

$$= 10 \times 2.0 \times 10$$

$$= 200 \text{ joules}$$

$$M_G = 1.0 \text{ kg};$$

$$h = 0.30 \text{ m}$$

$$E = 10 M_G h$$

$$= 10 \times 1.0 \times 0.30$$

$$= 3.0 \text{ joules}$$

**OPERATIONAL  
DEFINITION**

*Thermal energy of a system  
is measured by the mass of  
ice melted as the system  
comes to equilibrium with a  
mixture of water and ice.*

kilogram for 0.5 joule, and so on, as shown in Fig. 9.8b). As in the examples of thermal energy scales, experiments on the energy transfer among various systems of weights interacting with the earth lead to the conclusion that the two alternatives provide equal energy scales. A single mathematical model relates the energy in joules to the mass and height:  $E = 10 M_G h$  (Eq. 9.2). We will use this model and whichever of the two scales in Fig. 9.8 is more convenient. The relationship between the Calorie and the joule will be explained below (Section 9.4).

**9.3 The definitions of energy**

In this section we will introduce two operational definitions of energy that are easy to understand, can be used in practice with only moderate difficulty, and can be used easily in thought experiments. The comparison operation in each definition requires that the energy be transferred to a standard system that provides an energy scale.

**Using the thermal energy scale.** The first operational definition of energy makes use of the thermal energy scale derived from ice melting (Fig. 9.4). We have found that this scale is more convenient than the scale that derives from heating water to its boiling temperature, for this reason: systems at ordinary temperatures can transfer energy to ice and cause it to melt. Such systems, however, could not bring water to its boiling temperature, as would be required for use of the water-heating temperature scale (Fig. 9.3).

Applications of this operational definition are illustrated in Fig. 9.9. Because the water-ice mixture mentioned in the definition is at the freezing temperature of water, any system that is also at this temperature is already at equilibrium with the water-ice mixture and will not melt any ice at all. Systems at this temperature, therefore, have 0 Calories of energy.

**Negative energies.** What is the energy of a system whose initial temperature is *below* the freezing temperature? Fig. 9.9c shows this situation: a very cold chicken from a deep freeze interacts with the water-ice mixture. After the chicken comes to equilibrium with the water-ice mixture, their temperatures are the same; the temperature of the chicken has *increased*. The chicken has gained energy; the water-ice mixture has lost it, and some of the water has frozen and turned to more ice. Thus this process has transferred energy *from* the freezing water *to* the meat. You can weigh the amount of additional ice (say, 0.25 kilogram) and compute the number of Calories transferred from the water-ice mixture to the meat:  $0.25 \text{ kg} \times 80 \text{ Cal/kg} = 20 \text{ Calories}$ .

Now consider the following reasoning. When the meat finally is at the freezing temperature of water, it has 0 Calories. While the meat was warming up and some of the water in the mixture was freezing, energy was being transferred from the water (which was losing energy) to the meat (which was gaining energy). Originally, therefore, the meat must have had *less than* 0 Calories of energy; in other words, its energy was negative. Hence we say that the frozen meat had -20 Calories of energy. More generally, the energy of systems below 0° Celsius is described by a *negative* number of Calories according to our definition.



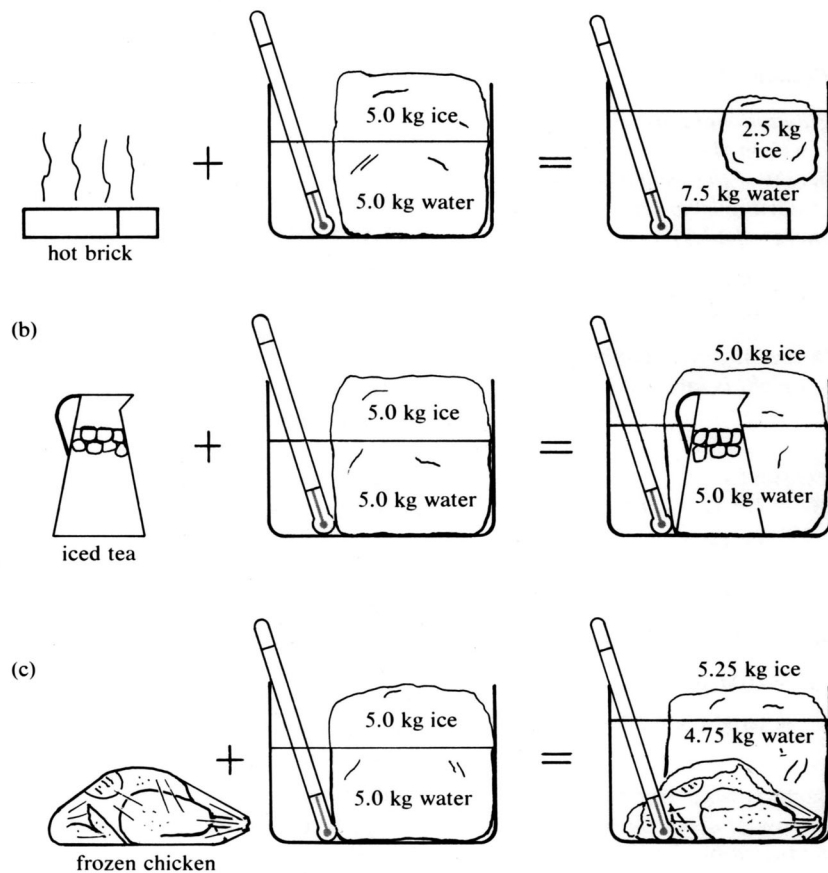


Figure 9.9 Application of the operational definition of energy based on ice melting. Sufficient water and ice must be in the system initially so that some of each remains in the system's final state. 1 Cal = energy to melt 0.0125 kg of ice.

(a) The brick melts 2.5 kilograms of ice. Its energy was 200 Calories.

(b) The iced tea melts no ice. Its energy was 0 Calories.

(c) The frozen chicken causes 0.25 kilogram of water to freeze. Therefore, the original energy of the chicken must have been less than zero (negative), or, more precisely, according to Eq. 9.1,  $-0.25 \text{ kg} \times 80 \text{ Cal/kg} = -20 \text{ Calories}$ .

#### OPERATIONAL DEFINITION

Mechanical energy of a system is measured by the height to which the system can raise a standard weight in gravitational interaction with the earth.

**Using the mechanical energy scale.** The second operational definition of energy makes use of the mechanical energy scale derived from the gravitational field and illustrated in Fig. 9.8. In applying this definition, a suitable coupling element has to be provided so that the energy of the system under study can be transferred completely to the standard weight with a 0.10 kilogram gravitational mass. In conducting real experiments that measure energy, you must minimize the limitations of the actual coupling element. Usually you will be carrying out thought experiments about energy transfer. Then you may assume that the coupling element is perfectly capable of transferring the energy in the desired way. Applications of this operational definition and the mathematical model of Eq. 9.2 are illustrated in Fig. 9.10.

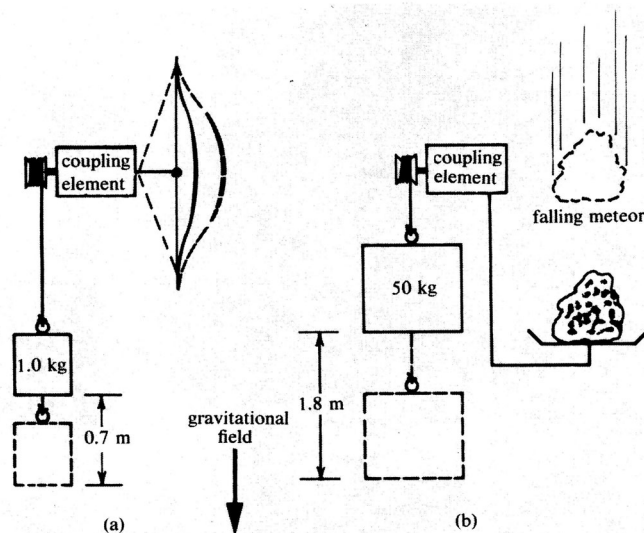


Figure 9.10 Applications of the operational definition of energy.  
 (a) The bow lifts 1 kilogram through 0.7 meter. Its energy was 7 joules.  
 (b) The falling meteor raises 50 kilograms through 1.8 meters. Its energy was 900 joules.

#### 9.4 Comparison of the joule and the Calorie

Since we have defined two energy scales, you may well ask how they are related. If it were possible to transfer energy from a raised weight to a water-ice mixture or vice versa, you could answer this question. Fortunately, the frictional interaction (Fig. 9.11) makes the necessary energy transfer possible. In particular, we can arrange the apparatus shown in Fig. 9.12 to achieve this energy transfer.

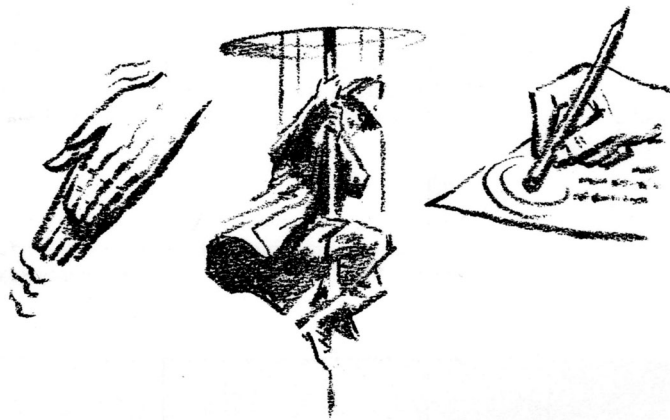


Figure 9.11 Examples of the frictional interaction. The temperature increase in these situations is evidence of friction.

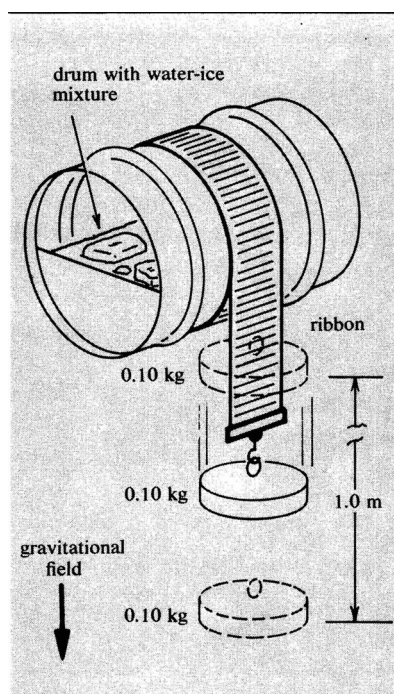


Figure 9.12 Apparatus for a thought experiment to find the number of joules in one Calorie.

The water-ice mixture is contained in a metal drum. A nylon ribbon is wrapped around the drum and has a standard weight hanging from it. The ribbon is pressed against the drum so that the frictional interaction is large enough to let the weight drop only very slowly. As the weight slowly drops through 1 meter, the ribbon rubs against the drum, transfers 1 joule of energy to the ice and water, and causes some ice to melt. This process is repeated until 0.0125 kilogram of ice has been melted. The number of times the process has to be repeated is the number of joules per Calorie.

Because of practical limitations, this specific experiment has never, to our knowledge, actually been carried out just as we have described it. The problem with the method we have described is that the joule turns out to be a very small unit of energy compared to the Calorie; thus, in the experiment above very little ice would be melted.

Nevertheless, over the last 100 years Joule and others have done many very similar experiments with specially designed equipment in which a measured quantity of mechanical energy is transferred, usually via friction, to a measured quantity of heat. Such experiments have established to a high degree of accuracy that the mechanical and thermal energy scales are equivalent with  $4186 \text{ joules} = 1 \text{ Calorie}$ . In this text, we will use the approximate value of  $4000 \text{ joules} = 1 \text{ Calorie}$ .

**Equation 9.3**  
 $1 \text{ Cal} = 4000 \text{ joules}$

### Summary

To measure how much energy is stored in a system, you must conduct an experiment in which the energy is transferred to a standard system that has been chosen to provide the energy scale. Two such standard systems have been selected for the formulation of two operational definitions of energy. In one definition, the standard system is a mixture of water and ice at its equilibrium temperature of  $0^\circ \text{ Celsius}$ . When

**Equation 9.1 (Thermal energy)**

$$\begin{aligned} \text{mass of ice melted (kg)} &= M_G \\ \text{energy increase of} \\ \text{system (Cal)} &= E \end{aligned}$$

$$E = 80M_G$$

( $M_G$  is a negative number if water freezes)

**Equation 9.2 (Mechanical energy)**

$$\begin{aligned} \text{gravitational mass of} \\ \text{the weight (kg)} &= M_G \\ \text{height the weight is} \\ \text{raised above starting} \\ \text{level (m)} &= h \\ \text{energy (joules)} &= E \end{aligned}$$

$$E = \left( \frac{M_G}{0.10} \right) h = 10M_G h$$

( $h$  is a negative number if the weight is lowered below starting level.)

energy is transferred to the system, some ice melts. When energy is transferred from the system, some water freezes. The unit of energy is the Calorie; 1 Calorie is the energy transferred when 0.0125 kilogram of ice melts or 0.0125 kilogram of water freezes (Eq. 9.1).

In the second definition, the standard system includes the earth, any convenient macro-domain object, and their gravitational field. When energy is transferred to this system, the object is raised to a higher level. When energy is transferred from the system, the object is lowered. The unit of energy is the joule. One joule is the energy transferred when an object with gravitational mass of 0.10 kilogram is raised or lowered by 1 meter (Eq. 9.2). Approximately 4000 joules of energy are equal to 1 Calorie.

**List of new terms**

energy scale  
Calorie  
joule

**List of symbols**

$E$  energy  
 $h$  height  
 $M_G$  gravitational mass

**Problems**

1. A 0.01 kilogram candle that burns up completely is observed to melt 1.4 kilograms of ice. Make a mathematical model that relates the energy stored in a candle-air system to the mass of the candle. Compare the use of this model to the procedure for finding the energy illustrated in Fig. 9.2.
2. Compare practical advantages and disadvantages of the energy scales illustrated in Fig. 9.3 and Fig. 9.4.
3. Explain why the legend in Fig. 9.5 can claim that subsystem A has 80 Calories initially.
4. Comment on the use of the gravitational mass (rather than inertial mass) in Eq. 9.1.
5. The energy scale in Fig. 9.4 is expanded to include these states of the water-ice system:
  - (a) 1 kilogram of ice, 9 kilograms of water;
  - (b) 9 kilograms of ice, 1 kilogram of water;
  - (c) 7.5 kilograms of ice, 2.5 kilograms of water;
  - (d) 3.7 kilograms of ice, 6.3 kilograms of water.
 What is the energy of the system in each state?
6. One kilogram of water at the boiling temperature is poured over a 1 kilogram block of ice. Describe the final state of this system, whose mass is 2 kilograms. What is the energy of this system?

7. Point out some advantages and/or limitations of the energy scale illustrated in Figs. 9.6 and 9.7 when compared to that in Fig. 9.8.
8. Invent and describe two mechanical energy scales other than the ones illustrated in the text.
9. Comment on the possibility of obtaining the mathematical model in Eq. 9.2 by thought experiments and/or by real experiments.
10. Apply the concept of "negative stored energy" (Fig. 9.8) to the measurement of mechanical energy. Describe a situation in which this concept might be used.
11. Describe and explain the advantages and limitations of using the three energy scales below:
  - (a) The unit of energy is the energy of a fresh flashlight battery. Two batteries have 2 units of energy, and so on.
  - (b) The unit of energy is the energy of a paper match-air system. Two matches (plus air) have 2 units of energy, and so on.
  - (c) The unit of energy is the energy of 1 kilogram of matter according to Einstein's relation (Eq. 8.11). First, calculate the magnitude of this unit when expressed in joules.
12. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

**Equation 8.11**

$$\begin{array}{ll} \text{energy (joules)} & = \Delta E \\ \text{change in mass (kg)} & = M \\ \text{speed of light (m/sec)} & = c \end{array}$$

$$\Delta E = M c^2$$

