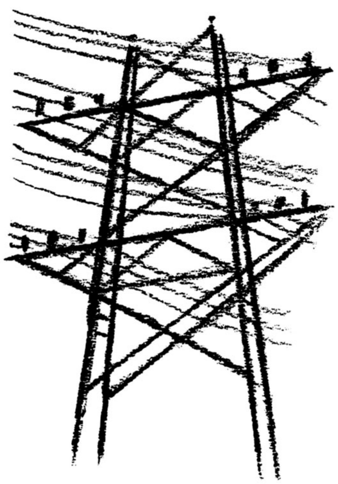
Chapter 12: Electric Current and Energy



Kilowatt-hours, volts, amperes—these are terms you frequently hear when household appliances are described or the electric bill is to be paid. What do they mean? Or you may sometimes have wondered how a light switch at the top of a flight of stairs could turn off the lamp that you turned on by a switch at the bottom.

Probably you associate all these matters with the mysteries of electricity, and you may even dismiss them as incomprehensible for this reason. Electric interactions appear mysterious because macro-domain effects such as the glowing of a light bulb are produced without detectable macro-domain causes. The situation is different from that of a fire heating water or a bow shooting an arrow, where both the energy source and the energy receiver could be easily identified.

As a matter of fact, electric interactions make possible the transfer of energy over very large distances. The light bulbs and appliances in your home receive energy from a power plant that may tap the chemical energy of coal, oil, or natural gas; the nuclear energy of uranium; or the gravitational field energy associated with the water stored behind a dam. The transfer of energy occurs along transmission lines—wires—that link the power plant to your home. Remarkably, the transmission line itself gives no visible evidence whether energy is at any moment being transferred or not.

In this chapter we will describe some of the conditions under which electric interactions transfer energy in technologically useful ways. To help you with visual images of the process, we will construct an analogue model and a micro-domain model for electrical phenomena. To help you make mathematical models for the process, we will introduce two variable factors describing it: the magnitude of the electric current, and the voltage of the energy source. These two variables can be explained qualitatively by applying them to an electric spark. The brightness of the spark is related to the magnitude of the current, while the distance the spark jumps is related to the voltage of the energy source.

Alessandro Volta (1745-1827) was born in the northern Italian town of Como. As professor of physics at Como and at the University of Pavia, he distinguished himself in research. His reputation among his contemporaries was enormous, and during a visit to Paris he was the personal guest of Napoleon. Volta's most important work was his study of the electric current flow resulting from contact between different metals. He was thus led to the invention of the voltaic pile and the voltaic battery (1800).

12.1 Electric current

Already in the eighteenth century Benjamin Franklin and others knew that electric charges were able to move along certain materials, especially metals, which were therefore called *electric conductors*. Early in the nineteenth century, Alessandro Volta invented the chemical *battery* made of two dissimilar metals and a conducting liquid (Fig. 12.1). Volta found that his battery gave only very short sparks compared to those obtainable by rubbing amber or glass, but the battery had an enormous capacity for sustained action in maintaining the spark; it could also cause frogs' legs to twitch extensively and could give repeated electric shocks to humans.

Operation of the battery required a continuous electrically conducting path between the two metals (called *electrodes*) of the battery (Fig. 12.2). Volta thought of his observations in terms of a circulating electric

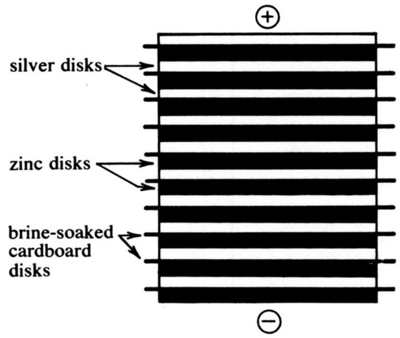


Figure 12.1 Chemical battery of zinc, silver, and brine-soaked cardboard disks constructed by Volta. This was called a voltaic pile. The zinc becomes electrically negative, the silver electrically positive. The zinc gradually dissolves.

Humphrey Davy (1778-1829), found in 1807 that electricity passing through a solution of sodium chloride (table salt) could separate it into the elements sodium (a metal) and chlorine (a gas); separating a compound into its elements with electricity in this way is known as electrolysis.

Michael Faraday (1791-1867), Davy's assistant and successor, made extensive studies of electrolysis, including the way in which electric charges pass through a conducting solution and the consequent chemical changes. By 1834, Faraday formulated what are now known as Faraday's laws of electrolysis.

fluid, which was later called an *electric current*. This is an extremely successful model, which helped to explain many other phenomena observed near wires connected to the battery electrodes. Because of this model's extraordinary success in explaining the need for a continuous conducting path, we tend today to think of electric currents as physical entities. Historic investigations that have further substantiated and elaborated the electric current model include Oersted's discovery of the interaction between magnets and electric currents, Davy and Faraday's investigations of the chemical effects of electric currents, and Joule's measurement of the heat produced during the passage of an electric current in certain conductors. Finally, the development of the electric light bulb by Edison resulted in giving the electric current concept household familiarity. For almost a hundred years before that, however, the effects of electric currents had been, as we have seen, investigated magnetically, chemically, and thermally, while light bulbs were unknown.

You may suspect that the electric current in Volta's apparatus is

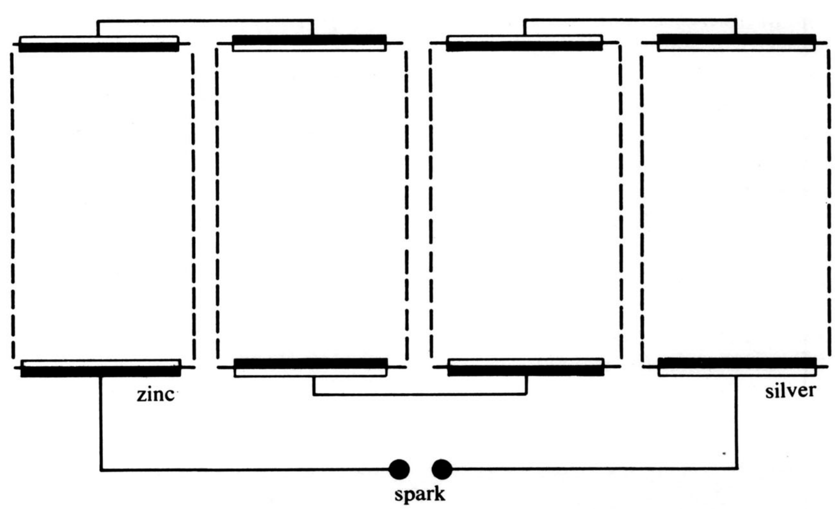


Figure 12.2 Four voltaic piles connected togehter cause an electric spark when the two conductors are almost touched together in air.The solid black lines represent metal wires, which provide a continuous conducting path along which the elctric charges can travel.

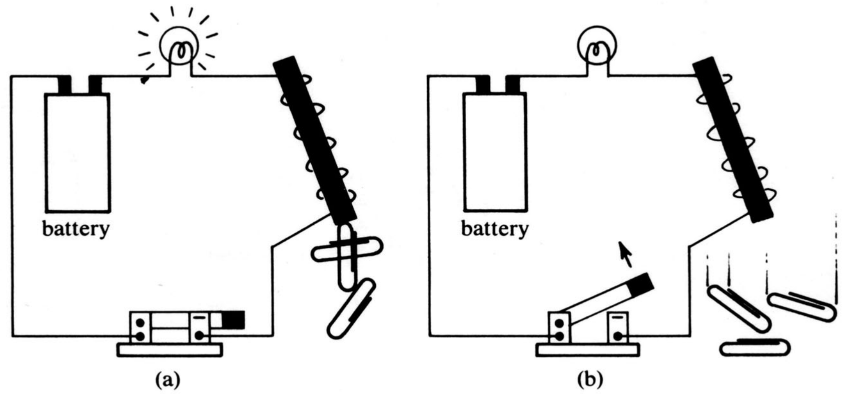


Figure 12.3 Evidence of the flow of an electric current, (a) When the switch (and thus the circuit) is closed, the bulb is lit, and the electromagnet picks up paper clips. (b) When the switch (and circuit) is open, the bulb is not lit, and the electromagnet does not hold paper clips.

related to the electric charges produced by rubbing and which are responsible for the phenomena of static (as opposed to current) electricity. The fact that the same metals and electric conductors participate in both phenomena convinced Volta that such a relation existed. This relation was implicit in Maxwell's incorporation of electric charge conservation into his electromagnetic wave theory: electric charges that disappeared from one place had to reappear somewhere else by passing through the intervening space as an electric current. Henry Rowland (1848-1901), finally, showed experimentally that electric charges produced on glass by rubbing do interact with a magnet when they move at a high speed relative to it. This observation showed conclusively that an electric current consists of electric charges in motion.

Electric circuits. An *electric circuit* is a system of conductors through which an electric current can flow. The most common conductors are metallic wires, but salt water or many other liquids can also serve, and even gases, such as the neon in a "neon light," are conductors under suitable conditions. Special objects, such as light bulbs, or motors, or electric heaters, which can operate in an electric circuit, are called *circuit elements*. A battery, an electric generator, or another energy source in a circuit is called an *electric power supply*.

A circuit element will operate only when there is a complete or *closed circuit* of electrical conductors linking it to an electric power supply. When the path is interrupted by a gap of air or some other non-conducting material, such as rubber or plastic, the system is called an *open circuit*. The lighting of a bulb, the operation of a motor, or the existence of a magnetic field near an electrical conductor is evidence that the circuit is closed. Conversely, the failure of the circuit elements to operate is evidence that the circuit is open. A switch is the most common device for opening and closing a circuit (Fig. 12.3).

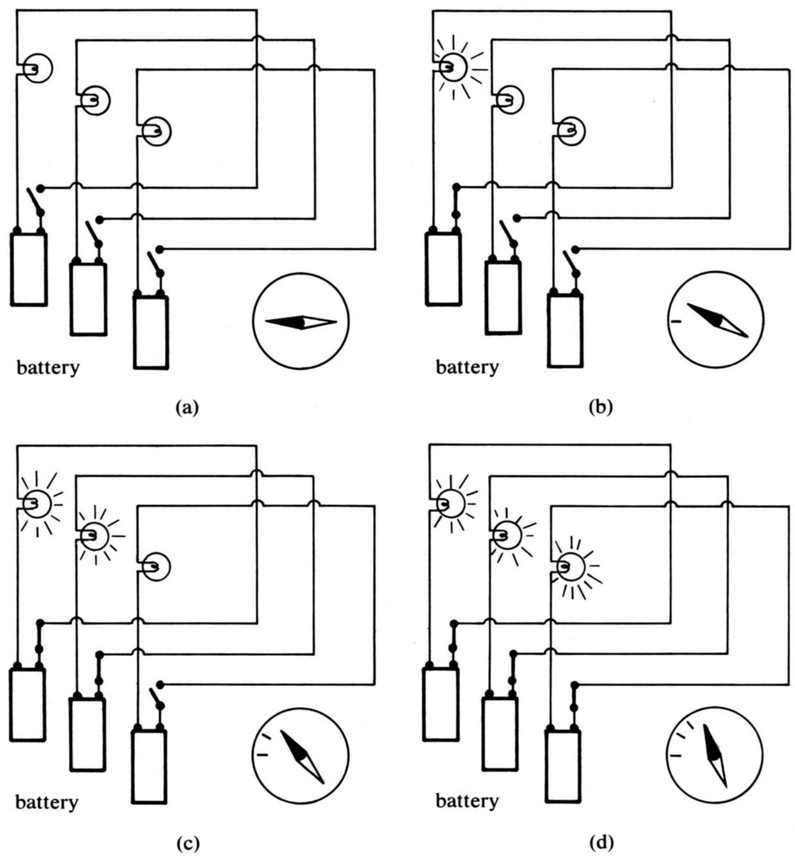


Figure 12.4 Diagrams suggesting how a compass dial may be calibrated to measure electric current. Three identical circuits are placed side by side, and the compass is held near the parallel wires,
(a) All circuits are open; the dial is marked for zero current,
(b) One circuit is closed; the dial is marked for one unit of current,
(c) Two circuits are closed; the dial is marked for two units of current,
(d) Three circuits are closed; the dial is marked for three units of current.

A closed circuit makes possible the transfer of energy from an electric power supply to one or more circuit elements. This energy transfer can take place over very large distances, as from Hoover Dam to Los Angeles, or from Niagara to New York City, or it can take place over short distances, as in a flashlight (from the battery to the bulb). Energy transfer occurs while the circuit is closed and an electric current is flowing; energy transfer ceases when the circuit is opened and the current ceases to flow. As we pointed out at the beginning of this chapter, the principal evidence of electric current flow and energy transfer is the operation of a circuit element; the connecting wires in the circuit themselves ordinarily give no such evidence. How can we unravel the mystery of what happens in the wires?

André-Marie Ampère (1775-1836) was educated in Lyons, France. Though his father was a political victim of the French Revolution, this apparently had no effect on Ampère's academic career and he became a member of the prestigious French Institute in Paris. Oersted's discovery in 1820 that an electric current-carrying wire affected a magnetic compass led to Ampère's equally important discovery of the magnetic forces exerted by currents on currents. From this discovery, Ampère developed the mathematical theory that describes these forces and that we know as Ampère's law.

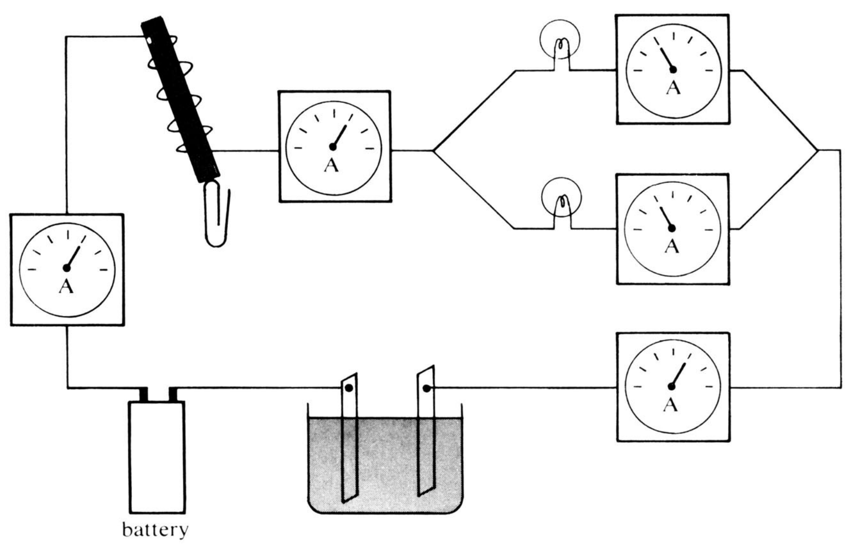
We will take three approaches to solving this problem. First, we will propose an operational definition of electric current that can be used to determine the current's magnitude. Second, we will construct analogue models in which water circulating through pipes corresponds to the current in the conductors. Third, we will use the models for atoms described in Chapter 8 to develop a micro-domain model for electric current.

Operational definition of electric current. Oersted discovered that there is a way of telling whether or not a current is flowing in a wire, for he found that a compass needle near an electrical conductor in a circuit is deflected when the circuit is closed; the needle returns to its original direction when the circuit is opened. The magnitude of the deflection can be used to measure the strength of the electric current. You can imagine placing a dial behind the magnetized compass needle and making marks for the deflections that indicate 1 unit of current, 2 units, and so on (Fig. 12.4). The commonly used unit of electric current, which has been rather arbitrarily chosen, is called the *ampere* after André-Marie Ampère, and the current measuring device (more sensitive and reliable than the one shown in Fig. 12.4) is called an *ammeter*. By inserting one or more ammeters into an electric circuit, you can measure and compare the electric current flowing in various parts of a closed electric circuit (Fig. 12.5).

Relation of electric charge and current. In Section 8.1 we described the faraday, a unit of electric charge. We defined one faraday as the

OPERATIONAL DEFINITION
Electric current is measured by the dial reading of a standard ammeter (shown in Fig. 12.4) Electric current is measured in units of amperes (amp).

Figure 12.5 The electric current in various parts of a closed electric circuit is indicated by ammeter readings.



Equation 12.1

electric charge delivered (faradays) $= \Delta q$
electric current (amp) $= \mathcal{J}$
time interval (sec) $= \Delta t$

$$\Delta q = 1.0 \times 10^{-5}\ \mathcal{J}\ \Delta t$$

combinations of amperes and seconds, which give 1 faraday:

$\mathcal{J} = 1$ amp, $\Delta t = 10^5$ sec
$\mathcal{J} = 1.7 \times 10^3$ amp, $\Delta t = 60$ sec
$\mathcal{J} = 10^5$ amp, $\Delta t = 1$ sec

The modern definition of the ampere is based on the electrolytic effect of an electric current on a standard solution of silver nitrate. One ampere is the electric current that deposits a specified amount of silver per second. This definition is used to check the accuracy of ammeters.

We can also express Equation 12.1 in another form, to show that electric current is essentially the same as the transfer, or flow, of charge per unit of time:

$$\mathcal{J} = (10^5)\frac{\Delta q\ (faradays)}{\Delta t\ (\text{sec})}$$

One proposal for a unit of electric current that was not adopted more generally:
"I have expressed my quantities of electricity on the basis of Faraday's great discovery of definite electrolysis; and I venture to suggest that that quantity of current electricity which is able to electrolyse a chemical equivalent expressed in grains in one hour of time, be called a degree."
James Prescott Joule
Philosophical Magazine, 1841

charge transferred when 1 gram of hydrogen is produced in the electrolysis of water. In this section we have defined electric current independently of electric charge, by using an operational definition in terms of an ammeter calibrated by the magnetic effect of the current on the direction of a compass needle (as in Figure 12.4). Electrolysis obviously involves *both* transfer of charge *and* an electric current; thus you may well ask exactly how the current (in amperes), is related to the charge (in faradays). Investigation shows that the amount of hydrogen produced in electrolysis of water, and therefore the electric charge delivered, is directly proportional both to the time of electrolysis and to the current strength.. You can construct a mathematical model (Equation 12.1 to the left) for this relationship once you determine what combination of amperes and seconds deliver a charge of 1 faraday, that is, produce 1 gram of hydrogen gas in electrolysis.

Water pipe analogue model for electric circuits. With the help of an ammeter, you can obtain visual evidence of an electric current in a conductor. The diagram in Fig. 12.5 illustrates the use of several ammeters to investigate the pattern of electric current in an electric circuit. Analogue models for electric circuits rely on the movement of macrodomain objects in circulating patterns to represent the electric current in wires. The flow of water through pipes is a very successful analogue model of this kind. The one-to-one correspondence between aspects of electric circuits and of the model is shown in Table 12.1.

Curiously, the flow of water even in transparent pipes or tubes shows some of the "mystery" of current flow in conductors, because it is visually undetectable. We have already pointed out that electric current flow in a wire cannot be seen without a tool such as a compass or an instrument like an ammeter. The same holds true for water. Since water is uniform, one bit of water looks like another and does not provide any visual reference point whose motion relative to the observer can be identified. To render the flow "visible," you have to mix in reference objects, such as bubbles or sand grains, which are carried along with the water and whose motion can be seen.

Water flow analogues for various circuit elements are illustrated in Fig. 12.6. Their operation can serve as evidence of water flow just as

TABLE 12.1 WATER FLOW ANALOGUE MODEL FOR ELECTRIC CIRCUITS

Electric circuit	*Water flow*
conductor	*pipe*
electric current	*flowing water*
electric charge	*stationary water*
electric power supply	*pump*
electric motor	*water wheel*
switch	*faucet or valve*
magnetic interaction	*(no analogy)*
ammeter	*flowmeter*
heating element	*capillary tube*

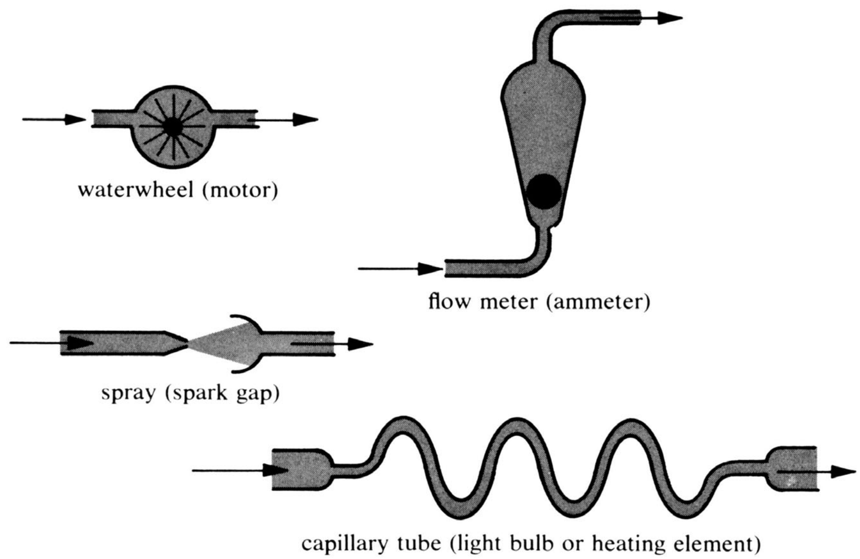


Figure 12.6 Water-operated devices analogous to certain electric circuit elements. In the flowmeter the water lifts the metal ball in the tapered chamber until there is enough space for the water to flow past. The faster the flow, the higher the ball. Thus the height of the ball can be used as a measure of the rate of flow.

the operation of electric circuit elements is evidence of electric current flow. "Seeing" the water flow directly, however, gives you an intuitive comprehension of water flow phenomena that is difficult to gain for electric circuit phenomena (Fig. 12.7).

Micro-domain model for electric current. A preview of some of the electrical phenomena described in this section was included in Chapter 8, where we presented evidence of the electrical nature of matter. According to the micro-domain models proposed there, matter consists of electrically charged constituents such as electrons (negative charge) and atomic nuclei (positive charge). These two constituents interact with one another and usually occur in combination in systems that may be electrically neutral (atoms or molecules) or electrically charged (ions). The charge of ions is positive or negative depending on whether the number of electrons is less than or greater than the total positive charge of all the nuclei in the ion.

"Electric fluids." With so many kinds of electrically charged constituents, it is easy to make micro-domain models for electric current. Current flows when there is motion of electric charges relative to the observer. In other words, you can think of electrons and ions as making up "electric fluids." When they accumulate on one body, it has a net electric charge; when they move through conductors, an electric current flows. These modern models, therefore, are similar to the models proposed by Franklin and others in the eighteenth century (Section 3.5). It is now recognized, however, that there are many different kinds of "electric fluids" because there are many kinds of ions.

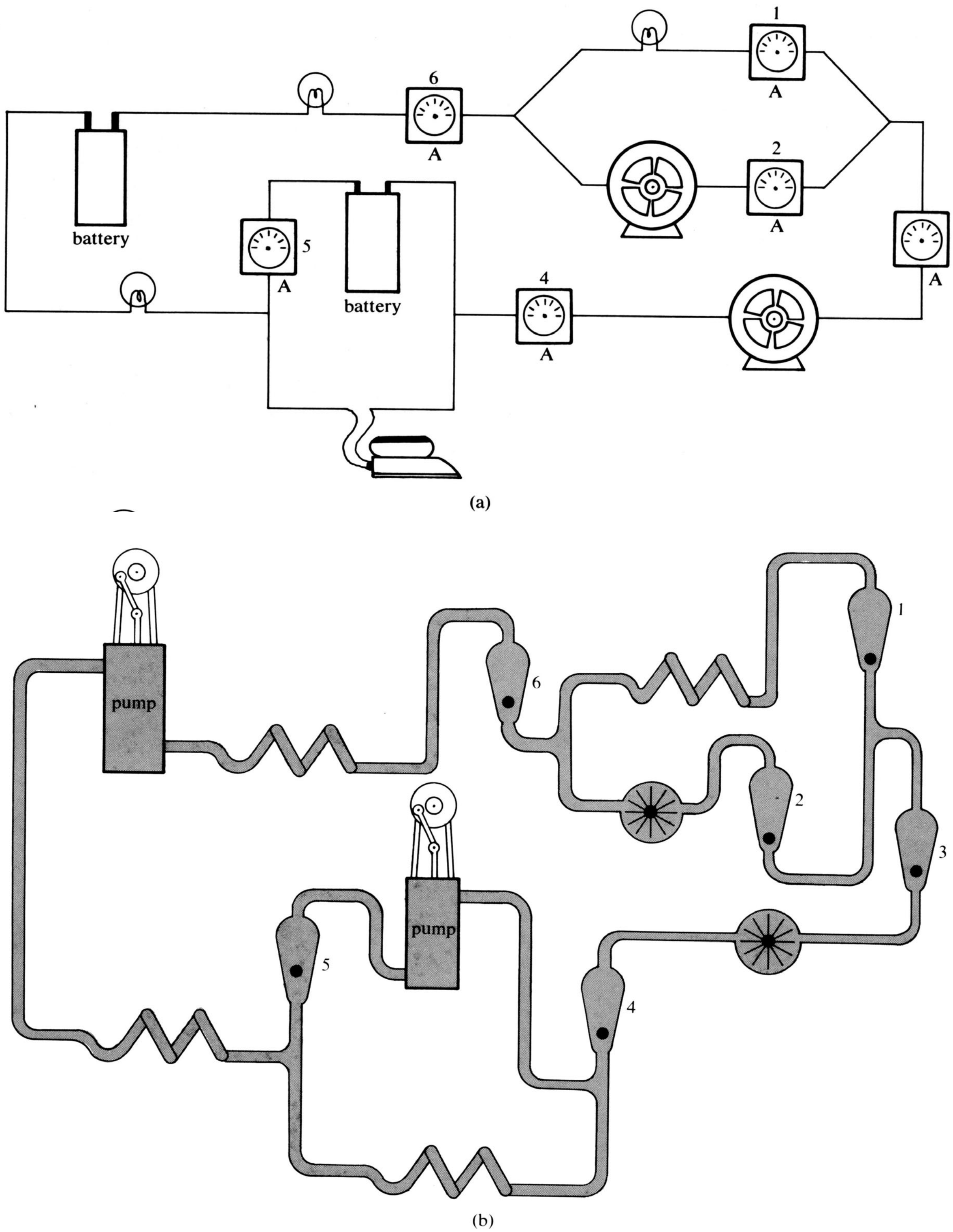


Figure 12.7 Analogue model for an electric circuit. (a) Electric circuit. (b) Water pipe analogue model.

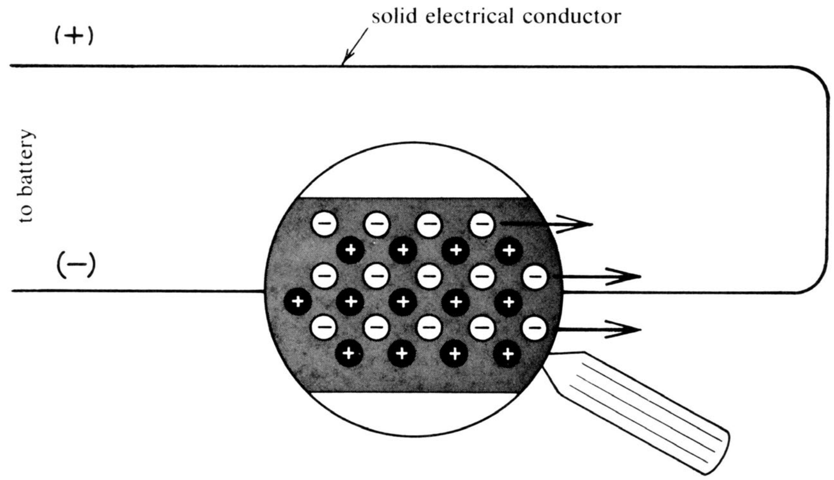


Figure 12.8 Micro-domain model for solid electrical conductors. Solid conductors contain stationary positive ions and a mobile negative "electric fluid" of electrons. The two kinds of charges are present in equal amounts, so that the conductor is electrically neutral.

There is a qualitative difference between the models for electric current in solid conductors (copper wire) and liquid or gaseous conductors (salt water, melted sodium chloride, gases at very high temperatures). In solid conductors, only electrons are in motion (Fig. 12.8). The shape of the solid material is maintained by the atoms and/or ions, which are bound together in a structure in mechanical equilibrium (Sections 4.5 and 11.7). In liquid and gaseous conductors, however, both ions (positive and negative) and/or electrons may be in motion (Fig. 12.9).

Forces on a steady current. According to these models, a *steady current* in an electrical conductor is the steady motion of electrons and/or ions, which we will call the "electric fluid." If you apply the Newtonian theory to this motion, you conclude that the electrons and ions must be

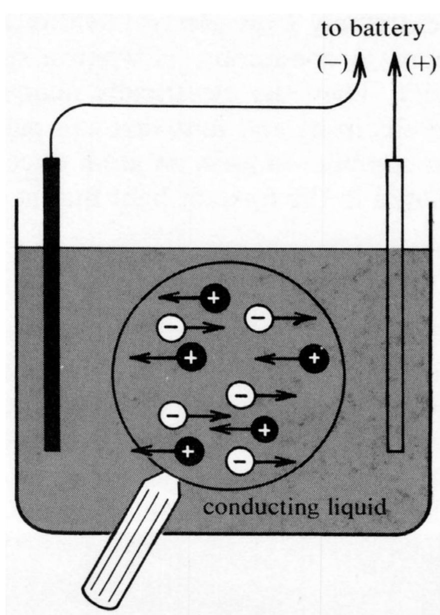


Figure 12.9 Micro-domain model for liquid or gaseous conductors. Liquid or gaseous conductors contain positive and negative mobile "electric fluids" of ions and/or electrons. The two kinds of charges are present in equal amounts so that the conductor is electrically neutral.

in mechanical equilibrium, because they are in steady motion (Section 11.2). Only when the electric current starts or stops is there a change of motion, which requires the action of a net force on the fluid.

Two partial forces play a particularly important role. One of these arises from the interaction-at-a-distance between the power supply and the electric fluid. The intermediary is an electric field set up by the power supply throughout the electric circuit. The other partial force arises from collisions between the moving electrons or ions and the stationary material of the electrical conductor in which the current flows. The combined effect of many collisions is to oppose the motion of the electric fluid relative to the conductor. This interaction is therefore analogous to the frictional interaction between two bodies sliding over one another (Section 11.8), but it occurs throughout the interior of the conductor and is not a surface effect.

When the electric current is steady, the two partial forces just described are equal in magnitude and opposite in direction. The net force acting on the electric fluid is zero. During an extremely short period of time (about 10^{-18} seconds) just after an electric circuit is closed, however, the force transmitted by the electric field is larger than the opposing force arising from collisions. The net force sets the electric fluid in motion and accelerates it until the opposing force, which increases in proportion to the speed of the electric fluid, is large enough to reduce the net force to zero. Similarly, during an extremely short time interval after a closed circuit is opened, the force transmitted by the electric field is smaller than the opposing force arising from collisions. Then the net force slows down the motion of the electric fluid and brings it to a halt.

Insulators and conductors. In this micro-domain model, *electrical conductors* are materials that contain a mobile electric fluid. Nonconductors or *insulators* are materials with no mobile electric fluid. Air, glass, and pure water are good examples of insulators. In insulators, all the electrons and ions are securely bound to one another and are not free to move even when a moderate electric field is applied. When an extremely large electric field is applied, however, an insulator may become a conductor, as when a spark jumps across an air gap in a circuit. Then the electrically neutral atoms gain energy from the electric field and are split into electrons and ions that can move freely. When the electrons and ions combine to form an atom once more, the energy of the system is radiated in the form of light that is seen as a spark.

Henry Cavendish (1731-1810) had identified these two key variable factors (current and voltage) long before Volta's, Oersted's, and Ampère's discoveries. In 1757 he wrote, "The strength of the shock depends rather more on the quantity of fluid [the current] which passes through our body than on the force with which it is impelled."

12.2 Voltage

In the introduction to this chapter, we referred to the two variable factors needed to describe an electric spark: the current, which is related to the brightness or intensity of the spark; and the voltage, which is related to the length of the spark. In the water analogue model, a water jet emerging from a hose represents a spark by a water jet (Fig. 12.10). You can recognize the two variable factors (current and voltage) in this analogue. The flow rate of water corresponds to the current, while the pressure by which it is ejected (and thus its length) corresponds to the

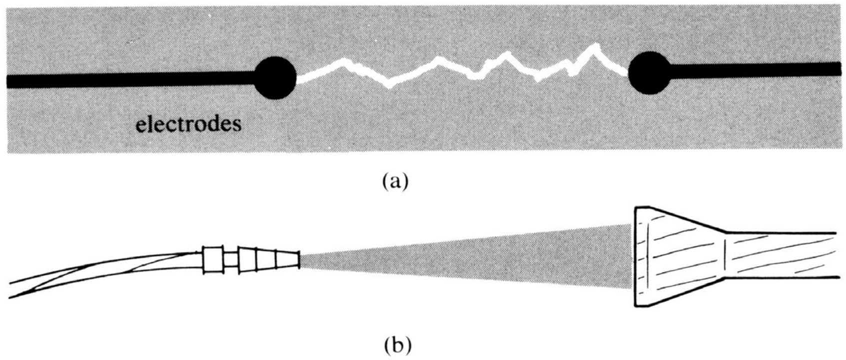


Figure 12.10 Water-jet analogue to an electric spark,
(a) The length of the spark is determined by the voltage,
(b) The length of the water jet is determined by the water pressure.

Equation 12.2

energy transfer (joules)	ΔE
voltage (volts)	$\mathcal{V}$
electric current (amp)	$\mathcal{I}$
time interval (sec)	Δt

$$\Delta E = \mathcal{V}\mathcal{I}\Delta t$$

FORMAL DEFINITION
Voltage is equal to the ratio of the energy transfer to the product of electric current and time interval.

Equation 12.3
(Definition of voltage)

$$\mathcal{V} = \frac{\Delta E}{\mathcal{I}\,\Delta t}$$

EXAMPLE
Energy transfer in a small flashlight in 1 minute:

battery voltage $\mathcal{V}=1.5$ *volt*
current $\mathcal{I}=0.5$ *amp*
time $\Delta t=60$ *sec*

energy transfer:
$\Delta E = 1.5\ \text{volt} \times 0.5\ \text{amp} \times 60\ \text{sec} = 45\ \text{joules}$

voltage. Voltage, therefore, is a sort of "electrical pressure" built up by an electric power supply and dissipated in the wires and circuit elements, just as a water pump creates pressure that drives the water through pipes.

Formal definition of voltage. While this discussion may clarify the meaning of "voltage," it does not suggest how an electrical "pressure gauge" or *voltmeter* may be constructed. To exploit the analogy further, we will relate the water pressure to energy transfer, which is a concept that *does* apply directly to electric circuits. Though we will not describe the details, it is possible to reason from water pressure to the force exerted by the pump, then to the work done by the pump, and ultimately to the energy transfer. The outcome of this procedure is to show that the energy transfer is proportional to the pressure times the flow rate of the water times the time of operation. The voltage of an electric power supply is therefore defined to make the energy transfer from the power supply equal to the voltage (in the unit called the *volt*) times the electric current times the time of operation (Eq. 12.2). The voltage concept is applicable to circuit elements (energy receivers) if the energy transfer to the circuit element is used. This desired relation of voltage and energy transfer leads to a formal definition of voltage (Eq. 12.3) that is applicable to power supplies (energy sources) and to circuit elements (energy receivers). An energy of 1 joule is transferred by a 1-volt power supply delivering 1 ampere for 1 second. A heating element releasing 1 joule per second with a current of 1 ampere operates at 1 volt. To make the voltage concept more concrete, we will explain how to measure the voltage of a battery.

Voltage of batteries. In a battery, a chemical reaction takes place in such a way that the chemical energy of the ingredients is transferred, by means of an electric circuit, to an energy receiver such as a light bulb, a motor, or a piece of wire that gets hot. Joule compared the thermal energy released during a chemical reaction when it took place with the ingredients together in an ordinary container and when the same

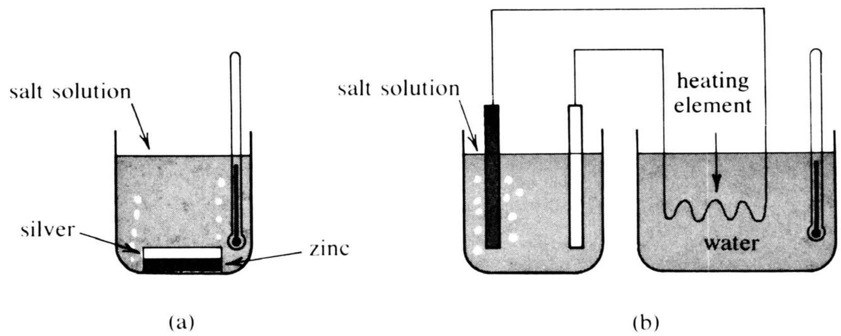
Figure 12.11 (to right) Reaction of zinc with salt solution in contact with silver.

(a) Silver and zinc are in direct contact. Chemical energy is converted to thermal energy as the zinc dissolves and the solution becomes warm.

(b) Silver and zinc are connected by an electrical conductor. Chemical energy is converted to electric field energy and then to thermal energy by the heating element in the separate container of water.

OPERATIONAL DEFINITION
Voltage of a battery can be measured by using the battery to transfer energy to a hot wire in cold water. The voltage is equal to the thermal energy transferred (joules) divided by the product of the electrical current (amperes) and the time elapsed (seconds). See Example 12.1.

"In 1843 I showed that the heat evolved by magneto-electricity is proportional to the [energy] absorbed, and that the [energy] of the electro-magnetic engine is derived from the [energy] of chemical affinity in the battery, [an energy] which otherwise would be evolved in the form of heat."
James Prescott Joule Philosophical Transactions, 1850



reaction took place in a battery and the electric current heated a wire (Fig. 12.11). He found that the energy values in the two experiments were closely equal and concluded that the electric circuit served as a passive coupling element that transferred the energy released in the chemical reaction but did not receive or contribute any additional energy.

By Joule's procedure, it is possible to measure the voltage of a battery. You only have to measure the current in the circuit to the hot wire, the time it operates, and the thermal energy transferred by the hot wire to cold water (Example 12.1). By doing many such measurements, you

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EXAMPLE 12.1 A battery delivers 0.5 ampere to a heating element for 1 hour (3600 seconds) and thereby raises the temperature of 0.10 kilogram of water by 13° Celsius. What is the battery voltage?

(a) Find the energy transfer in joules (Eq.10.2 and Eq 9.3).

$$\Delta E = CM_G \Delta T = 1\ \text{Cal}/^{\circ}\text{C/kg} \times 0.10\ \text{kg} \times 13^{\circ}\text{C} = 1.3\ \text{Cal} \approx 5200\ \text{joules}$$

(b) Find the voltage from Eq.12.3.

$$\mathcal{V} = \frac{\Delta E}{\mathcal{I}\Delta t} = \frac{5200\ \text{joules}}{0.5\ \text{amp} \times 3600\ \text{sec}} = \frac{5200}{1800} \approx 3.0\ \text{volts}$$

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find that the voltage of a battery with one pair of electrodes (Fig. 12.11b) is determined by the chemical reaction occurring in it and is not significantly affected by its size, shape, and even previous operation over short periods of time. Such a battery is called *one cell*, to distinguish it from a battery power supply in which many cells are connected in a chain, such as Volta used (Fig. 12.1). The voltage of a chain

Equation 12.4 (electrical cells connected in a chain, in "series")

battery voltage $= \mathcal{V}$
voltage of one cell $= \mathcal{V}_c$
number of cells $= n$

$$\mathcal{V} = n\,\mathcal{V}_c$$

Equation 12.5 (Ohm's Law, in terms of conductance)

electrical conductance $= \mathcal{C}$
voltage (volts) $= \mathcal{V}$
current (amps) $= \mathcal{I}$

$$\mathcal{I} = \mathcal{C}\,\mathcal{V}$$

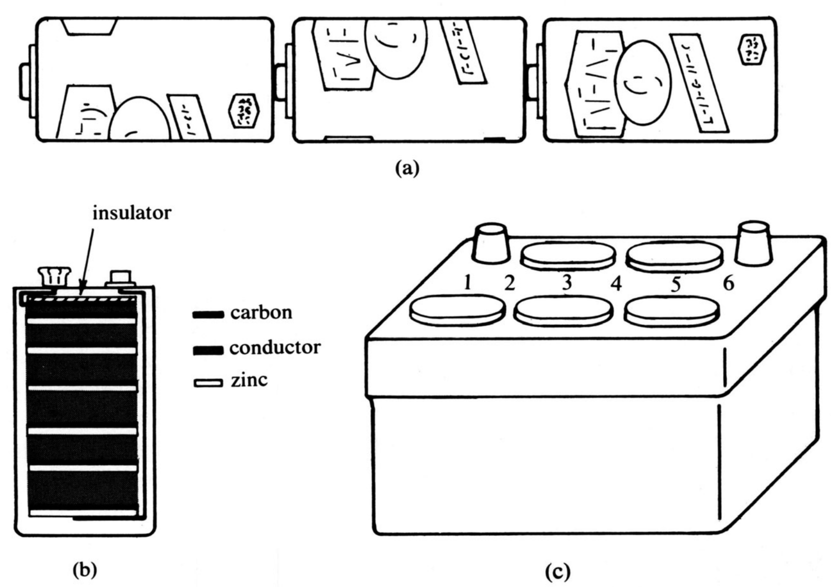
units for conductance: amp/volt

George Simon Ohm (1789-1854). Ohm's father was interested in mathematics and philosophy and prepared George for the University of Erlangen. After obtaining his degree, Ohm turned his attention to the voltaic battery and published several papers on electric conduction in a circuit. His work was rejected by the Berlin Academy, and Ohm retired from scientific life in discouragement. Posterity, however, has been kinder: one of his experimental results has come down to us as Ohm's law.

of cells equals the voltage of one cell times the number of cells (Eq. 12.4 and Fig. 12.12). A large battery consisting of one cell can release more energy than a small one, but it does so by delivering current for a longer time and not at a higher voltage.

Ohm's law and electrical conductance. Voltage and current in an electric circuit correspond to water pressure and flow rate in the analogue model. Now, you would expect that the water flow rate through a particular pipe would depend on the pressure applied by the pump: the greater the pressure, the greater the flow rate, and vice versa. You might therefore expect that electrical conductors would exhibit such a relation also. Indeed, George Ohm investigated the relationship and found that the electric current was directly proportional to the voltage of the power supply. The ratio of current to voltage, which is called the conductance of the conductor, depends on the conducting material, its shape, and its temperature. For example, the larger the conductor's cross-section, the greater its conductance; and the longer the conductor, the smaller its conductance. The mathematical model, that the current in a conductor is equal to the conductance times the voltage applied to the conductor, is called *Ohm's law* (Eq. 12.5 and Fig. 12.13a). Many conductors are well described by this model but there are also many exceptions. For

Figure 12.12 Electrical cells connected in a chain (in "series") to increase the voltage of the power supply,
(a) Three flashlight "D cells" each rated at 1.5 volts, combined to deliver 4.5 volts.
(b) Common rectangular battery, consisting of six 1.5-volt cells, rated at 9 volts.
(c) Automobile storage battery (consisting of six 2-volt lead-acid "wet" cells) rated at 12 volts.



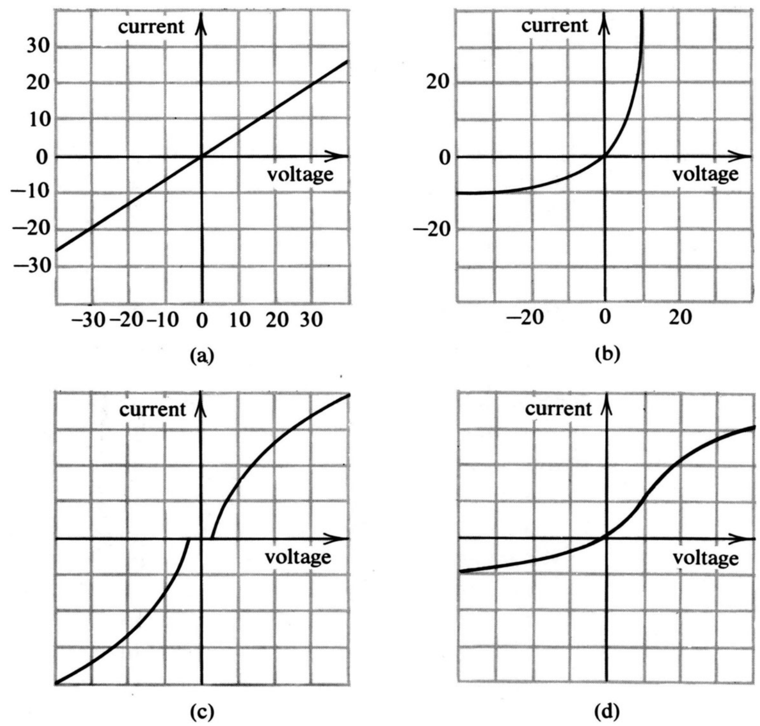


Figure 12.13 Relationship of voltage and current for various circuit elements.
(a) A circuit element described by Ohm's law (slope = conductance = 0.6 amperes per volt).
(b) A diode, which has a high conductance for positive voltage and a low conductance for negative voltage. (See Fig. 12.15)
(c) A solution of sodium chloride. (What occurs when current flows?)
(d) A radio tube.

example, Fig. 12.13 includes graphs of the relationship between current and voltage for four different systems; Ohm's law applies to only one of the graphs.

Ordinary materials show an enormous range of conductance values. It is customary to classify them into three groups: *conductors* (high conductance), *insulators* (extremely low conductance), and *semiconductors* with an intermediate conductance, but closer to that of conductors than insulators, (Table 12.2). The conductance of semiconductor materials is especially sensitive to temperature and to the presence of small amounts of impurities in the material.

Ohm's law and electrical resistance. When we described Ohm's law, we took the viewpoint that the applied voltage forced a current through the conductor. The conductance gave a measure of how much current was forced through. You can also take the viewpoint that the conductor resists the current flow by virtue of the collisions of the electrons and ions with the stationary material of the conductor. According to this viewpoint, the flow of an electric current in a conductor requires the application of a voltage that is proportional to the current. The ratio of voltage to current is called the *resistance* of the conductor. Ohm's law can be written in a form expressing this view (Eq. 12.6). Since the two

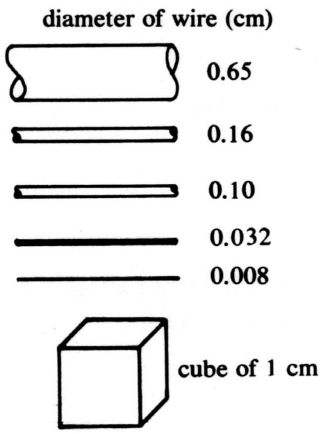


TABLE 12·2 CONDUCTANCE AND RESISTANCE

A. Wire, 1 meter long, circular cross section

Material	*Diameter (cm)*	*Conductance (amp/volt)*	*Resistance (volts/amp)*
copper	0.65	2000	5×10^{-4}
	0.16	120	8×10^{-3}
	0.10	48	2.1×10^{-2}
	0.008	0.30	3.3
aluminum	0.16	73	1.4×10^{-2}
iron	0.16	21	4.8×10^{-2}
tungsten			
(20° Celsius)	0.008	0.09	11
(3000° Celsius)	0.008	4.5×10^{-3}	220
nichrome	0.16	2.1	0.48
	0.032	0.083	12.1

B. Cube-shaped piece of material, 1-centimeter-long edges

Material	*Conductance (amp/volt)*	*Resistance (volts/amp)*
copper	6×10^{5}	1.7×10^{-6}
iron	1×10^{5}	1×10^{-5}
nichrome	1×10^{4}	1×10^{-4}
semiconductor	$10^{-3} - 10^{3}$	$10^{3} - 10^{-3}$
insulator (glass)	$10^{-14} - 10^{-10}$	$10^{14} - 10^{10}$

Equation 12.6 (Ohm's Law, in terms of resistance)

electrical resistance (volt/amp) = $\mathcal{R}$
voltage (volts) = $\mathcal{V}$
current (amps) = $\mathcal{I}$

$$\mathcal{V} = \mathcal{R}\mathcal{I}$$

Equation 12.7

resistance (amp/volt) = $\mathcal{R}$
conductance (volt/amp) = $\mathcal{C}$

$$\mathcal{R} = 1/\mathcal{C}$$

$$\mathcal{C} = 1/\mathcal{R}$$

equations state the same physical law (direct proportionality of current and voltage) in different forms, the conductance and the resistance of a conductor represent the same physical properties and can be directly calculated from one another (Eq. 12.7, Table 12.2, and Example 12.2). You might ask why we define two separate quantities that are so closely related. The reason is that they are each directly applicable to analyzing different types of electric circuits, which we will explain below. A circuit element described by Ohm's law is often called a *resistor*.

Voltmeters. We have so far not described any instruments for measuring voltage. When we discussed battery voltage, we did outline a procedure for measuring voltage by first determining thermal energy release (Example 12.1), but it is cumbersome and not useful for routine applications. After the discovery of Ohm's law, it became possible to calculate the voltage applied to a conductor from a measurement of the electric current with an ammeter (Fig. 12.14). The combination of an ammeter and a resistor with known resistance, therefore, serves as a voltmeter. The voltmeter dial must be calibrated once by using Joule's

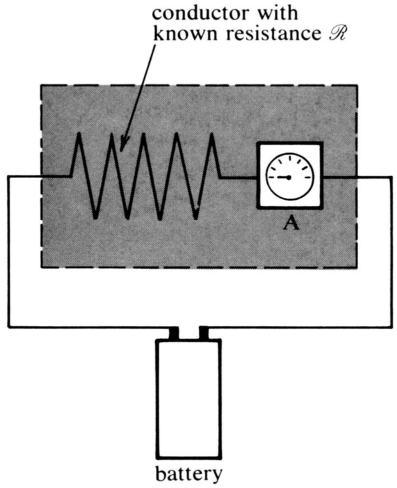


Figure 12.14 A conductor with a known large resistance is combined with an ammeter to serve as a voltmeter.

EXAMPLE: The resistance is 10^4 volts per ampere. The ammeter dial reads 4.5 x 10^{-4} ampere. What is the voltage?
10^4 volts/amp x 4.5 x 10^{-4} amp = 4.5 volts

procedure (Example 12.1), but thereafter it can be used simply and efficiently for voltage measurements.

EXAMPLE 12.2

(a) Find the resistance of the circuit element described in Fig. 12.13a. For a current of 10 amperes, the voltage is 17 volts.

$\mathcal{I} = 10\,\text{amp},\ \mathcal{V} = 17\,\text{volts}$

$$\mathcal{R} = \frac{\mathcal{V}}{\mathcal{I}} = \frac{17\,volts}{10\,amp} = 1.7\frac{volts}{amp};\ (\text{or } \mathcal{C} = \frac{1}{1.7} = 0.6 \text{ amp/volt, as in Fig 12.13a})$$

(b) A circuit element with a resistance of 5 volts per ampere is designed to draw an electric current of 0.6 ampere. What can you conclude? You can find the conductance (Eq. 12.7), voltage (Eq. 12.6), and energy transfer (Eq. 12.2).

Circuit elements that do not follow Ohm's Law (non-Ohmic elements). Modern electronic devices, such as vacuum tubes, diodes, transistors, and television picture tubes, are complex electric circuit elements that are not described by Ohm's law. The relationships of current and voltage are much more complicated. Diodes, for instance, are circuit elements that permit a large current to flow in one direction, but only a very small current in the opposite direction (Fig. 12.13b). In other words, they act somewhat as a one-way valve (Fig. 12.15). Vacuum tubes and transistors have more than two connections with an electric circuit. These devices can serve as "amplifiers" in that a small current flowing between two of the connections triggers a large current between two other connections.

Superconductors. Another class of circuit elements that are not described by Ohm's law is formed when certain metallic conductors are cooled to extremely low temperatures (-270° Celsius). They appear to

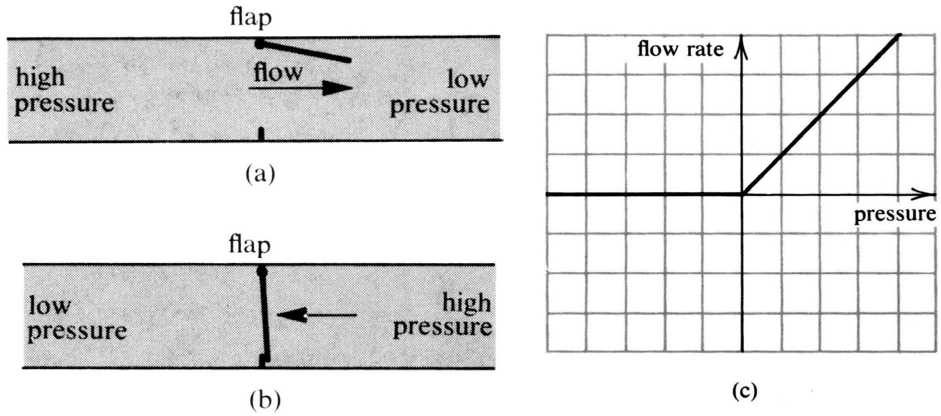
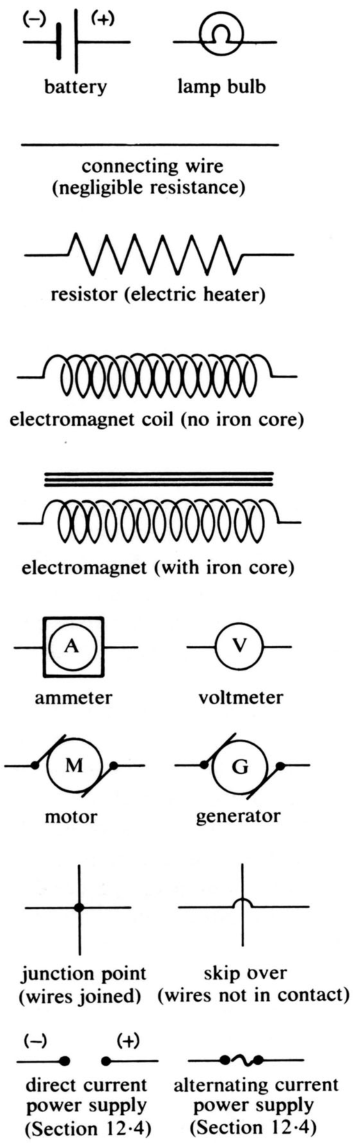


Figure 12.15 Water analogue model for a diode (one-way valve),
(a) High pressure on the left gives rise to a flow that opens the valve.
(b) High pressure on the right closes the valve,
(c) Graphical representation shows flow increasing with positive applied pressure (high pressure at the left) and zero flow for negative applied pressure (high pressure at the right).



Symbols for electric circuit elements, power supplies, and connections. The connecting wires are usually drawn in rectangular shapes, as much as possible along vertical and horizontal lines.

have zero resistance, so that zero applied voltage is necessary to maintain a steady electric current in them. Such materials are called *superconductors*. In a ring-shaped superconductor, an electric current that is injected continues to flow for months thereafter without benefit of a power supply. The micro-domain theory explains super-conductors as materials in which no collisions inhibit the motion of the electrons. The inertia of the electrons, therefore, results in a motion that persists. This is an example of uniform motion in the micro domain.

12.3 Analysis of simple circuits

The voltage and current concepts can be used to analyze electric circuits that include many circuit elements. To enable you to make diagrams of circuits, we have prepared a list of the generally accepted symbols that represent circuit elements.

Series and parallel connections. Two important principles of circuit analysis are derived from two especially simple types of circuit connections. In one, several circuit elements are connected end to end (Fig. 12.16). They are said to be connected *in series*. In the other one, several circuit elements are connected at two common terminals (Fig. 12.17). They are then said to be connected *in parallel.*

Series connection. The significance of the series connection is that all the circuit elements pass the same current, since there is no branch where part of the current could be diverted. The voltage applied to the entire system, however, can be divided into parts that are applied to each circuit element, in proportion to the energy transfer to each of them (Fig. 12.18). You should therefore think of a single current and partial voltages. For each resistor (circuit element that is described by

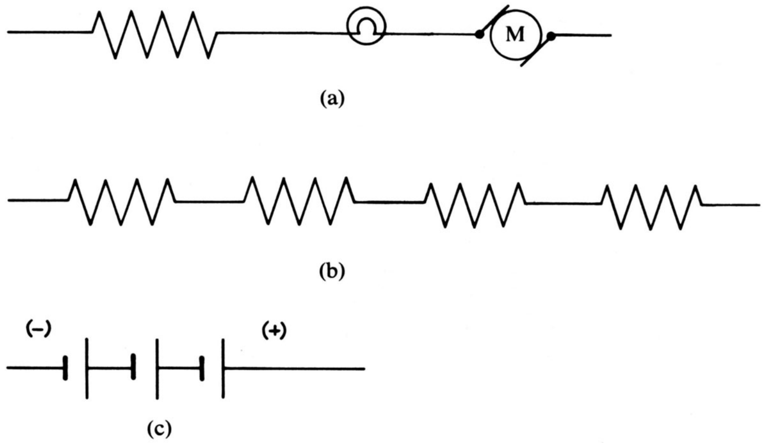


Figure 12.16 Electric circuit elements connected in series, (a) Resistor, lamp bulb, motor, (b) Four resistors, (c) Three batteries.

Equation 12·8

current through series connection	$\mathcal{I}$
partial voltages	$\mathcal{V}_1, \mathcal{V}_2, \ldots$
resistances of resistors	$\mathcal{R}_1, \mathcal{R}_2, \ldots$

$$\mathcal{V}_1 = \mathcal{R}_1\mathcal{I}$$
$$\mathcal{V}_2 = \mathcal{R}_2\mathcal{I}$$
$$\vdots$$

Ohm's law), the partial voltage is equal to the resistance times the current (Eq. 12.8). If Ohm's Law describes all the circuit elements connected in series, you can add the formulas for the partial voltages and arrive at Ohm's law for the entire circuit (Eq. 12.9a). You can see that the resistance of the entire circuit is equal to the sum of the resistances of all the circuit elements (Eq. 12.9b).

Parallel connection. The significance of the parallel connection is that the same voltage is applied to all the circuit elements, because they all are placed between the same two terminals. The current in the entire circuit can be divided into partial currents that flow through the several circuit elements (Fig. 12.19). You should therefore think of a single voltage and partial currents. For each circuit element that is described by Ohm's law, the partial current is equal to the conductance times the voltage (Eq. 12.10). If Ohm's Law describes all the circuit elements connected in parallel you can add the formulas for the partial currents

Equation 12·9

voltage applied to circuit	$\mathcal{V}$
resistance of series connection	$\mathcal{R}$

$$\begin{aligned}\mathcal{V} &= \mathcal{V}_1 + \mathcal{V}_2 + \cdots \\ &= \mathcal{R}_1\mathcal{I} + \mathcal{R}_2\mathcal{I} + \cdots \\ &= (\mathcal{R}_1 + \mathcal{R}_2 + \cdots)\mathcal{I} \qquad \text{(a)} \\ &= \mathcal{R}\mathcal{I}\end{aligned}$$

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \cdots \qquad \text{(b)}$$

Equation 12·10

voltage applied to parallel connection	$\mathcal{V}$
partial currents	$\mathcal{I}_1, \mathcal{I}_2, \cdots$
conductances of resistors	$\mathcal{C}_1, \mathcal{C}_2, \cdots$

$$\mathcal{I}_1 = \mathcal{C}_1\mathcal{V}$$
$$\mathcal{I}_2 = \mathcal{C}_2\mathcal{V}$$

Figure 12.17 Electric circuit elements connected in parallel, (a) Resistor, lamp bulb, motor, (b) Four resistors, (c) Three batteries.

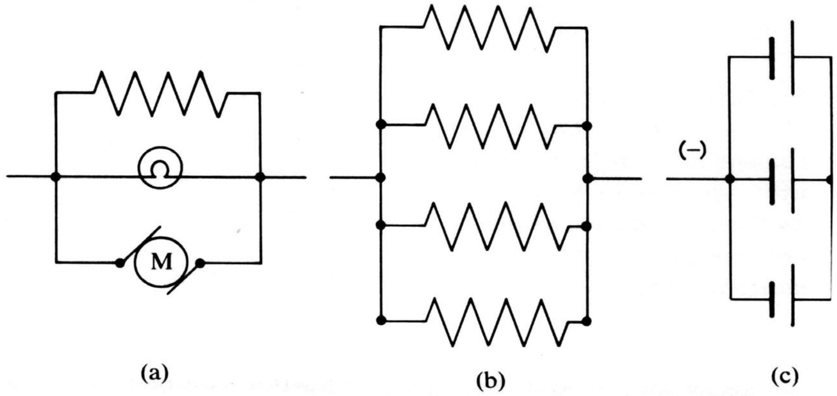
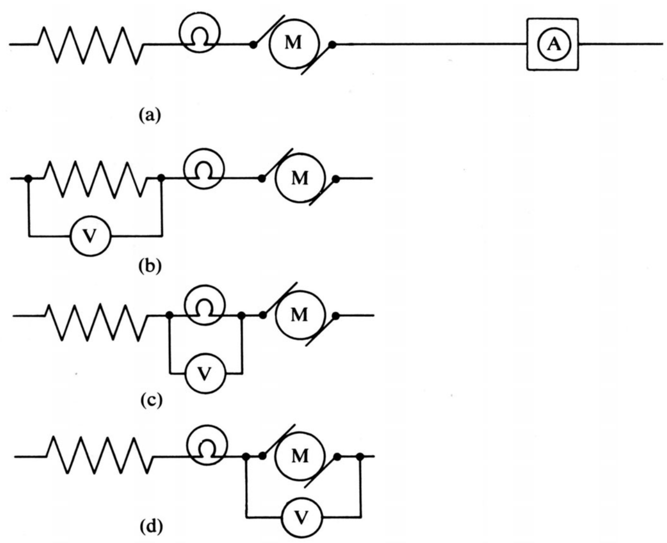


Figure 12.18 Current and voltage for a series connection.
(a) The current through the three circuit elements is measured by the ammeter connected in series. The same current must flow in all elements.
(b), (c), and (d) The partial voltage applied to each circuit element is measured by the voltmeter connected in parallel with that circuit element. Thus the total voltage is equal to the sum of the three individual voltages.
If all circuit elements follow Ohm's Law, Eq. 12.9 applies, and the total resistance is equal to the sum of the three individual resistances. However, in this circuit the motor does not follow Ohm's Law.



Equation 12·11

current through entire circuit $\mathcal{I}$
conductance of parallel connection $\mathcal{C}$

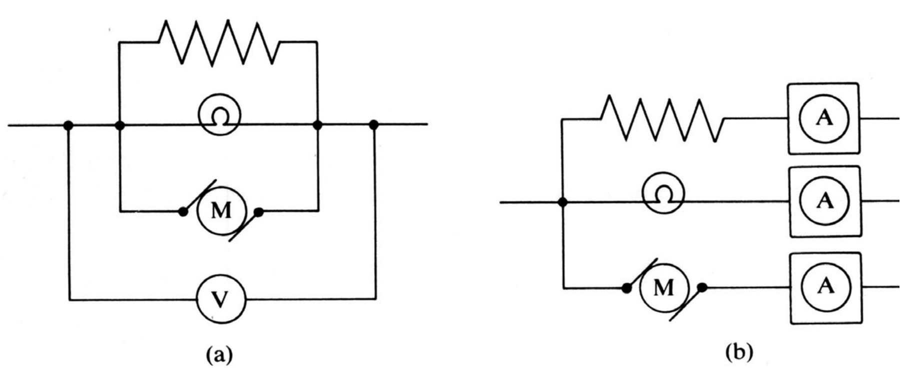
$$\begin{aligned}\mathcal{I} &= \mathcal{I}_1 + \mathcal{I}_2 + \cdots \\ &= \mathcal{C}_1\mathcal{V} + \mathcal{C}_2\mathcal{V} + \cdots \\ &= (\mathcal{C}_1 + \mathcal{C}_2 + \cdots)\,\mathcal{V} \qquad \text{(a)} \\ &= \mathcal{C}\mathcal{V}\end{aligned}$$

$$\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \cdots \qquad \text{(b)}$$

and arrive at Ohm's law for the entire circuit (Eq. 12.11a). You can see that for a parallel circuit, the conductance of the entire circuit is equal to the sum of the conductances of all the circuit elements (Eq. 12.11b).

Applications. Most real circuits that include a power supply and one or more circuit elements also include connecting wires whose resistance is very low compared to that of the circuit elements.

Figure 12.19 Voltage and current for a parallel connection,
(a) The voltage applied to the circuit is measured by a voltmeter in parallel with the circuit. Therefore, the voltage is the same for all circuit elements.
(b) The partial current in each circuit element is measured by an ammeter in series with that circuit element. The total current is the sum of the currents flowing through the individual circuit elements.



Since the wires are connected in series with the circuit element, the resistance of the circuit including the wires is only very slightly higher than that of the circuit element alone. It is customary, therefore, to make mathematical models in which the connecting wires are assigned zero resistance. When this is done, the partial voltage applied to a connecting wire and the energy transfer to it are zero (Eq. 12.8).

Since all the circuit elements in a series connection pass the same current, they all operate (or do not operate) simultaneously. Some Christmas tree light strings use a series connection. This is inconvenient in most domestic and industrial applications, however, where appliances or pieces of machinery are used individually. In these applications, therefore, all circuit elements are connected in parallel. Then each one operates under the same applied voltage and each one draws a partial current according to its requirements. The entire residence or factory, of course, draws a current that is the sum of all partial currents (Eq. 12.11a).

The series and parallel concepts can also be used to extend the entries for the conductances and resistances of pieces of wire in Table 12.2. Just think of a very long piece of wire as a certain number of 1-meter long segments in a series connection. Its resistance is therefore directly proportional to its length (Example 12.3). Similarly, a thick wire can be imagined as a bundle of thin wires (each of the same length) all connected in parallel. Thus, the conductance of a wire is directly proportional to its cross-sectional area (Example 12.4).

EXAMPLE 12.3. Using the value of resistance from Table 12.2, find the resistance of a 15-meter-long coil of 0.6 centimeter-diameter iron wire.

Solution: Think of the wire as 15 pieces, each 1 meter long, in a series connection.

$$\mathscr{R}_1 = \mathscr{R}_2 = \mathscr{R}_3 = \cdots = \mathscr{R}_{15} = 4.8 \times 10^{-2} \frac{\text{volt}}{\text{amp}}$$

By Eq. 12·9b,

$$\mathscr{R} = \mathscr{R}_1 + \mathscr{R}_2 + \cdots + \mathscr{R}_{15} = 15\mathscr{R}_1 = 15 \times 4.8 \times 10^{-2} \frac{\text{volt}}{\text{amp}}$$

$$= 0.72 \frac{\text{volt}}{\text{amp}}$$

EXAMPLE 12.4. Compare the conductance of two 1-meter-long pieces of nichrome wire that have diameters of 0.16 centimeter and 0.032 centimeter.

Solution: The diameters of the two wires are in the ratio of 0.16:0.032= 5:1. Since the area of a circle varies as the second power of the radius, the areas are in the ratio of 5^2:1 or 25:1. Thus, the thick wire

Equation 12·12

energy transfer (joules) ΔE
voltage (volts) $\mathcal{V}$
current (amp) $\mathcal{I}$
power (watts) $\mathcal{P}$
time interval (sec) Δt

$$\mathcal{P} = \frac{\Delta E}{\Delta t} = \mathcal{V}\mathcal{I}$$

Units of power:

$$1 \text{ watt} = 1 \text{ joule/sec} = 1 \text{ volt} \times \text{amp}$$
$$1 \text{ kilowatt} = 1000 \text{ watts}$$
$$1 \text{ horsepower} = 750 \text{ watts}$$

EXAMPLE

Electric toaster: $\mathcal{V} = 110$ *volts*, $\mathcal{I} = 8.2$ *amp*

$$\mathcal{P} = \mathcal{V}\mathcal{I} = 110 \text{ volts} \times 8.2 \text{ amp} = 900 \text{ watts}$$

Equation 12·13

energy of 1 *kilowatt-hour (kWh)*

$\mathcal{P} = 1$ *kilowatt* $= 1000$ *watts*
$\Delta t = 1$ *hr* $= 3600$ *sec*

$$\begin{aligned}\Delta E &= \mathcal{P}\Delta t \\ &= 1 \text{ killowatt} \times 1 \text{ hr} \\ &= 1 \text{ kWh} \\ &= 1000 \text{ watts} \times 3600 \text{ sec} \\ &= 3.6 \times 10^6 \text{ joules} \\ &\approx \frac{3.6 \times 10^6 \text{ joules}}{4.0 \times 10^3 \text{ joules/Cal}} \\ &= 900 \text{ Cal}\end{aligned}$$

EXAMPLE

Express the energy consumed by a 150-*watt lamp bulb, operating for* 10 *hr, in various units.*

$\mathcal{P} = 150$ *watts* $= 0.15$ *kilowatt*
$\Delta t = 10$ *hr*

$$\begin{aligned}\Delta E &= \mathcal{P}\Delta t \\ &= 0.15 \text{ kilowatts} \times 10 \text{ hr} \\ &= 1.5 \text{ kWh} \\ &= 5.4 \times 10^6 \text{ joules} \\ &\approx 1300 \text{ Cal}\end{aligned}$$

contains the same amount of nichrome as a bundle of 25 pieces of the thin wire, and we can consider the thick wire as equivalent to a bundle of 25 thin wires connected in parallel. This model leads to the prediction that the thick wire has 25 times the conductance of the thin wire, $\mathcal{C} = 25\ \mathcal{C}_1$. We can get the conductances from Table 12.2:

$$\mathcal{C} = 2.1 \text{ amp/volt}, \mathcal{C}_1 = 0.083 \text{ amp/volt}$$

$$25\mathcal{C}_1 = 25 \times 0.083 \frac{\text{amp}}{\text{volt}} = 2.1 \frac{\text{amp}}{\text{volt}}$$

12.4 Energy transfer

We have repeatedly pointed out that the electric current in a circuit acts as a passive coupling element that transfers energy from the electric power supply (battery, generator) to an energy receiver (lamp, motor). The quantity of energy transfer in a time interval is equal to the voltage times the current times the duration of the time interval (Eq. 12.2 used to define voltage). This relationship may be applied to the electric power supply to find the total energy transfer to the circuit, or it may be applied to each circuit element separately, to determine the energy transfer to that particular circuit element.

Electric power. The energy transfer in an electric circuit is proportional to the time of operation (Eq. 12.2). When an electric iron is operated for a whole hour, two times as much heat is produced as during half an hour. The significant quantity here is the rate of energy transfer ($\Delta E/\Delta t$), which is called the *electric power* (Eq. 12.12). The electric power, equal to the voltage times the current, is measured in *watts*. An appliance operating at a level of 1 watt for 1 second receives 1 joule of energy.

Most home appliances are rated in watts. Light bulbs range in power from 2.5 watts for a dim night-light to 100 watts for a good reading lamp to 500 watts for a brilliant floodlight. Electric toasters and electric irons usually are rated around 1 kilowatt. Motor-driven appliances such as washing machines and vacuum cleaners usually are rated at a few hundred watts.

Since the joule is a very small unit of energy, the much larger kilowatt-hour is used to describe the consumption of electric power. One *kilowatt-hour* is the amount of energy transferred when a 1-kilowatt appliance (such as a heater) operates for 1 hour. A kilowatt-hour is more than 3.5 million joules, or almost 1000 Calories (Eq. 12.13).

Direct and alternating current. In Section 12.1 we described electric current as a circulation of an "electric fluid" analogous to the circulation of water in pipes. The water in the analogue model is an intermediary in the transfer of energy. One way to accomplish energy transfer is shown in Fig. 12.20a, where a pump circulates the

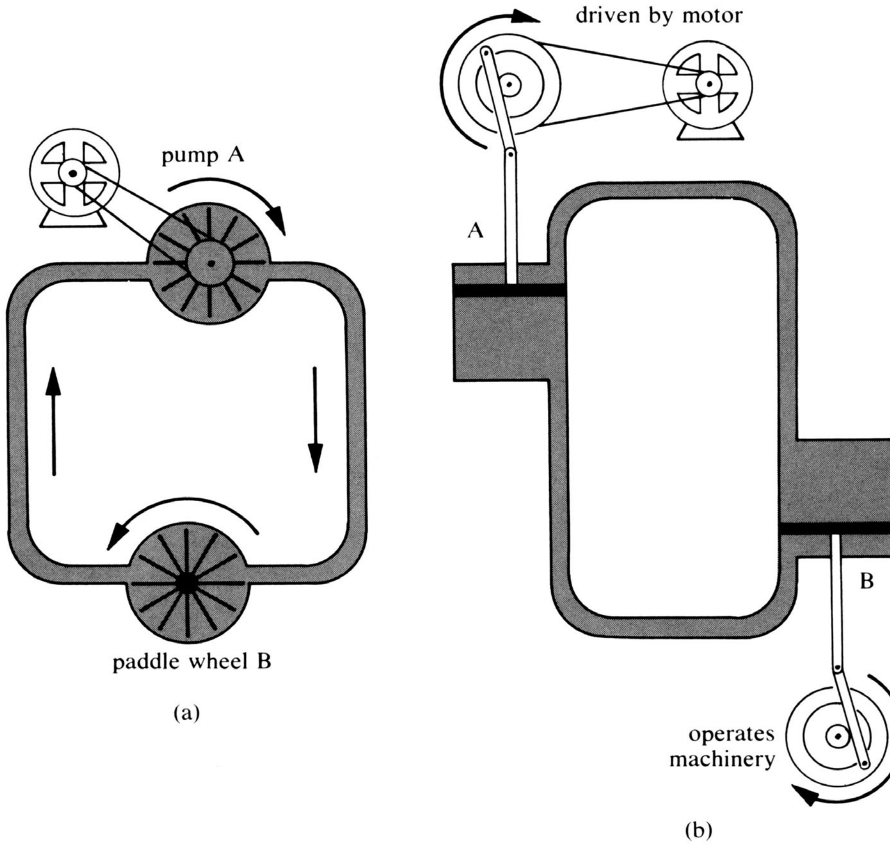


Figure 12.20 Energy transfer in a water pipe circuit,
(a) Circulating water transfers energy from pump A to paddle wheel B.
(b) Oscillating water transfers energy from piston A to piston B.

water and the water drives a paddle wheel. Another way to achieve energy transfer is shown in Fig. 12.20b, where one piston (driven by a motor) pushes the water in the pipes back and forth, and the water drives a second piston, which turns a wheel that can operate machinery. In the second system, the water does not actually circulate through the pipes, but moves alternately a short distance back and forth.

In an electric circuit, energy can be transferred in either of these two ways. In the first, the electric power supply establishes an electric field in one direction around the circuit and the current flows in that direction continuously; this is called *direct current (DC)*. Alternatively, the power supply establishes an electric field first in one direction and then in the other. The current, therefore, also alternates and is called *alternating current* (AC).

A chemical battery ordinarily gives rise to DC, because the type of metal in the battery electrodes determines the direction of the electric field. By an ingenious arrangement of switches, however, the battery connection can be reversed in a regular fashion, so as to produce AC (Fig. 12.21).

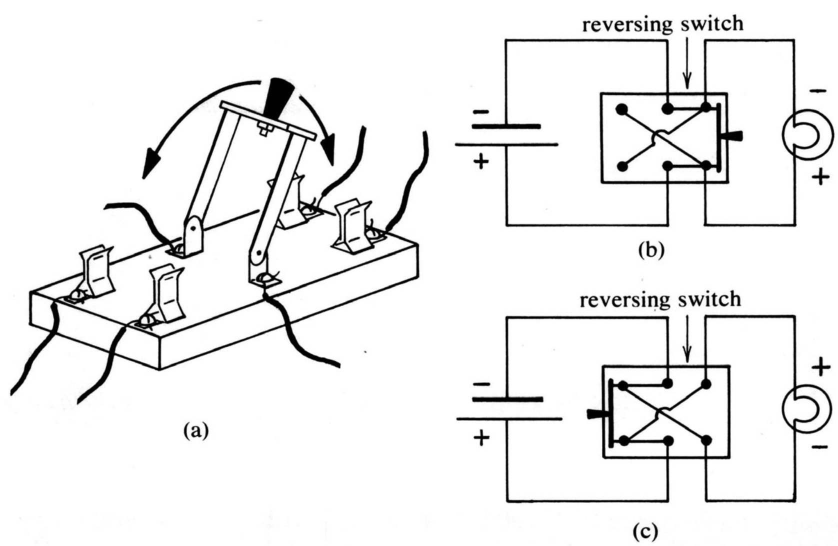
Electric generators can also produce either AC or DC, again depending on a suitable arrangement of the connections between the generator and the electric circuit. The important technological advantage of AC over DC is that the voltage and current of AC can be changed without appreciable energy loss (using a circuit element called a *transformer*). This cannot be done so simply with direct current. Electric power is most efficiently transferred over long distances (from generator to distribution) at high voltage and low current. Transformers then convert the AC to lower (and safer) voltages for residential and industrial use (usually 110 or 220 volts in the US). For this reason, alternating current is used in most industrial and residential applications. In the US, the alternating current goes through a complete cycle of flowing back and forth 60 times each second; it is therefore called 60-cycle alternating current.

Applications of alternating current. Motor-driven appliances can be designed for operation on either direct or alternating current. By referring to Fig. 12.20, you can readily see the need for matching the appliances correctly with the type of current. If the paddle wheel were to be subjected to "alternating" water flow, it would never advance significantly in either direction. Similarly, the piston would block a "direct" water flow once it had traveled to the end of its cylinder.

Appliances that produce thermal energy (lamp bulbs, heaters, electric irons, and so on) can be operated equally well on direct or alternating current. The reason for this is most easily recognized from the micro-domain model for electric current.

In Section 12.1, we pointed out that the micro-domain electric charges (electrons and ions) that comprise the "electric fluid" move at constant

Figure 12.21 A battery and a reversing switch produce alternating current,
(a) A reversing switch,
(b) The switch is closed in one direction.
(c) The switch is closed in the other direction.



speed. They are in mechanical equilibrium subject to two interactions: the electric field established by the power supply and the effect of their collisions with the particles making up the conductor in the MIP model for matter As a result of the collisions, the conductor particles gain kinetic energy, which manifests itself as thermal energy in the macro domain (Section 10.5).

The 60-cycle alternating current switches back and forth very slowly compared to the time required for the motion of the "electric fluid" to adjust to changes in conditions. Even though the electric field alternates and the fluid flow alternates, the mechanical equilibrium description of the fluid is approximately valid. The only significant difference from direct current is that the collisions with the conductor particles take place sometimes from one side, sometimes from the other. But this variation does not affect the transfer of energy and therefore the thermal energy produced in a heating element.

In spite of the difference between alternating and direct current, the mathematical models for energy transfer (Eq. 12.2) and electric power (Eq. 12.12) can still be used. Ohm's law (Eqs. 12.5 and 12.6) also applies to many conductors of alternating current (Example 12.5).

EXAMPLE 12.5. An electric toaster is to be designed with nichrome wire heating elements. Four heating elements are needed, one for each side of two slices of bread. The toaster will operate on 110-volt alternating current. About 1 kilowatt of power can be used.

(a) Find the electric current for 1 kilowatt.

(b) Find the resistance and conductance of the toaster.

(c) Design the toaster with four equal heating elements in parallel, so that it will operate partly, even though one element is broken.

(d) For comparison, design the toaster with four elements in series.

Solution:

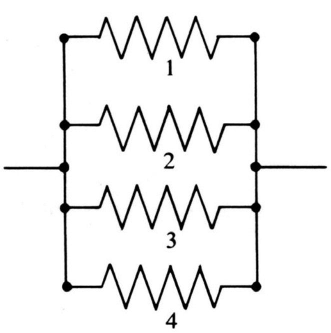
(a) $\mathscr{P} = 1000$ watts, $\mathscr{V} = 110$ volts

$$\mathscr{I} = \frac{1000 \text{ watts}}{110 \text{ volts}} = 9.1 \text{ amp}$$

(b) $\mathscr{I} = \mathscr{C}\mathscr{V}$

$$\mathscr{C} = \frac{\mathscr{I}}{\mathscr{V}} = \frac{9.1 \text{ amp}}{110 \text{ volts}} = 0.083 \frac{\text{amp}}{\text{volt}}$$

$$\mathscr{R} = \frac{1}{\mathscr{C}} = \frac{1}{0.083 \text{ amp/volt}} = 12 \frac{\text{volts}}{\text{amp}}$$



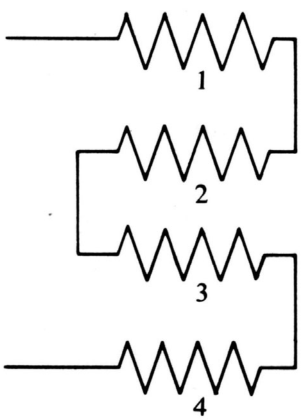
(c) Make the four elements equal, $\mathscr{C}_1 = \mathscr{C}_2 = \mathscr{C}_3 = \mathscr{C}_4$. For parallel connection, $\mathscr{C} = \mathscr{C}_1 + \mathscr{C}_2 + \mathscr{C}_3 + \mathscr{C}_4 = 4\mathscr{C}_1$.

$$\mathscr{C}_1 = \frac{1}{4}\mathscr{C} = \frac{1}{4} \times 0.083 \frac{\text{amp}}{\text{volt}} = 0.021 \frac{\text{amp}}{\text{volt}}$$

$$\mathcal{R}_1 = \frac{1}{\mathcal{C}_1} = \frac{1}{0.021\, amp/volt} = 48\, volts/amp$$

Consult Table 12.2 for properties of nichrome wire. One meter of 0.032-centimeter-diameter wire has a resistance of 12 volts per ampere. One 4-meter-long piece of wire therefore has 48 volts per ampere resistance. This is what is needed.

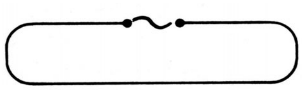
Each heating element is made of 4 meters of nichrome wire, 0.032 centimeter in diameter. The 4 meters of wire can be wound back and forth to radiate energy to one side of a slice of bread. The entire toaster (four elements) requires 16 meters of wire.



(d) Make the four elements equal, $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = \mathcal{R}_4$. For a series connection (shown at left), $\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4 = 4\mathcal{R}_1$.

$$\mathcal{R}_1 = \frac{1}{4}\mathcal{R} = \frac{1}{4} \times 12 \frac{volts}{amp} = 3 \frac{volts}{amp}$$

Consult Table 12.2 for properties of nichrome wire. One meter of 0.32-centimeter-diameter wire has a resistance of 12 volts per ampere, just what is needed for the entire toaster. Each of the four heating elements must, therefore, be made from ¼ meter or 25 centimeters (10 inches), which has a resistance of 3 volts per ampere. This is a very short piece of wire to use for radiating energy to a slice of bread. It would be better to use a much longer piece of the thicker nichrome wire with 0.16-centimeter diameter.



A short circuit

A "short circuit" (shown above) occurs when the power supply transfers energy solely to a conducting wire of low resistance, rather than to a circuit element. This results in a very high current,a sudden, large energy transfer, and a very hot, melted, or vaporized, wire. (See Example 12.6.) For obvious reasons, short circuits can be dangerous and should be avoided.

The "short circuit." In the description of series and parallel connections of circuit elements (Section 12.3), we pointed out that the resistance of the connecting wires is negligibly small. This statement is not true when the circuit is closed with no circuit element other than a conducting wire. Then the connecting wire is the energy receiver in the circuit and serves as a heating element. Its resistance can no longer be neglected, because it is the entire resistance of the circuit. Enormous power is delivered to the circuit under these conditions (Example 12.6). Such a circuit, which lacks a useful energy-receiving circuit element, is called a *short circuit*.

EXAMPLE 12.6. A "short circuit." A 6-foot-long extension cord is plugged into a wall outlet (110 volts) and develops a short circuit because the metal wires touch at a place of worn insulation halfway between its ends. Estimate the consequences if no fuse or circuit breaker is in the circuit. Extension cords contain a bundle of copper strands whose combined diameter is 0.10 centimeter and whose mass is 7 grams per meter.

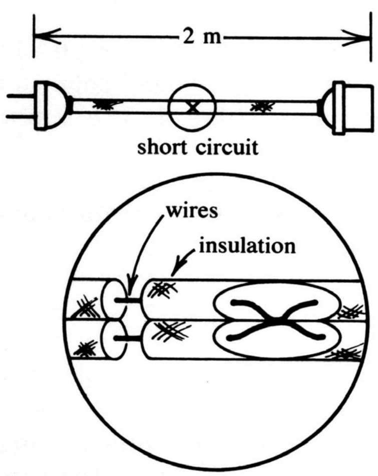
(a) Calculate the conductance of the wire.
(b) Calculate the current in the wire.

(c) Calculate the power delivered.

(d) Calculate the thermal energy produced in the wire in 1 second and the resulting temperature rise of the wire in 1 second. (Recall Eq. 9.3, 1 Cal = 4 x 10^3 joules; Eq. 12.13, $\Delta E = \mathscr{P}\Delta t$; Equation 10.2, $\Delta E = CM_G\Delta T$; and from Table 10.4, specific heat of copper, C = 0.093 Cal/deg C/kg).

Solution:

(a) The length of copper wire involved in the short circuit is about 6 feet or 2 meters. The resistance (Table 12.2 and Example 12.3) is



$$\mathscr{R} = 2 \times 2.1 \times 10^{-2} \frac{\text{volt}}{\text{amp}} = 4.2 \times 10^{-2} \frac{\text{volt}}{\text{amp}}$$

The conductance is

$$\mathscr{C} = \frac{1}{4.2 \times 10^{-2}\ \text{volt/amp}} \approx 24 \frac{\text{amp}}{\text{volt}}$$

(b) $\mathscr{I} = \mathscr{C}\mathscr{V} = 24 \dfrac{\text{amp}}{\text{volt}} \times 110 \text{ volts} \approx 2.6 \times 10^3 \text{ amp}$

(c) $\mathscr{P} = \mathscr{V}\mathscr{I} \approx 1.1 \times 10^2 \text{ volts} \times 2.6 \times 10^3 \text{ amp} \approx 2.9 \times 10^5 \text{ watt}$

(d) Find the thermal energy produced in 1 second.

$$\Delta E = \mathscr{P}\Delta t \approx 2.9 \times 10^5 \text{ watts} \times 1 \text{ sec} = 2.9 \times 10^5 \text{ joules}$$

$$\Delta E \approx \frac{2.9 \times 10^5 \text{ joules}}{4 \times 10^3 \text{ joules/Cal}} \approx 70 \text{ Cal}$$

Find the temperature rise.

$$\Delta E = CM_G\Delta T,\ \Delta T = \frac{\Delta E}{CM_G} \approx \frac{70 \text{ Cal}}{(0.093 \text{ Cal/deg/kg}) \times (0.014 \text{ kg})}$$

$$= \frac{70}{1.3 \times 10^{-3}} = 5.4 \times 10^4\,^\circ \text{ Celsius}$$

The predicted temperature rise is 54,000° Celsius. This absurd result indicates that the copper will melt and possibly vaporize after a fraction of a second if no other interaction occurs. Fuses (described below) are included in circuits to prevent this.

As you can see from Example 12.6, a short circuit can be dangerous. To reduce this hazard, current-limiting circuit breakers or fuses are connected in series in electric power circuits. The current may become excessive because of a short circuit or because too many appliances are connected in parallel and operated at the same time. A fuse includes a piece of wire that melts and thus opens the circuit when its temperature increases because too large a current flows in it. A circuit breaker includes a switch that opens the circuit when the current is too large. The switch is operated by either the magnetic field or the heating effects that accompany electric currents.

Summary

The flow of an electric current in an electric circuit achieves energy transfer from an electric power supply to an energy receiver. The power supply may consume chemical energy (as in a battery or an oil- or gas-fired generating plant); it may consume gravitational field energy (as in a hydroelectric installation); it may consume nuclear energy (as in a nuclear reactor power station); or it may consume kinetic energy of air (as in a windmill-driven generator). The energy receiver may be a light bulb, a heating element, an electric motor, or an electrolysis apparatus. Even though we have referred to these systems as energy receivers, some of them are in reality only coupling elements that transfer the energy to still other systems. Thus, the light bulb produces thermal energy and radiation in the space surrounding it; the motor produces kinetic energy or does work against friction; and so on. Only in electrolysis is there really a steady accumulation of chemical energy in the reaction products, in proportion to the energy consumed at the electric power supply.

Two variable factors are particularly useful to describe the operation of an electric circuit: the current and the voltage. In an analogue model where the circuit is compared to a system of water pipes, the current corresponds to the rate of flow of the water, the voltage corresponds to the water pressure. The electric power, which is the rate of energy transfer by the circuit ($\Delta E/\Delta t$), is equal to the current times the voltage (Eq. 12.12). In many circuit elements the electric current is directly proportional to the applied voltage. The mathematical model describing this relation is called Ohm's Law, which applies to both series and parallel circuits. Ohm's Law in terms of conductance (Eq. 12.5) is most convenient for solving parallel circuits; Ohm's Law in terms of resistance (Eq. 12.6) is most convenient for solving series circuits.

Equation 12.12

energy transfer (joules) = ΔE
voltage (volts) = $\mathcal{V}$
current (amps) = $\mathcal{I}$
power (watts) = $\mathcal{P}$
time interval (sec) = Δt

$$\mathcal{P} = \Delta E/\Delta t = \mathcal{V}\mathcal{I}$$

Equation 12.5 (Ohm's Law in terms of conductance)

conductance (amp/volt) = $\mathcal{C}$

$$\mathcal{I} = \mathcal{C}\mathcal{V}$$

Equation 12.6 (Ohm's Law in terms of resistance)

electrical resistance (volt/amp) = $\mathcal{R}$

$$\mathcal{V} = \mathcal{R}\mathcal{I}$$

Additional examples

EXAMPLE 12.7. A series circuit.

Two resistors, with resistances of 6 volts per ampere and 8 volts per ampere, are connected in series to a 10-volt power supply. Find the following items: (a) resistance and conductance of circuit; (b) current in circuit; (c) partial voltage applied to each resistor; (d) power delivered by power supply; (e) power delivered to each resistor.

Solution:

(a) $$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 = 6\,\frac{\text{volts}}{\text{amp}} + 8\,\frac{\text{volts}}{\text{amp}} = 14\,\frac{\text{volts}}{\text{amp}}$$

$$\mathcal{C} = \frac{1}{\mathcal{R}} = \frac{1}{14\ \text{volts/amp}} \approx 0.071\,\frac{\text{amp}}{\text{volt}}$$

(b) $$\mathcal{I} = \mathcal{C}\mathcal{V} \approx 0.071\,\frac{\text{amp}}{\text{volt}} \times 10\ \text{volts} = 0.71\ \text{amp}$$

(c) $\mathscr{V}_1 = \mathscr{R}_1\mathscr{I} \approx 6\ \frac{\text{volts}}{\text{amp}} \times 0.71\ \text{amp} \approx 4.3\ \text{volts}$

$\mathscr{V}_2 = \mathscr{R}_2\mathscr{I} \approx 8\ \frac{\text{volts}}{\text{amp}} \times 0.71\ \text{amp} \approx 5.7\ \text{volts}$

Check: $\mathscr{V} = \mathscr{V}_1 + \mathscr{V}_2 \approx 4.3\ \text{volts} + 5.7\ \text{volt} = 10\ \text{volts}$

(d) $\mathscr{P} = \mathscr{V}\mathscr{I} \approx 10\ \text{volts} \times 0.71\ \text{amp} = 7.1\ \text{watts}$

(e) $\mathscr{P}_1 = \mathscr{V}_1\mathscr{I} \approx 4.3\ \text{volts} \times 0.71\ \text{amp} \approx 3.1\ \text{watts}$

$\mathscr{P}_2 = \mathscr{V}_2\mathscr{I} \approx 5.7\ \text{volts} \times 0.71\ \text{amp} \approx 4.0\ \text{watts}$

Check: $\mathscr{P} = \mathscr{P}_1 + \mathscr{P}_2 \approx 3.1\ \text{watts} + 4.0\ \text{watts} = 7.1\ \text{watts}$

EXAMPLE 12.8. A parallel circuit.

The two resistors of Example 12.7 are connected to the same power supply in parallel. Find the same items as in Example 12.7.

Solution:

(a) In a parallel connection, we can add the conductances of the resistors (Eq. 12.11b).

$$\mathscr{C}_1 = \frac{1}{\mathscr{R}} = \frac{1}{6\ \text{volts/amp}} \approx 0.17\ \frac{\text{amp}}{\text{volt}}$$

$$\mathscr{C}_2 = \frac{1}{\mathscr{R}_2} = \frac{1}{8\ \text{volts/amp}} \approx 0.13\ \frac{\text{amp}}{\text{volt}}$$

$$\mathscr{C} = \mathscr{C}_1 + \mathscr{C}_2 \approx 0.17\ \frac{\text{amp}}{\text{volt}} + 0.13\ \frac{\text{amp}}{\text{volt}} = 0.30\ \frac{\text{amp}}{\text{volt}}$$

$$\mathscr{R} = 1/\mathscr{C} = \frac{1}{0.30\ \text{amp/volt}} \approx 3.3\ \frac{\text{volts}}{\text{amp}}$$

(b) $\mathscr{I} = \mathscr{C}\mathscr{V} \approx 0.30\ \frac{\text{amp}}{\text{volt}} \times 10\ \text{volts} = 3.0\ \text{amp}$

(c) $\mathscr{I}_1 = \mathscr{C}_1\mathscr{V} \approx 0.17\ \frac{\text{amp}}{\text{volt}} \times 10\ \text{volts} = 1.7\ \text{amp}$

$\mathscr{C}_2 = \mathscr{I}_2\mathscr{V} \approx 0.13\ \frac{\text{amp}}{\text{volt}} \times 10\ \text{volts} = 1.3\ \text{amp}$

Check: $\mathscr{I} = \mathscr{I}_1 + \mathscr{I}_2 \approx 1.7\ \text{amp} + 1.3\ \text{amp} = 3.0\ \text{amp}$

(d) $\mathscr{P} = \mathscr{V}\mathscr{I} \approx 10\ \text{volts} \times 3.0\ \text{amp} = 30\ \text{watts}$

(e) $\mathscr{P}_1 = \mathscr{V}\mathscr{I}_1 \approx 10\ \text{volts} \times 1.7\ \text{amp} = 17\ \text{watts}$

$\mathscr{P}_2 = \mathscr{V}\mathscr{I}_2 \approx 10\ \text{volts} \times 1.3\ \text{amp} = 13\ \text{watts}$

Check: $\mathscr{P} = \mathscr{P}_1 + \mathscr{P}_2 \approx 17\ \text{watts} + 13\ \text{watts} = 30\ \text{watts}$

List of new terms

electrical conductor	voltage	electric power
battery	volt	watt
electrode	voltmeter	kilowatt
electric current	cell	kilowatt-hour
battery	Ohm's law	direct current (DC)
electric circuit	conductance	alternating current (AC)
circuit element	semiconductor	transformer
electric power supply	resistance	heating element
closed circuit	resistor	short circuit
open circuit	superconductor	fuse
ammeter	series connection	circuit breaker
ampere	parallel connection	
insulator		

List of symbols

Δq	electric charge transfer	$\mathcal{V}$	voltage
ΔE	energy transfer	$\mathcal{R}$	electrical resistance
Δt	time interval	$\mathcal{C}$	conductance
$\mathcal{I}$	electric current	$\mathcal{P}$	power

Problems

1. (a) Determine what types of energy sources (for example, coal, oil, natural gas, nuclear, falling water, wind, solar) are used by the electric utility serving your area.
   (b) Determine from what distance electric power is imported into your area in substantial amounts.
   (c) Determine to what distances electric power is exported from your area in substantial amounts.

2. Carry out library research to investigate the early history of electric currents and batteries as studied by Luigi Galvani and Alessandro Volta.

3. (a) Touch your tongue to the two electrodes of a fresh 9 volt battery (shown in Fig 12.12b) and describe your observations. Then do the same for a "worn-out" battery.
   (b) Drop a worn-out 9-volt rectangular (shown in Fig. 12.12b) battery into a glass of table salt dissolved in water and describe your observations. Use an ionic model to explain what you see.

4. Investigate and describe the closed circuit in an operating flashlight. Identify the action of the switch.

5. Extend Table 12.1 to include energy transfer, voltage, resistance, and Ohm's law.

6. Any phenomenon that includes circulation of something that is

conserved can be used as an analogue model for an electric current flowing in a circuit. Invent such a model of your own, or use one of these ideas: (i) a conveyor belt takes coal from a stockpile to a furnace; (ii) skiers "circulate" up and down a ski slope; (iii) traffic circulates through the streets in a city. In your example, set up the correspondence as it is done for the water pipe analogue in Table 12.1. Indicate where the model is inadequate.

7. Six ammeters in Fig. 12.7 are identified by numerals. Find as many relations as you can among the currents passing through these ammeters, and explain your reasons. (Example: ammeters 3 and 4 indicate equal currents, because there is no circuit branch between them where current could be diverted or added.)

8. Explain why the "negative ion fluid" and the "positive ion fluid" in a liquid conductor move in opposite directions (Fig. 12.9).

9. Identify shortcomings of the micro-domain model for electric current presented in Section 12.1.

10. Imagine a "battery" in which hydrogen and oxygen combine to form water. Such a battery is called a fuel cell. Calculate the voltage such a fuel cell would deliver. (Hint: Use the chemical energy of the fuel system given in Table 10.3, the definition of the faraday in Section 8.1, and the relation of electric charge and current in Eq. 12.1.)

11. Use the definition of voltage in Eq. 12.3 and the quantity of chemical energy change occurring in a chain of cells connected in series to derive Eq. 12.4 for the voltage of such a chain as compared to the voltage of one cell.

12. Use the definition of voltage in Eq. 12.3 and the quantity of chemical energy change occurring in cells connected in parallel (as in the diagram to the left) to relate the voltage of this battery to the voltage of one cell.

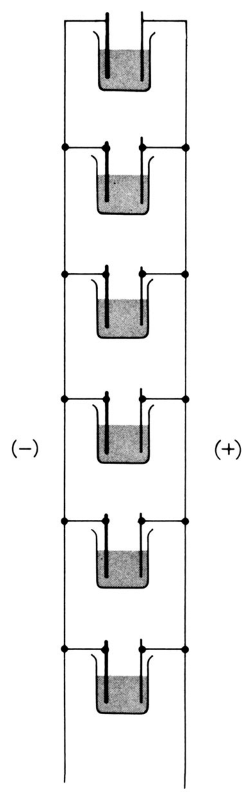


Diagram for problem 12.

13. Make a diagram and describe the operation of water analogue models for the systems in Problems 11 and 12.

14. The standard dry cell (flashlight battery) has a voltage of 1.5 volts. Make a diagram to show how you would make a 9-volt transistor radio battery out of single dry cells.

15. (a) Find the voltages of the batteries diagramed in Fig. 12.22 (below) in terms of the single cell voltage.

(b) Describe the operation of water analogue models for the systems in Fig. 12.22.

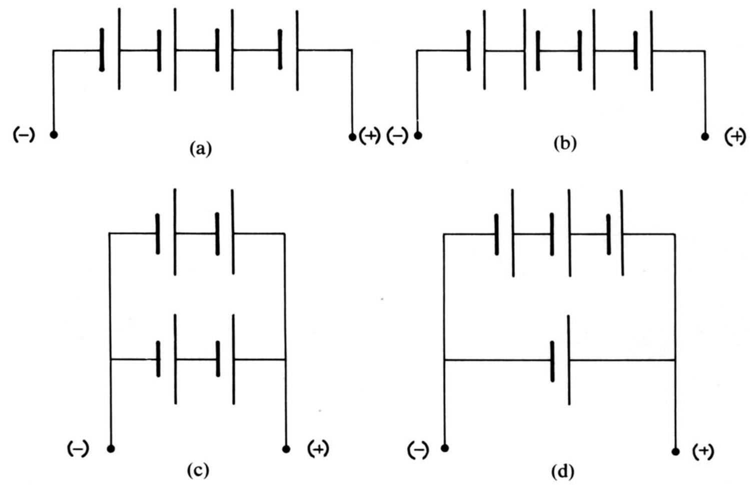


Figure 12.22 Each battery symbol represents a single standard dry cell of 1.5 volts (Problems 15, 16).

16. Find the total energy that can be delivered by each of the batteries in Fig. 12.22. Use the energy of one dry cell as the unit of energy. Explain your answers briefly.

17. A 12-volt automobile storage battery is advertised to have a capacity of 100 ampere-hours. Find the battery's energy when it is completely charged. Express the result in kilowatt-hours, joules, Calories, and ice-melting capacity (Section 9.2). The ampere-hour is a unit of electric charge; one amp-hour is the amount of charge transferred (Δq) by 1 ampere flowing for 1 hour.

18. Explain the "gap" in the current-voltage graph for a sodium chloride (table salt in water) solution (Fig. 12.13c).

19. Even though a diode is not described by Ohm's law, you can describe it approximately as having a high "forward conductance" (for positive voltage) and a low "backward conductance" (for negative voltage). Draw a graph for a circuit element that has two different conductances and estimate numerical values of these conductances for the diode in Fig. 12.13b.

20. The following data are observed for an electric circuit element.

current:	1.8	3.0	4.6	6.2	8.6	(amp)
voltage:	8.0	13.5	19.2	27	35	(volts)

(a) Represent the data by a graph.
(b) Describe the data by Ohm's law as well as you can. Find the resistance and conductance.
(c) Evaluate the adequacy of the Ohm's law model for this circuit element.

21. (a) Explain why the resistor in a voltmeter should have a large resistance (Fig. 12.14).
(b) Define what is meant by "large resistance" in terms of the applications of the voltmeter.

22. An ammeter has a dial that is calibrated to indicate up to 5.0×10^{-3} ampere. The ammeter is to be combined with a resistor to construct a voltmeter.
(a) What is the resistance of the resistor if the dial is to indicate up to 5.0 volts?
(b) What is the resistance of the resistor if the dial is to indicate up to 150 volts?

23. Use the three water-pipe analogue models illustrated in Fig. 12.23 to carry out the following.
(a) Draw a qualitative graph of the relation of pressure and flow rate (as in Fig. 12.15) for each of models I, II and III.
(b) Describe the electric circuit element that might be represented by each water analogue, if there is one.
(c) Invent a water analogue model for a radio tube (Fig. 12.13d).

24. You are given four resistors, each with a resistance of 10 volts per ampere.
(a) Connect them in such a way (make a diagram) that the resulting circuit has the lowest possible resistance. What is the resistance?
(b) Connect them in such a way (make a diagram) that the resulting circuit has the largest possible resistance. What is the resistance?

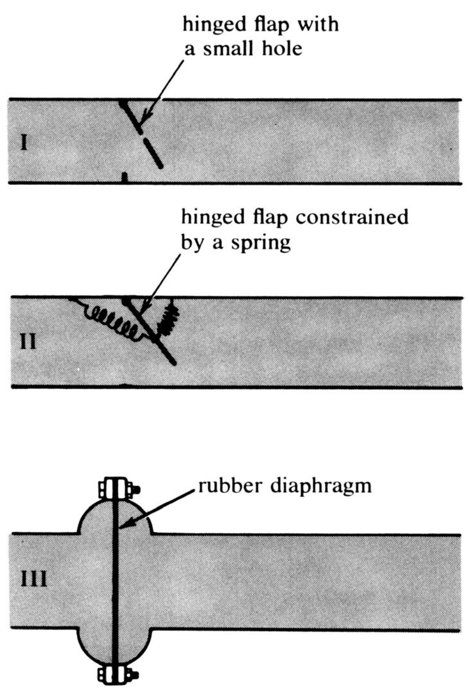


Figure 12.23 Water pipe devices that may serve as analogue models for electric circuit elements (Problems 23, 37).

(c) Invent as many other different circuits as you can by connecting the four resistors, make a diagram for each circuit, and find its resistance.

25. Compare the conductances of the 0.16-centimeter-diameter and 0.008-centimeter-diameter copper wires (Table 12.2) in the light of the model that represents a thick wire as a bundle of thin wires with the same total cross-sectional area. (See Section 12.3, Ex. 12.4.)

26. Make a mathematical model that relates the resistance of a piece of wire to its length, cross-sectional area, and the resistance of a piece of the same material in a standard shape. (Note: the most commonly used "standard shape" is a cube with 1-centimeter-long edges, Table 12.2B.)

27. Electric toasters are usually made with nichrome ribbon heating elements. In one toaster, the ribbon was 0.085 centimeter wide and 0.010 centimeter thick. What is the diameter of a circular nichrome conductor with close to the same resistance for the same length?

Note for Problems 28-33 about light bulbs. The white-hot electric conductor in a light bulb is a piece of fine tungsten wire called the filament. (If you are curious, break open a light bulb—wrapped in a towel to confine glass splinters—and examine the filament with a magnifier. You may find that the wire is wrapped into a spiral, or helical, shape.) While the bulb operates, it is in a steady state (Section 4.6), with electric power being consumed at the same rate as radiant energy is emitted. Light bulb filaments are designed to operate near 3000° Celsius. You can assume that the electric power supply voltage in all problems is 110 volts.

28. (a) Find the electric current passing through a 60-watt bulb under normal operating conditions. Assume that 110 volts is applied to the bulb.
(b) Find the resistance of the filament and calculate the length of tungsten wire (0.008-centimeter diameter) that must be used in the filament? (See Table 12.2 for tungsten.)
(c) Find the electric current passing through a 60-watt light bulb when it is cold (at room temperature, just after it is turned on).
(d) Find the power delivered to a 60-watt light bulb when it is cold.

29. Prepare a qualitative description of the process whereby a 60-watt light bulb arrives at a steady state very shortly after it is turned on.

30. About 2% of the electric power consumed by a light bulb becomes visible light. What happens to the rest?

31. To manufacture a light bulb, after the resistance and length of the tungsten filament wire for a light bulb has been determined (Problem 28, parts a and b), a suitable piece of wire must be selected and then twisted into a shape so it will attain its steady-state operating temperature near 3000° Celsius. Enumerate some of the variable factors that influence the operating temperature and indicate the direction of the influence.

32. Describe what will happen when two 60-watt light bulbs are connected in series and then connected to a 110-volt power supply. Explain.

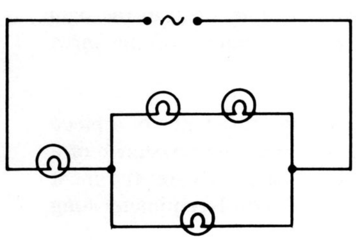


Diagram for Problem 33(a)

33. Describe the relative brightness of the light bulbs in these connections: (a) a 200-watt bulb and a 60-watt bulb in parallel; (b) a 200-watt bulb and a 60-watt bulb in series; (c) four 60-watt bulbs connected as in the diagram to the left.

34. A student carries out his studies using a lamp with two 60-watt light bulbs. The lamp is turned on for 5 hours every evening. Assuming that the energy cost is 13 cents per kilowatt-hour, estimate the monthly electric bill incurred by his studiousness.

35. (a) Examine and report on the electric power rates of your local utility company.
(b) Compare the cost of the energy for maintaining one young adult for one year (10^6 Calories), when the energy is obtained from the following sources (see Table 10-6): (i) electric utility; (ii) gasoline; (iii) a diet consisting of 30% sirloin steak, 30% American cheese, 30% rye bread, and 10% sugar; (iv) your actual diet (that is, the approximate amount you spend per year on food).

36. (a) Conduct an experiment to estimate the energy used to toast two slices of bread.
(b) Estimate the energy transfer to the toast according to a model in which the only difference between toast and bread is that water has been removed by evaporation (see Table 10.5).

37. (a) Compare the operation of the water pipe device in Fig. 12.23, part III, when it is subject to "direct current" with its operation on "alternating current" (Fig. 12.20).
(b) Describe an electric circuit element for which this water pipe device is an analogue model.

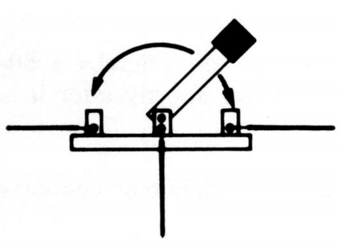


Diagram for problem of 38

38. (a) Devise a circuit whereby a staircase light can be turned off and on from two locations. (Hint: Use "double throw" switches, as in diagram to left.)
(b) Extend the procedure to the control of one light from three or more locations. If necessary, invent switches that will help solve the problem.

39. Interview four or more children (ages 8-12) to investigate their concepts of electric circuits and the functioning of electric circuits. (Suggestions: provide the children with parts from which simple circuits can be assembled—batteries, bulbs, wire, wire coils and nails for electromagnets, salt water for electrolysis. Observe their activities and ask for explanations.)

40. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

Bibliography

M. Born, *The Restless Universe*, Dover Publications, New York, 1957.

I. B. Cohen, *Benjamin Franklin's Experiments*, Harvard University Press, Cambridge, Massachusetts, 1941.

M. Faraday, *Experimental Researches in Electricity*, Dover Publications, New York, 1966.

A. Holden, *Conductors and Semi-Conductors*, Bell Telephone Laboratories, Murrary Hill, New Jersey, 1964.

R. A. Millikan, *Electrons (+ and -),* University of Chicago Press, Chicago, Illinois, 1947.

S. P. Thompson, *Michael Faraday*, Cassell, London, 1901.

J. Tyndall, *Faraday as a Discoverer*, Crowell, New York, 1961.

Articles from Scientific American. Some or all of these, plus many others, can be obtained on the Internet at http://www.sciamarchive.org/.

G. de Santillana, "Alessandro Volta" (January 1965).

G. A. Hoffman, "The Electric Automobile" (October 1966).