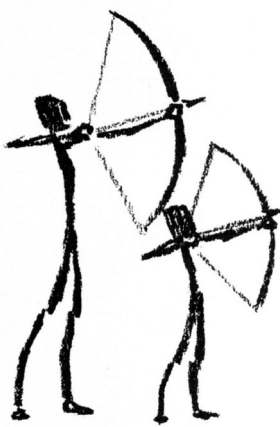


Chapter 11:

Force Displacement and Energy

The archer bending a longbow, the ski tow pulling a skier, the fireman sliding down a brass pole, the housekeeper pulling a magnet off the front of a refrigerator, and the child hopping on a pogo stick are all examples of energy transfer from among the phenomena in our environment. The archer serves as the energy source for the elastic energy of the longbow, with the string acting as the coupling element. The gravitational field of the fireman and the earth serves as energy source for the brass pole and the fireman's hands, with the frictional interaction transferring the energy. And so on.

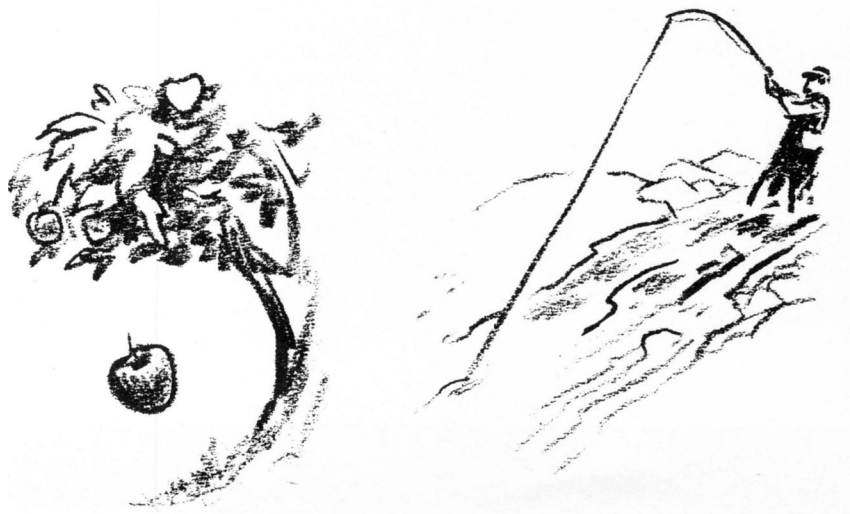
11.1 Factors in energy transfer



We will now analyze thought experiments of these two examples in more detail, with a view toward making a mathematical model of the energy transfer being accomplished by the interaction. It is clear that the more the bow is bent by displacement of the center of the string, the more elastic energy it stores (Fig. 9.1). The elastic energy can be measured roughly, though not in the joule unit, by the distance the arrow travels when it is released. It is also clear that a child's bow, which can be bent by a weaker arm, stores less energy even though it may be bent to the same extent. It appears, therefore, that a mathematical model for the energy of the bow and the energy transfer to the arrow must take into account two factors: the displacement of the string tied to the bow and the strength of interaction required to bend the bow.

An analysis of the fireman sliding down his brass pole leads to the same conclusion. The taller the pole, the more thermal energy will be created in the brass pole and the fireman's hands. Also, the more massive the fireman, the more tightly he will have to cling to the pole if he is to avoid breaking his legs on impact with the floor, and the hotter his

Figure 11.1 Examples of forces. The earth exerts a gravitational force on the falling apple. The bent fishing rod exerts an elastic force on the fishing line.



hands will become. Both the displacement of the fireman and the strength of his interaction with the pole must therefore be taken into account in a mathematical model for energy transfer in this example.

Force and work. We have mentioned the notion of interaction strength before. In Chapter 3 it came up as determining the magnitude of the response to interaction of an object or system that has inertia. We will introduce force as the measure of interaction strength that is appropriate to the archer bending his bow, the fireman sliding down his pole, the ski lift, and so on—that is, whenever energy transfer accompanies the displacement of an interacting object. This energy transfer has been given a name of its own; it is called work. In this chapter, therefore, we will make a mathematical model that relates work to force and displacement.

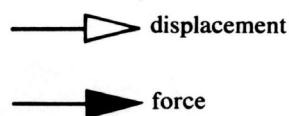
11.2 Interaction and force

The word "force" is used in everyday language to signify compulsion, either physical or mental. In scientific use, the word "force" has a narrow and quite specific meaning distinct from its everyday meaning. Rather than attempting to present a concise formal definition of force, we will use examples to show how the concept of force grows out of the more general concepts of interaction and energy.

The force concept. Isaac Newton (1642-1727) introduced the concept of force in his brilliant formulation of a theory for the motion of rigid bodies. In this theory, which we will present in Chapter 14, Newton ascribed the changes in motion of a body to a net (unbalanced) force acting on the body. Newton realized that several interactions might compensate for one another and produce no change of motion, as when the hands hold the bow and arrow ready for shooting. Only when the hand relaxes its hold does a net force exerted by the string on the arrow set the latter in motion.

Examples of force are illustrated in Fig. 11.1. You can see in Fig. 11.1 that a force has to be described not only by a magnitude (or strength) but also by a direction in space. Thus, the gravitational force exerted by the earth on an apple acts downward; the bent fishing rod exerts an upward-directed force on the line; the flowing creek water exerts a downstream force on the line; and so on. In this respect the force concept resembles the relative position and displacement concepts, which were described by a distance and a direction in space (Sections 2.1 and 2.3). Force will therefore be represented by a boldface letter **F** in print, by a letter **F** with an arrow over it in writing, and by an arrow in a diagram. The magnitude of the force is represented by the symbol $|\mathbf{F}|$, where the vertical bars symbolize that only the numerical strength of the force is important and that the direction of the force is to be disregarded. Forces can be combined (added, multiplied by numbers, and so on) in the same way that displacements are combined by arithmetic operations or by diagrammatic manipulation of the arrows representing them.

Force arrows will be drawn differently from position and displacement arrows.



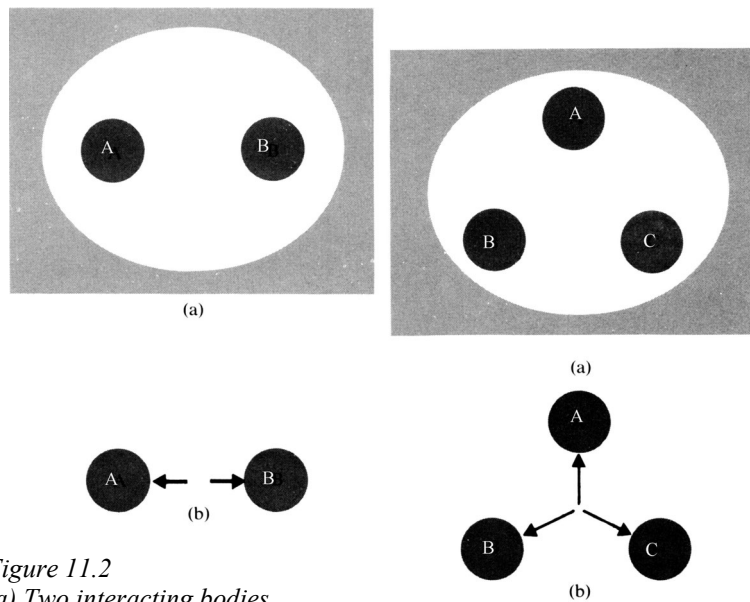


Figure 11.2

(a) Two interacting bodies
 (b) The two forces of interaction, represented by arrows. One force acts on each body.

Figure 11.3

(a) Three interacting bodies
 (b) The three forces of interaction, represented by arrows. One force acts on each body.

The Newtonian theory requires us to think about interaction between objects in a different way than we have so far. We have emphasized the *mutuality* of the interaction and have drawn your attention to the entire system of interacting objects, including the "field" which transmits the interaction. However, in the Newtonian point of view, we must focus our attention on one body at a time in order to study its motion. We think of interaction between two objects A and B as simply two forces: one net force acting on A (exerted by B), and a second net force acting on B (exerted by A). These two forces are indicated by arrows in Fig. 11.2. In a three-body system there are three forces, as shown in Fig. 11.3; one net force acts on A and is exerted by the subsystem composed of B and C, another net force acts on B and is exerted by the subsystem composed of A and C, and the third net force acts on C and is exerted by the subsystem A and B. In a four-body system there are four net forces, and so on. The net force acting on an object in a system of other objects is always exerted by a subsystem that includes all the *other* objects but not the object itself. In Newtonian theory, an object never exerts a net force on itself.

Combination of forces. You may find it difficult to visualize the net force exerted on body A by the subsystem composed of B and C in Fig. 11.3. However, you probably find it easier to think of the net force exerted by just one body B on A in Fig. 11.2. The sun-earth-moon system

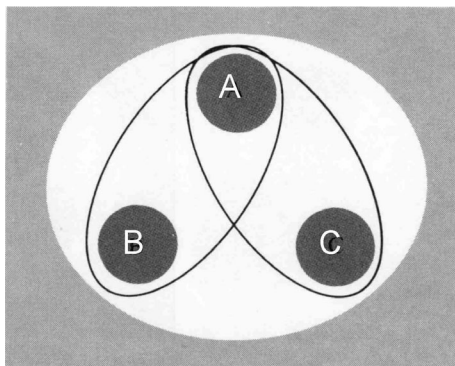


Figure 11.4 Two overlapping subsystems of the three-body system A-B-C.

is an example in which the moon is subject to the net force exerted by the sun-earth subsystem.

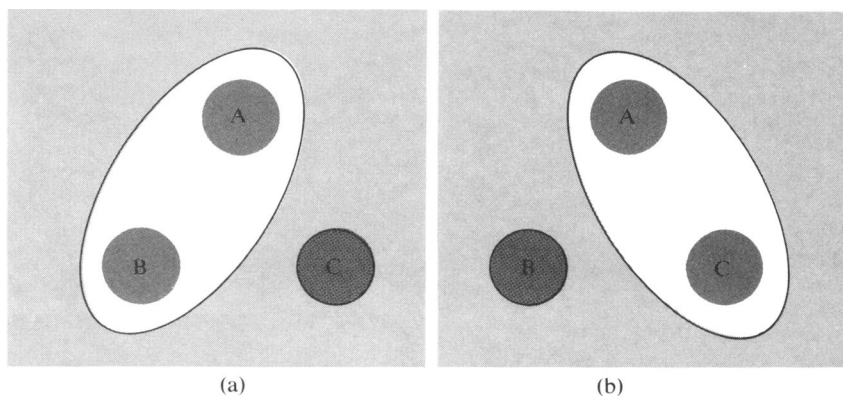
Now, the Newtonian method for dealing with such a three-body system is to simplify it by imagining the three-body system to be made up of three overlapping two-body subsystems (Fig. 11.4). There is an interaction within each two-body subsystem, and therefore a force exerted on A by B (in the A-B subsystem) and a force exerted on A by C (in the A-C subsystem) (Fig. 11.5). In the Newtonian theory it is assumed that each of these interactions is *unaffected by the presence of the third body*. In the astronomical example above, it is assumed that the earth-moon interaction is not affected by the sun, and the sun-moon interaction is not affected by the earth. This step may or may not seem appropriate to you, but it is central to grasping Newtonian theory.

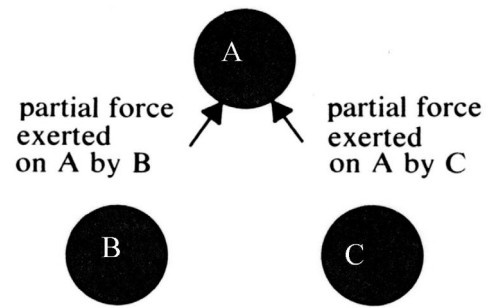
The next step is to represent the two partial forces acting on A (exerted by B and by C) by arrows (Fig. 11.6a). The last step in finding the net force on A is to combine all the partial forces according to the procedure described in Section 2.3 for the addition of displacements.

Figure 11.5 Analysis of forces in a three-body system.

(a) The interaction of A and B in the three-body system (Fig. 11.4) is assumed equal to the interaction of A and B in the two-body subsystem.

(b) The interaction of A and C in the three-body system (Fig. 11.4) is assumed equal to the interaction of A and C in the two-body subsystem.



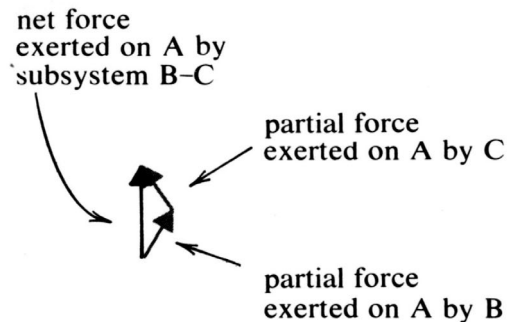


(a)

Figure 11.6 Partial and net forces in a three-body system.

(a, above) Newtonian diagram for partial forces exerted on A by B and by C separately.

(b, below) Addition of arrows that represent the partial forces acting on A to find the net force on A.



(b)

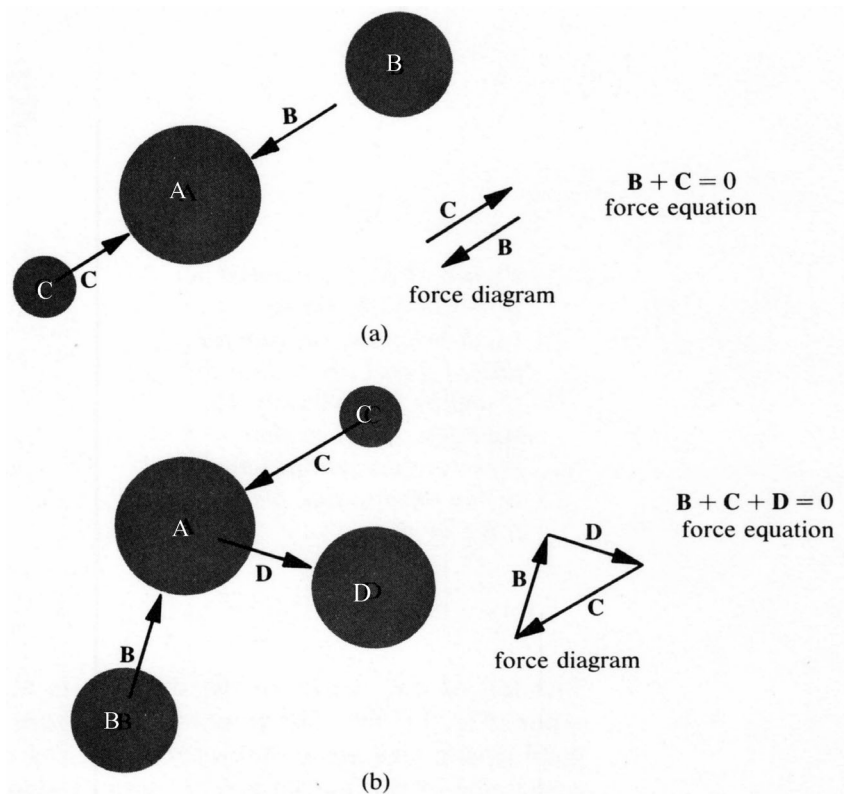
For an example of a body in equilibrium and in motion at the same time, think of your luncheon tray on a jet airliner moving at a constant 600 miles per hour relative to the ground. (What happens when the motion of the airliner changes suddenly?)

The tail of one arrow in the diagram is attached to the head of the other (Fig. 11.6b). The overall arrow from the tail of the first to the head of the second is the net force acting on body A. The net force is the sum of the partial forces, with directions and magnitudes of the partial forces being taken into account. The combination of forces is described further at the end of this section.

Mechanical equilibrium. If a body is not subject to a net force, then its motion is steady and does not change. Such a body is said to be in mechanical equilibrium, and the partial forces acting on it are called balanced. The apples on the tree, the fisherman's boots, and the steadily flowing brook water in Fig. 11.1 are in mechanical equilibrium. The falling apple, the turbulent brook water, and a boy swinging nearby are not. What about the tip of the fishing rod?

If a body in equilibrium is at rest, it remains at rest; if the body in equilibrium is in motion, it remains in steady motion with a constant speed in the same direction. This is not really a new result, because one kind of evidence of interaction described in Section 3.4 was the change in motion of interacting objects. In that section we pointed out that a body might not exhibit any change in motion, even though it is subject to interactions, if the interactions compensate. Alternate ways of describing this condition are mechanical equilibrium, a zero net force, and the balance of the partial forces acting on the body.

Figure 11.7 Body *A* is in mechanical equilibrium. Symbols **B**, **C**, and **D** represent the forces exerted on *A* by the corresponding bodies,
 (a) Two balanced forces of repulsion,
 (b) Three balanced forces.



Addition of partial forces. The net force is absent (or equal to zero) if the arrow representing it in a diagram has zero length. This condition is achieved if the partial forces are represented by arrows that form a closed chain, with the head of the last arrow reaching the tail of the first arrow (Fig. 11.7). If only two partial forces are acting on a body in mechanical equilibrium, then the two must be represented by arrows of equal length and opposite direction (Fig. 11.7a). Three partial forces form a triangular diagram (Fig. 11.7b), and so on. Thus Fig. 11.7 illustrates sets of partial forces that are balanced. A body that is not in mechanical equilibrium (i.e., it is subject to a net force) can be brought to equilibrium by the application of an additional force that is equal in magnitude and opposite in direction to the net force.

Origin of the forces. This discussion of mechanical equilibrium has referred to a single body and the partial forces acting on it. The origin of the forces was not mentioned. Nevertheless, forces are a measure of interaction strength, and you should be aware of the interactions that operate even though you temporarily concentrate your attention on the motion or lack of motion of one particular body in the system. In particular, the interaction will affect the motion of the other bodies in the system.

Operational definition of force. The calculation of the net force acting on an object is based on measurements of the partial forces, and such measurements are in turn based on an operational definition of

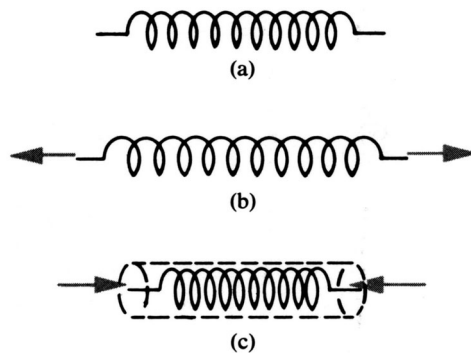


Figure 11.8 Springs can be used to measure forces,
 (a) A spring not subject to forces,
 (b) The same spring deformed by stretching,
 (c) The same spring deformed by compression. (Compressed springs are usually kept inside a tube to prevent them from bulging to the side.)

force. We will now construct such an operational definition by selecting a force measuring device, calibrating it in standard units of force called *newtons*, and describing a procedure for using the device.

Any system that gives reliable and reproducible visual evidence of the action of a force can be used as force measuring device. A bow can be used, or a spring, or a rubber band. All these systems show a deformation when subject to a force, and return to their undeformed state when the force ceases to act. They are examples of elastic systems and will be discussed in greater detail in Section 11.6.

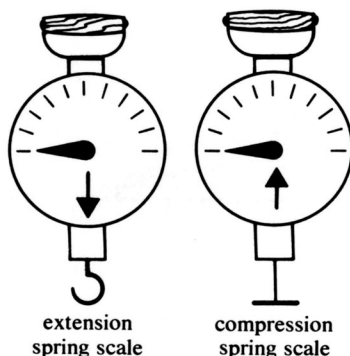
Standard spring scale. For convenience, we will select springs as force measuring devices. Springs may be deformed by stretching or by compression (Fig. 11.8). To calibrate a spring, we attach objects of various masses to the end of the spring and allow them to hang freely and vertically (no swinging) in the gravitational field of the earth. Such an object is subject to two partial forces: the downward force exerted by the earth via the gravitational field and the upward elastic force exerted by the spring. After such an object is allowed to come to mechanical equilibrium (that is, to stop bouncing), the spring is stretched just enough so that the force it exerts is equal in magnitude to the gravitational force.

It would be natural to choose a 1 kilogram mass as the standard object for one unit of force. For historical reasons we will not do this, but instead, we will choose a weight with a gravitational mass of 0.10 kilogram. This is the same standard object used to define the joule in Section 9.2. We will explain this apparently arbitrary choice in Chapter 14.

The unit of force on our scale is called a *newton*. The scale is marked 1 newton when a 0.10 kilogram weight hangs on the spring, 2 newtons when two such weights hang on the spring, and so on (Fig. 11.9). The spring with a scale is called a *spring scale*.

Definition. The operational definition of force employs a spring scale calibrated in newtons. When the spring scale is used to measure the net force acting on a body, it is attached to the body and used to hold it in mechanical equilibrium (Fig. 11.10). The magnitude of the net force is then equal to the scale reading. The direction of the net force is the direction in which the spring scale is extended. For the sake of simplifying the diagrams, we will henceforth represent spring scales by dials without numerical scale indications. The scale reading will be printed near the dial.

OPERATIONAL DEFINITION
 Force is measured by a standard spring scale. The magnitude of the force is indicated by the scale reading. The direction of the force is indicated by the direction of the spring scale.



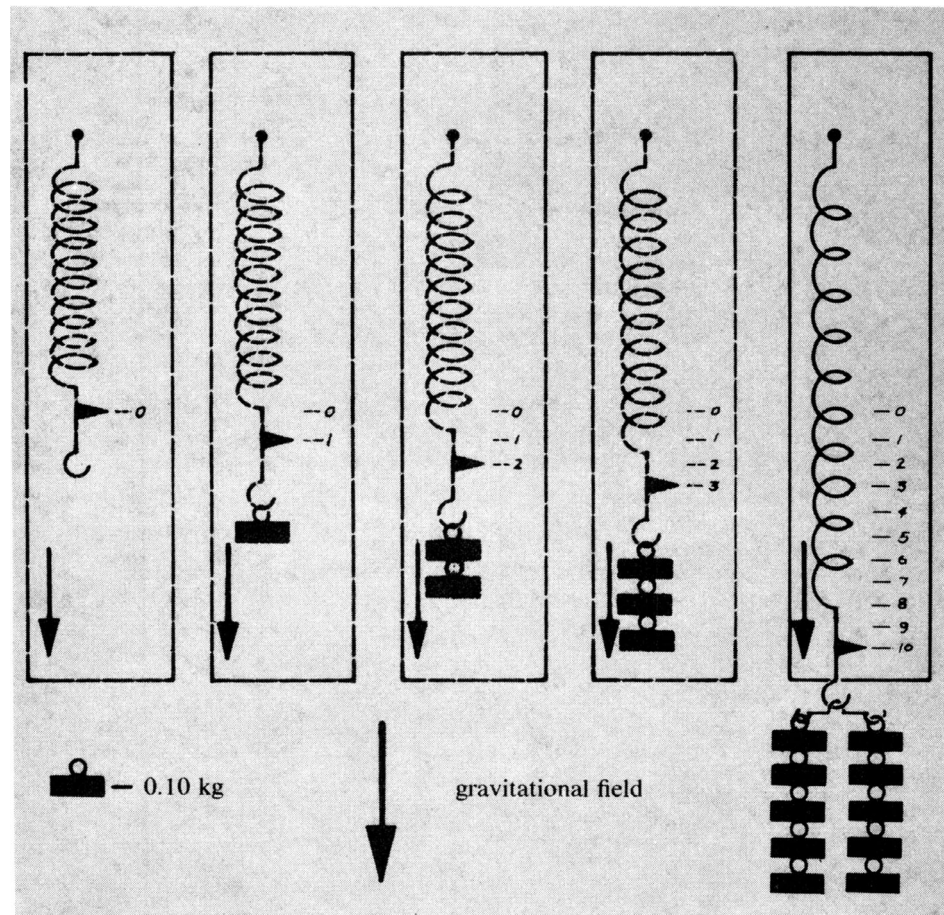


Figure 11.9 Calibration of a standard spring scale with weights having a mass of 0.10 kilogram. The arrow represents the direction of the force exerted on the spring scale.

Partial forces. The spring scale can also be used to measure partial forces acting on a body, like the force exerted on the fishing rod by the fisherman's left hand in Fig. 11.1. To do this, you only have to let the spring scale, instead of the man's left hand, hold the fishing rod in the same position. The force exerted by the spring scale then replaces and is equal to the force exerted by the hand. Other examples of the substitution method for measuring partial forces are shown in Fig. 11.11. Frequently these "measurements" are only carried out in thought experiments and the actual data come from the interpretation of indirect evidence.

Figure 11.10 The net force acting on the wagon in the absence of the spring scale is 2.1 newtons to the right. The partial forces acting on the wagon are exerted by the mouse (to right), the table (up), the earth (down), and the spring (to left).

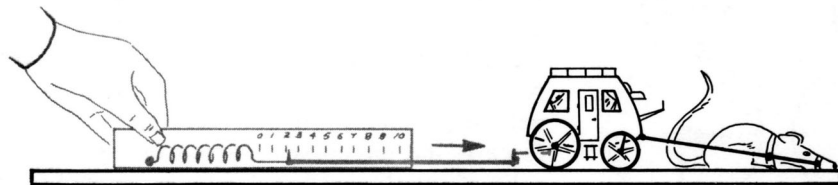
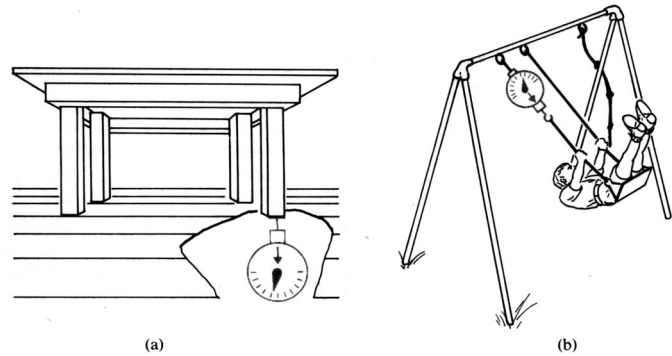


Figure 11.11 The substitution method for measuring partial forces.
 (a) Compression spring scale is used to measure the partial force exerted by the floor on one table leg.
 (b) Extension spring scale is used to measure the partial force exerted by the swing frame on the rope.



Addition of partial forces. Now that we have introduced a procedure for measuring forces, we can repeat some of the earlier explanations with quantitative illustrations. It is usually helpful to begin by making a force diagram, with an arrow for each partial force; the length of each arrow is scaled to the magnitude of the force (see Figs. 11.6 and 11.7). Since this is a force diagram and not a diagram or map of the position of the bodies on which the forces act, the force arrows can be drawn anywhere, as long as they have the correct magnitude and direction. To find the net force, draw one force, then draw the second with its tail attached to the head of the first, the third with its tail attached to the head of the second, and so on, until all forces have been added. Then draw an arrow from the *tail* of the first force to the *head* of the last one (Fig. 11.6b); this is the net force.

An alternative method of adding forces using arithmetic is to find and add the corresponding *rectangular components* of the forces. This approach requires the introduction of a rectangular coordinate frame (Fig. 11.12). The x component of the net force is then the sum of the x components, and the y component of the net force is the sum of the y components (Eq. 11.1). We recommend that you choose the x- or y-coordinate axes in the same direction as one or more of the forces (Fig. 11.13). If you do this, the components of these forces are especially easy to find. The components of forces not in the direction of the axis may be found graphically or with the sine and cosine functions.

It is also possible to use the method of addition of forces to find an

Equation 11.1

(Forces are defined in Fig. 11.12.)

$$\mathbf{F}' + \mathbf{F}'' + \mathbf{F}''' = \mathbf{F}$$

$$[1, 1] + [-2, 4] + [-3.5, 0] = [-4.5, 5.0]$$

$$\mathbf{F} = [-4.5, 5.0] \text{ newtons}$$

Figure 11.12 Three partial forces \mathbf{F}' , \mathbf{F}'' , \mathbf{F}''' and the net force \mathbf{F} with the following rectangular components:

$$\mathbf{F}' = [1.0, 1.0] \text{ newtons}$$

$$\mathbf{F}'' = [-2.0, 4.0] \text{ newtons}$$

$$\mathbf{F}''' = [-3.5, 0.0] \text{ newtons}$$

$$\mathbf{F} = [-4.5, 5.0] \text{ newtons}$$

(see Eq. 11.1).

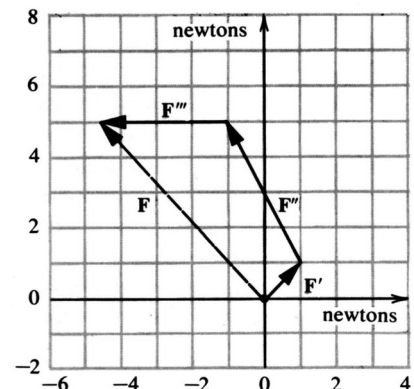
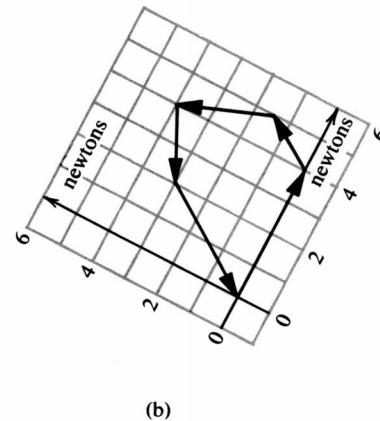
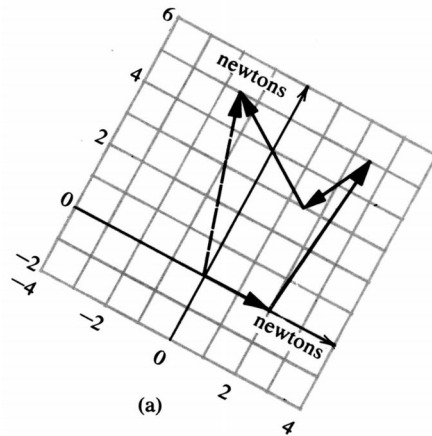


Figure 11.13 Examples of the addition of forces:

(a) partial forces:
 $[2.0, 0.0]$ newtons
 $[0.5, 5.0]$ newtons
 $[-1.0, -2.0]$ newtons
 $[-3.0, 2.0]$ newtons
 net force:
 $[-1.5, 5.0]$ newtons

(b) partial forces:
 $[4.0, 0.0]$ newtons
 $[1.0, 1.5]$ newtons
 $[-1.0, 2.5]$ newtons
 $[-2.0, -1.0]$ newtons
 $[-2.0, -3.0]$ newtons
 net force:
 $[0.0, 0.0]$ newtons

Note: The coordinate axes are purposely oriented at an angle, so that the x-axis is in the same direction as one of the forces. This is a good way to simplify the calculation. In fact, this technique of setting up the coordinate axes to fit the problem can be a powerful aide in problem solving.



unknown partial force if the net force is known. This problem is similar to the earlier one in which, given the first part of a sailing trip and the destination, we showed how to use subtraction of displacements to find the displacement needed to complete the trip (Section 2.3, Fig. 2.19). A problem of this kind with forces is worked out below (Fig. 11.14).

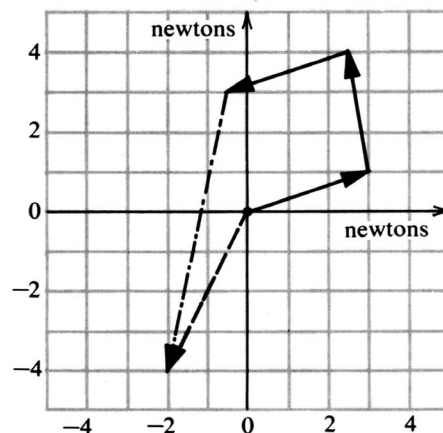


Figure 11.14 Example of how to find one partial force when the net force and the other partial forces are known.

net force: $[-2.0, -4.0]$ newtons
 partial forces:
 $[3.0, 1.0]$ newtons
 $[-0.5, 3.0]$ newtons
 $[-3.0, -1.0]$ newtons
 missing partial force :
 $[-7.5, -7.0]$ newtons

11.3 The gravitational force

Equation 11.2

force of gravity (at
surface of earth)
(magnitude, newtons)
 $= |\mathbf{F}_G|$
gravitational mass (kg)
 $= M_G$

$$|\mathbf{F}_G| = \frac{M_G}{0.10} = 10M_G$$

EXAMPLES:

(a) Force of gravity on
a person.

$$M_G = 60 \text{ kg}$$

$$\begin{aligned} |\mathbf{F}_G| &= 10M_G \\ &= 10 \times 60 \text{ newtons} \\ &= 600 \text{ newtons} \end{aligned}$$

(b) Force of gravity on
quarter pound
(4-ounce) hamburger.

$$\begin{aligned} M_G &= 0.11 \text{ kg} \\ |\mathbf{F}_G| &= 10M_G \\ &= 10 \times 0.11 \text{ newtons} \\ &= 1.1 \text{ newtons} \end{aligned}$$

Equation 11.3

gravitational intensity
 \mathbf{g} at the surface of
the earth.

$$\begin{aligned} \mathbf{g} &= (10 \text{ newtons / kg,} \\ &\quad \text{downward}) \\ |\mathbf{g}| &= 10 \text{ newtons / kg} \end{aligned}$$

The gravitational interaction between the earth and objects near its surface is very important for all of us who dwell on the earth. You can measure the force of gravity exerted by the earth on any object free of other interactions by hanging the object from a spring scale. Then the force exerted by the spring scale compensates the force of gravity and holds the object in mechanical equilibrium.

We have already described how you may use a plumb line to define the direction of vertical (p. 15). In comparing this with the definition of force, you see that the vertical is precisely the direction of the force of gravity exerted by the earth on any nearby object. If you were to carry out the force measurement at various places on the earth, you would find that the direction is toward the center of the earth and that the magnitude is the same.

We will use the boldface symbol \mathbf{F}_G for the force of gravity on an object. The directions of \mathbf{F}_G at various places on the earth's surface are shown in Fig. 11.15. The magnitude of the force of gravity on an object with mass of 0.10 kilogram is, by definition, 1 newton. The force on other objects is described by the following mathematical model: $|\mathbf{F}| = 10M_G$ (Eq. 11.2).

These spring scale measurements can be made with the object at rest or in steady, or uniform, motion relative to the earth's surface (Fig. 11.16). However, if the motion of the object changes, then a spring scale does not measure the net force (Fig. 11.17). These statements can be tested experimentally.

Gravitational intensity. The strength of the gravitational interaction is close to 10 newtons per kilogram, regardless of the shape, composition, or other properties of the object, and it is directed vertically downward. The symbol \mathbf{g} is used to denote the gravitational force per unit mass, and it is called the *gravitational intensity* (Eq. 11.3). The relation of gravitational force, gravitational intensity, and mass is given by

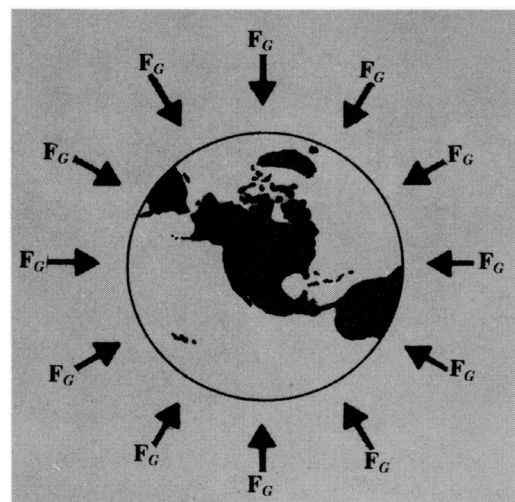


Figure 11.15 The force of gravity at various locations on the earth.

Equation 11.4 (force of gravity at surface of earth)

force of gravity
(newtons) = F_G
gravitational mass
(kg) = M_G
gravitational intensity
(newtons/kg) = g
at surface of earth:
 $g = (10 \text{ newtons/kg, downward})$

$$F_G = g M_G$$

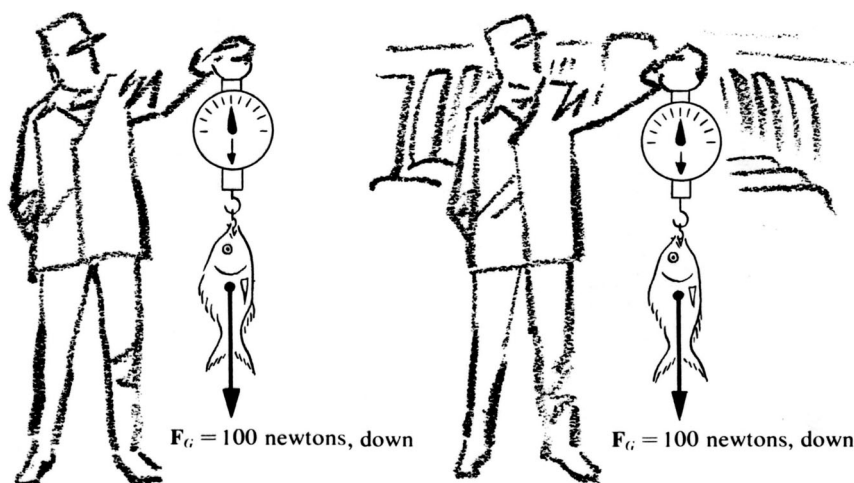


Figure 11.16 (above) The force of gravity acting on the fish is measured, properly, as 100 newtons whether it is being weighed inside a train moving at constant speed in a straight line (at right) or at rest in the station (at left).. In both cases, the fish and scale are moving at constant velocity relative to a reference frame fixed on the surface of the earth. The fish is in mechanical equilibrium in both situations. The essential similarity of these two situations is the basis of Newton's first law and will be explained in more detail in Section 14.2.

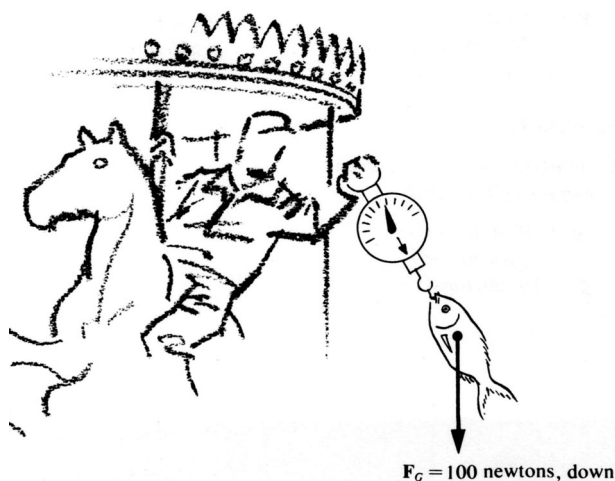
TABLE 11.1
GRAVITATIONAL
INTENSITY (g)

Location	gravita- tional intensity (g , new- tons/kg)
earth's surface	10
sun's surface	270
moon's surface	1.7

Eq. 11.4, which is a restatement of Eq. 11.2. On the sun and the moon, the gravitational intensity has different values than it does at the earth's surface (Table 11.1).

You may wonder whether the fact that the force of gravity is proportional to the gravitational mass is a law of nature, or whether it is a result of our definition of the magnitude of the force. Actually, it is the latter. After all, we calibrated the spring scale by hanging objects on it while they were subject only to the earth's gravitational field and we marked force units in proportion to the mass, 1 newton for every 0.10 kilogram.

Figure 11.17 (to right)
The force of gravity acting on the fish is not measured correctly by the spring scale under the conditions shown. As the carousel turns, the velocity of the fish is not constant but is changing in direction (though not magnitude). As a result, the fish is accelerating and not in equilibrium.



11.4 Work

WORK IN PHYSICS

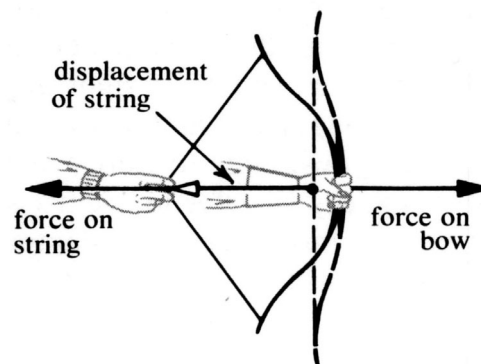
Energy transfer resulting from a force acting on an object that is displaced in the direction of the force.

The concept of *work* represents a key connection between two different styles of thought and two different types of theory: 1) A "holistic," "systems," or "field theory" approach based on concepts such as energy, waves, and fields (gravitational, electric and magnetic). These concepts all apply to a system as a whole or to an extended region of space. 2) A Newtonian, particle-based approach, based on breaking a system down into its parts (each piece reduced to a simple mass-point at a defined position in space), analyzing the interactions between the various particles, identifying all the forces acting on any individual particle, finding the net force on that particle, and repeating this procedure for each particle in the system so as to build up an understanding of the system as a whole.

In Chapters 1-10, we explained the concepts of interaction and energy and how to apply them in many situations. In particular, we have emphasized how the interaction of a system of objects can be thought of as arising from various types of "fields," (gravitational, magnetic, electric and so on). We have focused on interactions between objects as phenomena in which all of the objects participate, as part of a system in a shared, mutual way. This has involved, to a large extent, a "holistic" or "systems" style of thinking, as described above.

In this Chapter, we have introduced the concept of "force," which requires thinking about the interacting objects individually and then two-by-two so as to identify and add up all of the forces acting on a particular object. This style of thinking is very different from much of what we have explained so far, but it is essential to understanding Newtonian theory. This theory, in turn, provides a powerful tool with which to analyze many of the systems we have already encountered, leading to additional models and increased understanding.

More specifically, we will now show how work connects the Newtonian concept of force (defined in Section 11.2) with the concept of energy (a more "systems-oriented" idea, as defined in Sections 9.2 to 9.4). We will first provide a general context for this new concept; we will then define work formally as well as with a mathematical model, explain in detail how to calculate it, and show that this calculation yields the same numerical result as do our earlier calculations of energy changes.



*Figure 11.18
Energy transfer to the bow occurs during displacement of the center of the string, where the archer's hand exerts a force.*

FORMAL DEFINITION

Work is the product of the magnitude of the force and the displacement component in the direction of the force.

Equation 11.5 (Work)

$$\begin{aligned} \text{work (joules)} &= W \\ \text{force magnitude} &= |\mathbf{F}| \\ \text{displacement magnitude} &= \Delta s \\ \text{in force direction} &= \Delta s_F \\ (m) &= \Delta s_F \\ W &= |\mathbf{F}| \Delta s_F \end{aligned}$$

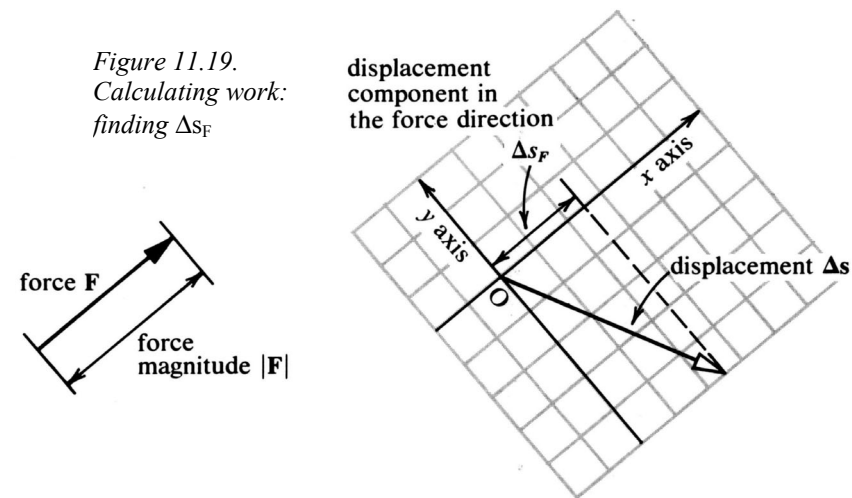
Detailed procedure for calculating work (W), illustrated in Figure 11.19 to right.

1. Identify the force and draw the force arrow, \mathbf{F} , pointing in approximately the correct direction. Find the magnitude of the force in newtons. (Don't draw the coordinate frame first; the calculation is simpler if you do that in the next step!)
2. Now draw the coordinate frame with the x-axis parallel to the force arrow;
3. Identify the direction and magnitude of the displacement ($\Delta \mathbf{s}$);
4. Draw $\Delta \mathbf{s}$, the displacement arrow, on the coordinate frame with its tail starting at the origin;
5. Draw the dashed line from the point of $\Delta \mathbf{s}$ parallel to the y-axis until it crosses the x-axis;
6. Draw and measure (or calculate) the length of Δs_F (the x-component of $\Delta \mathbf{s}$).
7. Calculate W from Eq. 11.5 ($W = |\mathbf{F}| \Delta s_F$).

We gave an informal definition of work above in Section 11.1: the energy transfer that "accompanies the displacement of an interacting object." To be more precise, work refers to the energy transfer resulting from a force acting on an object that is *displaced in the direction of the force*. For example, when the archer bends his bow, one hand interacts with the center of the string and one hand interacts with the center of the bow. Both hands exert forces, one on the string, the other on the bow. But only the center of the string is displaced; the arm holding the bow is rigid (Fig. 11.18). The energy transferred from the archer to the bow and string system is called the work done by the force his hand exerted on the string. The force exerted on the bow did no work because the center of the bow was not displaced.

Definition of work. We now present a more formal definition of work that can be used in calculations: *Work is the product of the magnitude of the force and the displacement component in the direction of the force.*

This definition is restated to the left as a mathematical model (Equation 11.5). The overall procedure for calculating work is to identify the force, find its magnitude, identify the displacement, find its component in the force direction, and multiply these two numbers. This procedure is illustrated in figure 11.19 and summarized in the left margin.



There are two important points to keep in mind when using the above definition to calculate work. First, *both* force and displacement have a direction in space. The work done depends on the relative direction of the force and the displacement and does not change if both arrows are reoriented without changing their relationship in space.

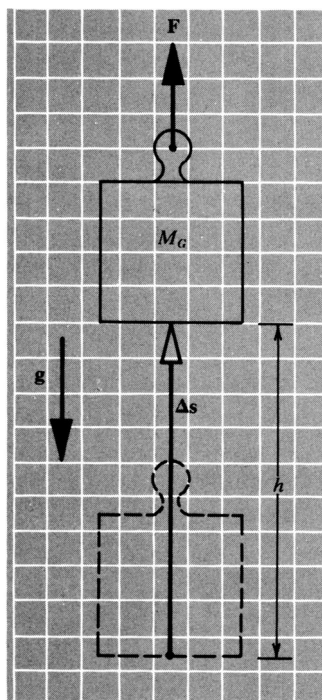
Second, the work concept can be associated with any force, net or partial. When several forces are acting on a body, you can use the definition to find the work done by each partial force and you can find the work done by the net force. In the example of the bent bow, for instance, the net force on the bow and string was zero; hence, it did zero work. The partial force acting on the bow also did zero work, because the center of the bow

was not displaced. Only the partial force acting on the string did work. This example shows that the work done by the net force acting on a system may *not* be equal to the sum total of the work done by all the partial forces acting on the same system.

Work is a numerical quantity measured in joules; it does *not* have a direction in space. Zero work is done when the displacement is zero. Zero work is also done when the displacement is at right angles to the force, for then the displacement component in the direction of the force is zero.

We now must verify that this definition of work really represents energy transfer in accord with at least one of the operational definitions of energy explained in Chapter 9. This is most easily accomplished for the definition based on raising an object in the gravitational field, in which energy is measured in joules (Eq. 9.2, $E = 10M_Gh$, Fig. 9.8).

Example 11.1 below shows how to calculate the work done when a weight is raised or lowered and the energy stored in the gravitational field is changed. You can see that this result is in agreement with the mathematical model (Eq. 9.2) based on the operational definition. From now on we will use Eq. 11.5 to describe energy transfer whenever this occurs in the form of work, and we will not use the operational definition of energy in joules except for illustrative purposes.



EXAMPLE 11.1. How much work is done when a weight is raised? Is the work equal to the energy transfer described by Eq. 9.2?

Solution: We must find the force and the displacement to be able to use the definition of work, Eq. 11.5. The important step is to think of raising the weight so slowly that it is always very close to mechanical equilibrium and does not acquire kinetic energy. Then the force acting on the weight is equal in magnitude, opposite in direction, to the force of gravity (Eq. 11.4),

$$|\mathbf{F}| = |\mathbf{F}_G| = |\mathbf{g}| M_G = 10 M_G$$

The displacement (see diagram to left) has the magnitude h and is directed along the force. Hence the displacement component in the force direction is equal to h ,

$$\Delta s_F = h$$

The work done is $W = |\mathbf{F}|\Delta s_F = |\mathbf{g}| M_G h = 10 M_G h$ in agreement with Eq. 9.2.

Gravitational field energy. The fireman sliding down the pole, the ski tow pulling the skier, and the child hopping on the pogo stick are all phenomena in which energy stored in the gravitational field is either increased or decreased as the height of an object is increased or decreased.

One point that is sometimes a source of difficulty in dealing with gravitational field energy is the absence of a natural reference state in which the energy stored in the gravitational field is zero. It is always possible to increase the energy in the gravitational field by raising an object near the earth, and it is possible to decrease this energy by lowering the object.

Because there is no natural reference state, you may choose the most convenient state as reference state. Usually you should choose the state with the lowest gravitational field energy that occurs in the phenomenon, as when the fireman is on the ground floor or the skier is in the valley (Fig. 11.20). Then you can ascribe positive energy values to the states of greater gravitational field energy than the reference state, as when the fireman is

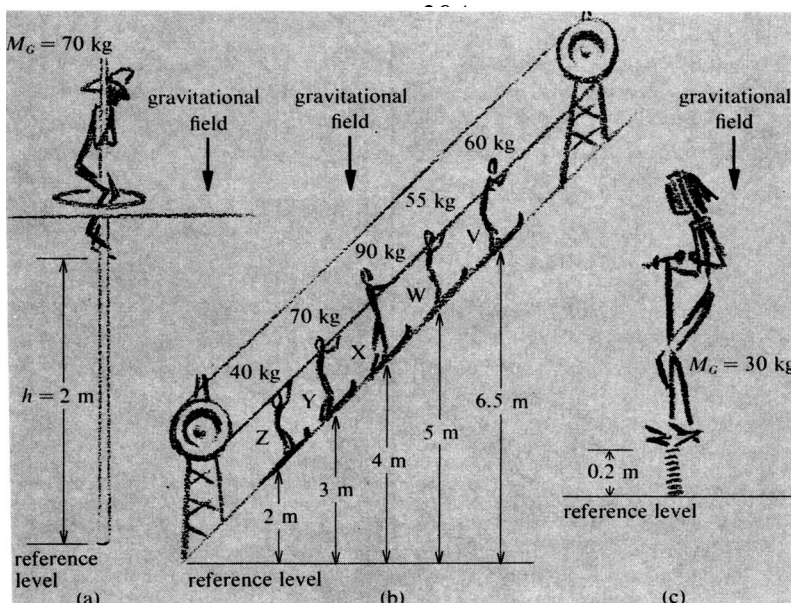


Figure 11.20 The gravitational field energy (see Eq. 11.6) is calculated relative to the specific reference levels shown in the diagram.

(a) Fireman: $E_G = |g|M_G h$
 $= 10 \times 70 \times 2$
 $= 1400 \text{ joules}$

(b) Skier Y: $E_G = |g|M_G h$
 $= 10 \times 70 \times 3$
 $= 2100 \text{ joules}$

Skier V: $E_G = |g|M_G h$
 $= 10 \times 60 \times 6.5$
 $= 3900 \text{ joules}$

(c) Child: $E_G = |g|M_G h$
 $= 10 \times 30 \times 0.2$
 $= 60 \text{ joules}$

Equation 11.6

gravitational field energy
(joules) $= E_G$
gravitational mass (kg) $= M_G$
height (m) $= h$
gravitational intensity
(newtons/kg) $= g$

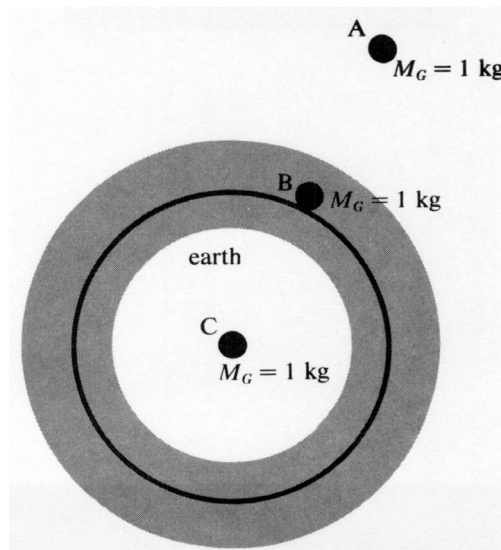
$$E_G = |g|M_G h \quad (a)$$

at the earth's surface:
 $E_G = 10 M_G h \quad (b)$

upstairs or the skier is on his way up the hill. States that have less gravitational field energy than the reference state are described by negative energy values, as when the fireman is in the cellar of the firehouse. The height of the object in the system is measured from the level of the reference state (reference level).

Since the definition of the mechanical energy scale made use of gravitational field energy, you can obtain a mathematical model for the field energy directly from Eq. 9.2. The result is stated in Eq. 11.6. The height that occurs in this formula is measured from the chosen reference level as described above. The mathematical model works well for objects near the surface of the earth, as illustrated in Fig. 11.20. However, when an object is displaced far above or below the surface of the earth (Fig. 11.21), you

Figure 11.21 Cross-sectional view of the earth. The model for gravitational field energy (Eq. 11.6) breaks down when the object is displaced to locations where the gravitational intensity is different from its value at the earth's surface. The model is applicable in the shaded region (body B). It is not applicable at the location of bodies A or C.



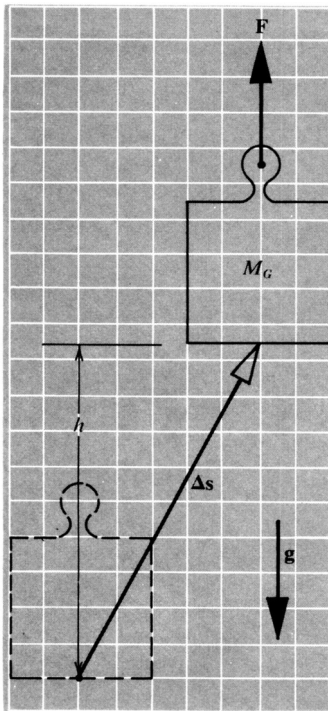


Figure 11.21 Raising a weight obliquely (along a diagonal). The work done is $10 M_G h$, which depends on the vertical height ($h = \Delta s_F$) and not the diagonal displacement (Δs).

Charles Augustin de Coulomb (1736-1806) was a French army officer of engineers who spent several years in the West Indies until failing health forced his retirement to France. After his return, he won a prize from the French Academy for a paper, *Theorie des Machines Simples*, in which the law of friction was announced. Coulomb's most important work, however, was his measurement of the forces of electrical attraction and repulsion, and his formulation of the mathematical model that describes them.

should expect the model to break down because the model in Eq. 11.4 for the force of gravity breaks down under those conditions.

Is the work done on an object near the surface of the earth always compatible with Eq. 11.6? In Example 11.1 we examined the work done on an object that was raised vertically and found agreement. Is agreement also obtained when the object is displaced to the side or obliquely?

Since the force of gravity is in the vertical direction, only the vertical component of the displacement is used to calculate the work done by the force of gravity (Example 11.2). The horizontal component of the displacement does not contribute to this work. Consequently, the horizontal component of the displacement does not change the gravitational field energy. This result is in accord with the commonsense expectation that only raising or lowering of objects affects the gravitational field energy and that sliding an object sideways (for example, a skier walking on level ground) does not affect the gravitational field energy.

EXAMPLE 11.2. How much work is done when an object is raised obliquely (that is, along a diagonal), rather than straight up, as in Example 11.1?

Solution: This problem is very similar to Example 11.1. Again, we move the weight so slowly that it is always very close to mechanical equilibrium and does not acquire kinetic energy. Then the force acting on it is equal in magnitude, and opposite in direction, to the force of gravity (Eq. 11.4),

$$|\mathbf{F}| = |\mathbf{F}_G| = |\mathbf{g}| M_G = 10 M_G$$

The displacement (see Fig 11.21 to left) is not directed along the vertically upward force. We can, however, find the rectangular components of the displacement (Fig. 11.21). The vertical component is equal to h ,

$$\Delta s_F = h$$

The work done is

$$W = |\mathbf{F}| \Delta s_F = |\mathbf{g}| M_G h = 10 M_G h.$$

This is the same result as found in Example 11.1.

11.5 Electrical force and energy

The interaction of electric charges was described in Section 3.5 and was applied in a crucial way in the construction of models for atoms in Chapter 8. We will now describe the magnitude of the force between electrically charged objects. Charles Augustin de Coulomb investigated this definitively with a very delicate spring balance.

By carrying out many experiments with the electrically charged objects closer together and farther apart. Coulomb constructed a mathematical model in which the electrical force of interaction varies inversely as the second power of the distance R between the charged objects (Fig. 11.22). He also found, by testing objects with various quantities of electric charge, that the mathematical model must include a dependence of the force on the

Equation 11.7 (Coulomb's Law)

$$\begin{array}{lcl} \text{electrical force} & & \\ \text{(newtons)} & = & \mathbf{F} \\ \text{electrical charge} & & \\ \text{(faradays)} & = & q_1, q_2 \\ \text{distance between} & & \\ \text{bodies (m)} & = & R \end{array}$$

$$|\mathbf{F}| = 8.4 \times 10^{19} \frac{q_1 q_2}{R^2}$$

"... the mutual attraction of the electric fluid which is called positive on the electric fluid which is ordinarily called negative is in the inverse ratio of the [second power] of the distances . . ."

Charles Augustin de
Coulomb

Memoires de l'Academie
Royale des Sciences,
1785

two charges: the force is proportional to the product of the two charges q_1 and q_2 . When the dependence on the distance and charge are combined, we obtain the model for the magnitude of the force described in Eq. 11.7, called *Coulomb's law*. The direction of the electrical force is along the line from one charged body to the other. It is attractive or repulsive, depending on whether the charges are opposite charges or like charges (Section 3.5).

The very large factor (8.4×10^{19}) in Coulomb's law means that 1 faraday is a charge that exerts enormous forces on charged objects nearby. Laboratory experiments in which objects become electrically charged, therefore, produce objects with very small charges (as measured in faradays).

The energy stored in electric fields may be calculated from the work that electrical forces can do when a charged object is displaced. Unfortunately, the electrical force changes so rapidly with displacement of the object (Eq. 11.7) that a simple average value to use in calculating the energy is not evident. We will therefore not make a mathematical model for the electrical energy, but you may like to consult a more advanced text where this is done.

Because the electrical forces are so large, electric charges are rarely and not conveniently separated from a macro-domain quantity of matter. Macro-domain electric fields are not easy to produce and do not furnish a technologically useful type of energy storage. In the micro domain, however, electric fields provide an extremely significant type of energy storage, because of their role in the many-interacting-particles model for matter (Sections 4.5, 8.1, and 11.7).

11.6 Elastic energy and elastic force

Elasticity. We have used the archer's bow as an example of a system that can store energy. The spring in the standard spring scale also can store energy when it is deformed from its equilibrium configuration by interaction with a weight or some other object. When released, it will spring back to its free equilibrium configuration. Such a system is called an *elastic system*. The energy stored by an elastic system when it is in its deformed configuration is called *elastic energy*. The force it exerts is called an *elastic force*. When an elastic system is deformed,

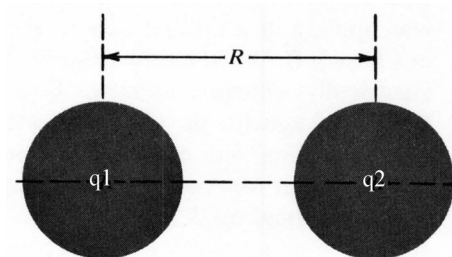
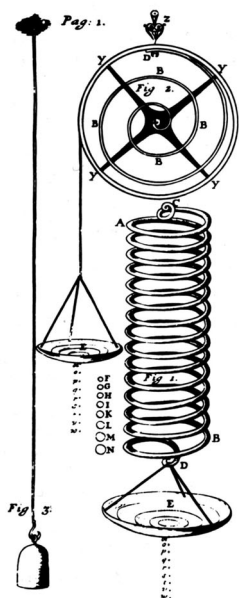


Figure 11.22 Conditions of Coulomb's experiments. Two bodies with electric charges q_1 and q_2 are separated by a distance R . The force is directed along the line joining the two bodies.



Hooke's apparatus, from his Lectures Cutlerina, 1674–1679

Equation 11-8

elastic force magnitude (newtons)	$ \mathbf{F} $
elastic deformation (m)	Δs
force constant (newtons/m)	κ
$ \mathbf{F} = \kappa \Delta s$	

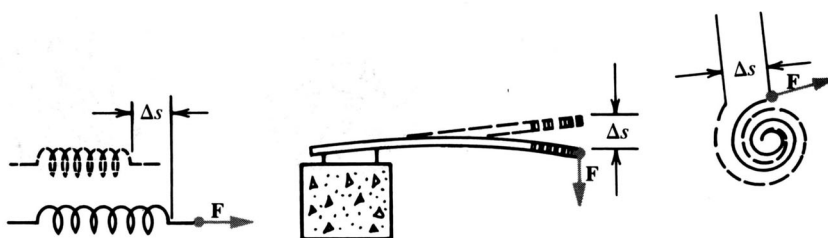
the deforming force does work and transfers energy to the elastic system where it is stored as elastic energy. When the elastic system springs back, the elastic force does work and the energy is returned to the objects in the environment with which the elastic system is interacting. Bow and arrow, a toy car with a spring motor, or a watch with a spring are good examples of systems where energy is stored in an elastic sub-system.

Many solid objects exhibit *elasticity*, that is, a tendency to spring back after they are deformed. The deformation may occur in any one of many different ways. A coil spring, for example, may be coiled up more tightly. A fishing rod or a bow may be bent away from their original straight shapes. A piano wire may be stretched as the instrument is tuned. A paper clip may be opened slightly when it is slipped over a stack of papers. Branches of a tree may sway in the wind—even skyscrapers and bridges may sway in the wind.

Elastic limit. By contrast, there are many solid materials, such as clay and putty, which are inelastic. If they are deformed into a new shape, they retain that shape; they do not spring back. You know from everyday experience, however, that even elastic systems cannot be deformed indefinitely. If the paper clip is opened too far, it will remain in its new configuration. If the fishing rod is bent too far, it snaps. The smallest force that produces a permanent alteration of shape is called the *elastic limit* of the material.

Hooke's Law. You may remember that the spring scale in Section 11.1 had the marks for equal force increments at almost equal intervals (Fig. 11.9). This experimental result suggests that the spring can be described by the mathematical model of Eq. 11.8, in which the force applied to produce a deformation of distance Δs from the equilibrium configuration is proportional to the distance as shown in Fig. 11.23. Robert Hooke observed this property of many elastic bodies in the seventeenth century and proposed Eq. 11.8 for their description. The formula is therefore called Hooke's Law. The constant κ (Greek "kappa") in the formula is called the *force constant* and depends on the material and shape of the elastic body.

Figure 11.23 Definition of Δs , the deformation distance of the movable end of a spring, for three different spring shapes.



Robert Hooke (1635-1703) became Robert Boyle's assistant after study at Oxford. In 1662, he became curator of experiments to The Royal Society and professor of geometry at Gresham College. Like many men of great ability, Hooke tended to become involved with too many things and thus found it difficult to finish anything. Nevertheless, he made a number of substantial contributions to science. He was an early exponent of the wave theory of light and was recognized by Newton as having been among the first to suggest the law of gravitation. In making observations of the structure of cork with microscopes he built, Hooke was the first to use the word "cell." Hooke is, however, best remembered for his law governing the compression and extension of elastic systems.

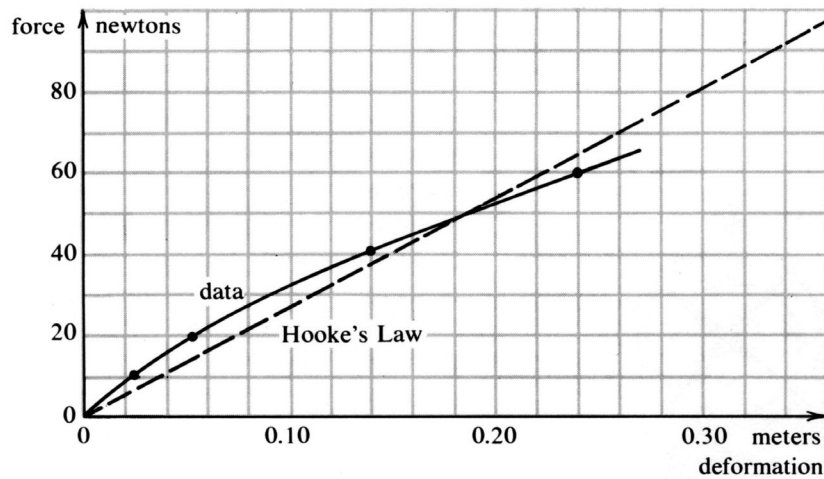
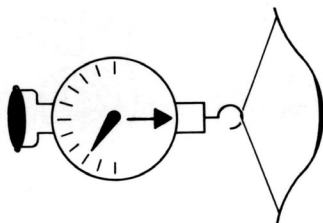


Figure 11.24 Graph of the data for the elastic force of a bow (Table 11.2, below). The dots connected by the solid, curved line represent the experimental data. The straight (dashed) line is calculated from Hooke's law ($|\mathbf{F}| = 270\Delta s$). The relatively small discrepancy between the solid and dashed lines demonstrates that Hooke's law describes the performance of this bow quite well.

According to Hooke's law, the deformation returns to zero when the force on the elastic body is removed; that is, the body springs back completely. This model, therefore, clearly does not apply to bodies that are stressed beyond the elastic limit, for these do not spring back completely. The model also does not apply to rubber bands, as you can verify by trying to provide a rubber band with a calibrated scale in the way we did for the standard spring scale. The scale marks do not fall at equal intervals. In spite of its limitations, Hooke's law is extremely useful because it applies to many examples and is so simple (Table 11.2 and Fig. 11.24).

Elastic energy. Hooke's law enables us to derive a mathematical model for the elastic energy stored in a deformed body. The natural reference state (zero elastic energy) is the undeformed state. We need only calculate the work that is done by the force that deforms the elastic

TABLE 11.2 ELASTIC FORCE OF A BOW

Displacement (Δs , meters)	Force* ($ \mathbf{F} $, newtons)	Force** ($ \mathbf{F} $, newtons)
0.000	0	0
0.025	10	7
0.055	20	15
0.14	40	38
0.24	60	65

* Experimental data (using apparatus shown in margin to left).

** Calculated from Hooke's law: $|\mathbf{F}| = 270 \Delta s$ (with $\kappa = 270$ newtons per meter).

body to a distance Δs , because this work is equal to the stored elastic energy (Eq. 11.9). We therefore would like to combine the mathematical model for work (Eq. 11.5) with Hooke's law (Eq. 11.8). Unfortunately, these two cannot be combined readily because the elastic force varies proportionally to the deformation, but the problem can be solved approximately (Example 11.3). The elastic energy stored in the deformed body varies as the second power of the displacement: doubling the displacement stores four times the energy, tripling the displacement stores nine times the energy, and so on (Example 11.4).

EXAMPLE 11.3. Construct a mathematical model for the elastic energy of a system described by Hooke's law. Use Eqs. 11.5, 11.8, and 11.9.

Solution: Hooke's law, Eq. 11.8 gives the magnitude of the force that deforms the elastic body. This force varies from zero to a maximum value of $\kappa\Delta s$ when the body is deformed. As an approximation, describe the force by a constant "average" value, halfway between the minimum and the maximum,

$$|\mathbf{F}| \approx \frac{1}{2} \kappa \Delta s \quad (1)$$

The displacement of the end of the spring on which the force acts is parallel to the force (Fig. 11.23). Hence the displacement component is equal to the deformation distance,

$$\Delta s_F = \Delta s \quad (2)$$

The mathematical model for the elastic energy is found by using these results in Eq. 11.9:

$$E = W = |\mathbf{F}| \Delta s_F \approx (\frac{1}{2} \kappa \Delta s) \Delta s = \frac{1}{2} \kappa (\Delta s)^2 \quad (3)$$

EXAMPLE 11.4. Calculation of the elastic energy of a bow using the value of κ in Table 11.2 (270 newtons per meter) and the formula derived in Example 11.3.

$$E = \frac{1}{2} \kappa (\Delta s)^2 = \frac{1}{2} \times 270 (\Delta s)^2 = 135 (\Delta s)^2$$

Displacement (m)	Energy (joules)
0.00	0.0
0.05	0.34
0.10	1.4
0.15	3.0

11.7 Elastic energy and the MIP model for matter

Let us now return to consider elastic energy and ask the question, what happens to the energy when a system is deformed beyond the elastic limit? Clearly, the energy that was transferred to the system will

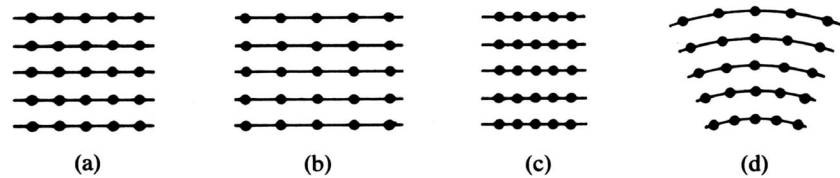


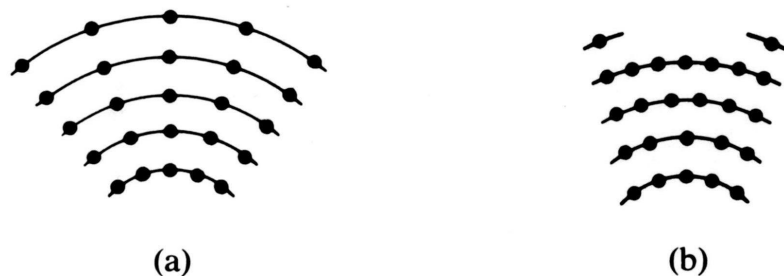
Figure 11.25 MIP model for an elastic specimen of 25 particles, (a) The specimen is unstressed, (b) The specimen is extended, (c) The specimen is compressed, (d) The specimen is bent.

not be simply returned to the environment because the system does not spring back completely. You can easily do an experiment with a paper clip (especially a 2-inch-long one), bending it rapidly back and forth (beyond the elastic limit) and touching the bent portion with your finger or above your upper lip to observe its temperature. It gets quite warm! You conclude that at least some of the energy becomes thermal energy. The same thing happens when the sidewall of an automobile tire, particularly an under-inflated one, is repeatedly flexed as the tire turns. It gets quite hot!

MIP model for an elastic system. To obtain an understanding of how elastic energy is transformed into thermal energy, we make an MIP model for elastic systems. We know that particles in solid materials are in a more or less regular arrangement that depends on the crystalline structure of the material. The electrons and nuclei of adjacent particles interact with one another via an electric field in such a way as to maintain the shape of the entire piece of material. The particles are in an equilibrium arrangement relative to one another, and are spaced at certain equilibrium distances from one another.

Now, if we bend, stretch or compress the material and thus change its shape, we are also changing the inter-particle distances and the micro-domain electric field. Figure 11.25 shows four arrangements of a system of 25 particles schematically. Elastic energy is stored in the electric field of the deformed arrangement. This energy is released by the system when the specimen is permitted to spring back to equilibrium. In

Figure 11.26 An elastic specimen is bent beyond the elastic limit.
(a) Particle arrangement before elastic energy is lost,
(b) Particle arrangement after elastic energy is lost.



other words, what we call elastic energy in the macro domain is, according to this model, stored in micro-domain electric fields.

Elastic energy loss. Let us now consider what is likely to happen when the specimen is bent so far that it remains deformed (Fig. 11.26). Some particles are so very far apart that others from adjacent rows are attracted into the intervening spaces. This means that the particles can move toward a new equilibrium arrangement (Fig. 11.26b), which is compatible with a bent shape. As the particles move toward their new equilibrium positions, they gain kinetic energy at the expense of the electric field energy. Through collisions among the particles, the kinetic energy is shared among many of them, so that the specimen comes to a higher temperature in a macro-domain description (Section 10.5). This is a theory to explain why the paper clip gets hot.

Elastic versus chemical and phase energies. The situation we have here, where elastic energy is stored in micro-domain electric fields, is similar to the earlier example of chemical and phase changes, where energy was also gained by or lost from micro-domain electric fields. The major difference is that the particle displacements in an elastic deformation are so correlated among all the particles that the macro-domain shape of the entire system is altered: it is stretched, compressed, or bent. Therefore the elastic energy can be transferred by the elastic forces doing work in the macro domain, as when the elastic energy stored in the longbow is transferred to kinetic energy of the arrow. During a phase change, however, the displacements of the various particles are not correlated so as to produce a net displacement or work in the macro domain. The energy released during a phase change is therefore transferred to (or from) micro-domain kinetic energy, that is, macro-domain thermal energy.

The MIP model for "rigid" bodies. So far we have described elastic systems, such as springs, longbows, and fishing rods, which can be given large, easily visible deformations. The MIP model, however, leads you to expect that all solids would exhibit elastic behavior, even though the magnitude of the deformation may be very small if the interactions holding the particles in their equilibrium arrangement are very strong. Fortunately, indirect evidence of elasticity is furnished by many observations, for example, a glass marble bouncing on a steel plate (Fig. 11.27) or a drinking glass bouncing on a wood floor.

The bouncing of the glass marble and of other "rigid" bodies will make you aware that all rigid bodies are really elastic, can suffer deformation, and can store elastic energy. Thus, when the micro-domain particles are slightly displaced from their equilibrium arrangement, work is done and energy is stored; this energy is released when the particles spring back after the external stresses are removed. This fact is usually not appreciated because the elastic displacements under ordinary conditions are too small to be detected. All evidence, however,

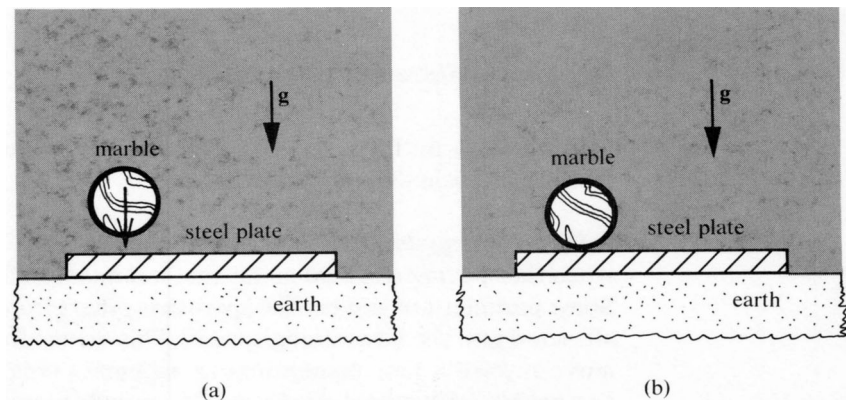


Figure 11.27 A glass marble is dropped and hits a steel plate,
 (a) The marble just before impact. The gravitational field energy of the elevated marble-earth system has been transferred to kinetic energy,
 (b) The marble at impact. The gravitational field energy is zero, and the kinetic energy is zero. Where is the energy of the system?

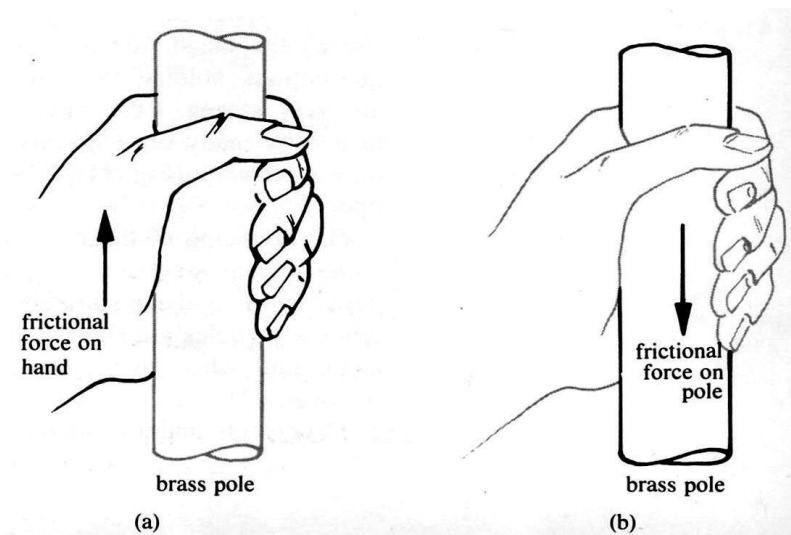
indicates that the elastic deformation and the elastic force of so-called rigid bodies are related by Hooke's law (Eq. 11.8), just as they are for springs, diving boards, and other objects that suffer visible elastic deformations. The MIP model for rigid bodies and their elasticity also helps to explain how it is possible for sound waves to propagate through such bodies (Section 7.1).

11.8 Frictional force

We have already had occasion to refer to frictional interactions and the force of friction on several occasions. The interaction of the fireman with the brass pole was the most recent example. Others are the

Figure 11.28 Friction between the hand and the brass pole.

(a) The hand slides downward relative to the pole. The force of friction on the hand is directed upward and thereby opposes this motion,
 (b) The pole slides upward relative to the hand. The force of friction on the pole is directed downward and thereby opposes this motion.



interaction of your shoes with the floor while you walk, the interaction of an eraser with the paper on which it is used, and even the interaction of the archer's fingers with the arrow when he is ready to shoot.

Properties of friction. Friction occurs when two surfaces are in contact. The frictional force *opposes* relative motion of the two surfaces. When two pieces of sandpaper are rubbed on one another, the interlocking of irregularities on the two surfaces clearly causes friction. Why do even very smooth surfaces give rise to friction when the two surfaces are pressed together? Later we will describe a theory of friction that answers this question.

Friction between the fireman's hand and the brass pole, like other interactions, is described by two forces, one exerted on the hand, the other on the pole (Fig. 11.28). The direction of each frictional force is such as to drag the other surface along with it and to reduce relative motion of the two surfaces. Its magnitude is determined by the condition of the surfaces and the tightness with which they are pressed together. In one simple and fairly successful mathematical model for friction, the magnitudes of the frictional forces are directly proportional to the force pressing the two surfaces together.

Whether or not the two surfaces actually are displaced relative to one another depends on the magnitude of the frictional force in relation to the other forces that are acting. If the frictional forces on the hands of the fireman are equal to the force of gravity acting on him, he will be in mechanical equilibrium and will not slide down. If he slightly relaxes his hold and thereby decreases the frictional force, he will slide. The same is true of an eraser interacting with a piece of paper. Pressed lightly, the frictional force is small and the eraser glides over the paper easily. Pressed very heavily, the paper may tear and move along with the eraser because the frictional force is larger than the forces holding the paper fibers together.

Energy transfer by friction. If the two interacting surfaces are not displaced relative to one another, the frictional forces do no work. When there is some sliding, however, the frictional force acts on a body that is being displaced and work is done. According to the definition of work in Eq. 11.5, the work done is equal to the force of friction times the relative displacement of the two surfaces.

What happens to the energy? From your experience of rubbing your hands together on a cold day, you know that thermal energy is produced in the interacting objects, which may be your two hands, the fireman's hands and the brass pole, the eraser and the piece of paper, and so on. In the example of the eraser and the piece of paper, eraser crumbs are produced and the paper tends to be worn thin; here the shape of the interacting objects is also being changed, which requires some energy. The same is true when sandpaper is rubbed on wood or a grindstone is used to sharpen a knife.

A theory of friction. As a matter of fact, energy transfer during friction can be explained by the use of the same MIP model that we

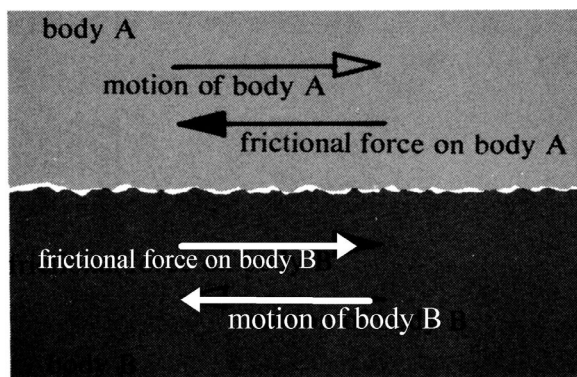


Figure 11.29
Micro-domain
irregularities
interlock and
thereby bring
about the forces of
friction opposing
relative motion.

introduced to explain elasticity. In the micro domain all surfaces have irregularities. When two surfaces slide over one another, very strong interactions occur at the interlocking irregularities (Fig. 11.29). The irregularities are stressed beyond their elastic limit and are deformed, with the production of thermal energy as explained in Section 11.7.

Applications. The effects of friction are often undesirable. Friction increases the energy necessary to operate machinery, and it causes wear of the moving parts. To reduce this waste, we use wheels and ball bearings and/or lubricants to minimize friction. Wheels and ball bearings roll rather than slide, whereas lubricants form a film between the interacting surfaces and prevent their irregularities from interlocking.

In many circumstances, however, friction is very desirable. When walking, driving a car (not on ice!), tying shoelaces, and holding a drinking glass, friction is indispensable in that it prevents relative motion of the interacting surfaces. The same is true of thread in fabric and nails that hold boards together. Under other conditions, frictional forces are used to do work to achieve energy transfer. For example, when you press on the brake pedal of a car, the brake shoe presses against the brake drum (or disc), and the kinetic energy of an automobile is transferred to the thermal energy of the brake linings, or when an orbiting spacecraft must be slowed down to return to earth, friction with the air converts the kinetic energy of the craft to thermal and phase energy of the heat shield.

Friction in liquids and gases. So far we have described friction between solid surfaces. Liquids and gases also exhibit friction in that relative motion of their parts is opposed by interactions among those parts. When water in a bowl is stirred near the edge, soon all the water in the bowl is rotating. Honey, though a liquid, is almost impossible to stir. Friction in liquids is called *viscosity*. Honey has high viscosity, water has low viscosity. Lubricating oils must have enough viscosity so they are not squeezed out completely from between the surfaces they are to lubricate.

Gases, too, have viscosity, although much less than liquids. Gases, therefore, make excellent lubricants, but they must be continually sup-

plied to the lubricated surfaces because they escape rapidly. Even though air has very low viscosity, its motion past the surface of a plane flying at supersonic speeds or of a reentering spacecraft creates a great deal of frictional drag. Special materials must be used to withstand the high temperatures produced by the high rate of production (through friction) of thermal energy at an airplane's surface.

Summary

The central concept around which this chapter revolved is the Newtonian force. Instead of stressing the mutuality of the interaction among the objects in a system, Newton selected the objects' motions for detailed study one at a time. In Newton's theory, a net force acts on every body whose motion is changing. For Newton, the net force was the cause of the change in motion. A body in steady motion in a straight line is subject to a zero net force; such a body is said to be in mechanical equilibrium.

The net force acting on an object in a complex system is the sum of partial forces exerted on it by each of the other objects in the system. Each of these partial forces is assumed to be affected neither by the presence of the other objects nor by the other partial forces. A body in mechanical equilibrium may be subject to no interaction at all, or (more likely) it is subject to a set of partial forces that compensate for one another. No body ever exerts a force on itself.

Four kinds of forces were explained in some detail: the gravitational force, the electric force, the elastic force, and the frictional force. A numerical measure of force, which is described by a magnitude and a direction in space, is obtained with a spring scale calibrated in newtons. The gravitational force on a body is of special importance for all dwellers on the earth. Near the earth's surface it is described by the mathematical model in Eq. 11.4. Its direction is vertical and its magnitude is very close to 10 newtons per kilogram of gravitational mass of the body, regardless of the body's other properties.

Eq. 11.5 defines the work done by a force acting on a moving body. It is equal to the magnitude of the force times the moving body's displacement component along the force direction. The work is the energy transferred by the action of the force. When a body falls freely, for instance, the work done by the gravitational force transfers gravitational field energy to kinetic energy. Work done in the elastic deformation of a solid body is stored as elastic energy. Work done by the frictional force produces thermal energy at the surface where friction occurs.

List of new terms

force	gravitational intensity	elastic limit
net force	work	force constant
spring balance	elasticity	friction
newton		

Equation 11.4 (force of gravity)

$$\begin{aligned}
 \text{force of gravity (newtons)} &= \mathbf{F}_G \\
 \text{gravitational mass (kg)} &= M_G \\
 \text{gravitational intensity} &= \mathbf{g} \\
 \text{(newtons/kg)} &
 \end{aligned}$$

$$\mathbf{F}_G = \mathbf{g} M_G$$

Near surface of earth:
 $\mathbf{g} = 10 \text{ newtons/kg, and}$
 $\mathbf{F}_G = 10M_G \text{ (downward).}$

Equation 11.5 (Work)

$$\begin{aligned}
 \text{work (joules)} &= W \\
 \text{force magnitude} &= |\mathbf{F}| \\
 \text{(newtons)} & \\
 \text{displacement} & \\
 \text{component in force} & \\
 \text{direction (m)} &= \Delta s_F
 \end{aligned}$$

$$W = |\mathbf{F}| \Delta s_F$$

List of symbols

F	force	R	distance (between charges)
F_G	force of gravity	κ	force constant
g	gravitational intensity	M_G	gravitational mass
W	work	h	height above reference level
Δs_F	displacement component in the force direction	E_G	gravitational energy
q	electric charge		

Problems

- Identify four examples of forces (elastic, gravitational, pressure, or frictional) in Fig. 11.1 in addition to the ones listed in the caption.
- Give two examples from everyday life of bodies that are in mechanical equilibrium relative to one reference frame and are being accelerated relative to another.
 - Describe the partial forces acting on the bodies in your examples.
 - Relate the partial forces (qualitatively) to the net force in each reference frame.
- Comment on the assumption that the interaction between two bodies is unaffected by the nearby presence of a third, according to Newton's theory, as follows.

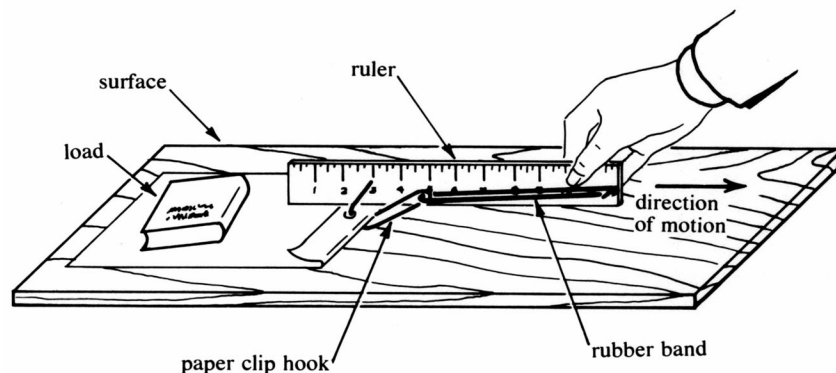
 - Do you find this assumption reasonable on the basis of your experience with inanimate objects?
 - Do you find this assumption applicable to human social interactions?
- Enumerate the partial forces acting on each of these objects in Fig. 11.1:

 - the fisherman's left boot;
 - falling apple;
 - tip of the fishing rod.
- Draw an approximate force diagram for all the partial forces acting on one of the objects in Problem 4.
- Measure the spring extensions produced by various numbers of weights in Fig. 11.9 and draw a graph relating these two variable factors (extension and number of weights).
- Use a rubber band to make a "spring" scale (not necessarily calibrated in newtons).
 - Measure the rubber band extensions produced by various numbers of weights as you calibrate the rubber band scale.
 - Draw a graph relating the two variable factors in (b).
 - Is the rubber band described by Hooke's law?

8. Describe how the operation of a spring scale may be affected by the force of gravity acting on the elastic system in the spring scale itself.
9. Imagine the spring scale in Fig. 11.10 to be connected to the mouse's harness instead of to the wagon. What reading would you expect it to show? Explain your answer.
10. (a) At what stage of the child's swing would you expect the spring scale reading in Fig. 11.11b to be largest? Explain.
(b) At what stage would you expect the spring scale reading in Fig. 11.11b to be smallest? Explain.
11. Make an analogy between the spring scale and a thermometer. Discuss this analogy, using gravitational field energy as the analogue of thermal energy. What are the analogues of temperature and specific heat? Discuss this analogy critically.
12. Use a spring scale (bathroom scale with 1 pound equal to 4.5 newtons) to estimate the net force on yourself under the conditions in your bathroom, and then in an elevator while it is
 - (a) starting upward;
 - (b) coming to a stop on its way up; and
 - (c) moving uniformly. Describe and discuss your observations.
13. Calculate the reading on your bathroom scale if you were to weigh yourself (a) on the surface of the sun and (b) on the surface of the moon. (See Table 11.1.)
14. Propose one or more operational definitions of "work."
15. Calculate the work that is done when a 90 kilogram fireman slides 3 meters down a brass pole.
16. Calculate the gravitational field energies of skiers X and W in Fig. 11.20.
17. Calculate the gravitational field energy of the fireman-earth system (Fig. 11.20) when the fireman is in the firehouse cellar, 1.8 meters below ground floor level.
18. Describe how you might expect the mathematical model in Eq. 11.6 to fail for objects A and C in Fig. 11.21.
19. Your hands do work when they knead a piece of clay or dough. What happens to the energy transferred in this process?
20. Test a diving board to determine whether Hooke's Law describes its deformation. Use people as weights in this experiment. Find the force constant if the diving board satisfies Hooke's law.

21. Find an elastic system that is described by Hooke's law.
 - (a) Measure the deformation displacement and the elastic force, plot a graph of these two variables, and determine the force constant.
 - (b) Calculate the elastic energy and plot a graph of its relation to the deformation displacement.
22. Carry out the experiment with the paper clip described in Section 11.7. Describe and compare the effect you observe with paper clips of various sizes, shapes and materials.
23. Describe four everyday observations that give evidence that "rigid" bodies are really elastic systems.
24. Use your rubber band scale (Problem 7) to estimate the force of friction between a piece of paper and a smooth surface when various loads are placed on the paper (Fig. 11.30).
 - (a) Plot a graph to show the relationship between the force of friction and the force of gravity acting on the load, for two different surfaces.
 - (b) Make approximate mathematical models in algebraic form for the graphs in (a), if possible.
25. The Niagara Falls have a height of 50 meters. Estimate the maximum temperature rise in the water as gravitational field energy is converted to thermal energy at the Falls.
26. Interview four or more children (ages 8-12) to find their explanations for the temperature rise produced by rubbing your hands together.

Figure 11.30 Rubber band "spring" scale is used to measure the frictional force exerted by the table surface on the piece of paper (Problem 24).



27. Find the partial force exerted by the rope tow on each of the skiers in Fig. 11.20. Assume that the frictional force acting on the skis is negligible. (Hint: The snow exerts a force at right angles to the skis if there is no friction.)
28. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

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