Chapter 13: Objects in Motion

Galileo was the first student of nature to observe and describe carefully the motion of objects, including falling bodies. He recorded their speeds and the changes in speed that occurred when they fell (increasing speed) or encountered resistance to motion (decreasing speed). Galileo also realized that the direction of motion was of great significance. For instance, projectiles tend to curve downward, but they maintain their motion in the horizontal direction even while they first rise higher, level off, and finally arch down to the ground.

When we defined the speed of relative motion in Eq. 2.1, we referred only to the distance traveled, not to its direction. In this chapter, therefore, we will introduce a new concept, the *velocity* of a moving object, which takes into account the direction of motion as well as the speed. Like position, displacement, and force, *velocity* will be denoted by a boldface letter in the text (**s**, **Δs**, **F**, **v**) and by an arrow in a diagram. The operations of arithmetic apply to velocity in the same way as to displacements (Section 2.3).

Physical quantities, such as displacement and velocity, which must be described by both a numerical magnitude and a direction in space are examples of a class of mathematical quantities called *vectors*. To help you recognize and manipulate vectors, we are listing all the ones used in this text in Table 13.1. The table includes the algebraic and diagrammatic symbols, the algebraic symbol for the magnitude, and the text section where the quantity is defined. Note the arrows with a variety of heads to suggest close relationships among the quantities. When you are writing by hand, the boldface notation cannot be used; instead, the algebraic symbol for a vector is written with an arrow over it ($\vec{v}$) to remind the reader of the directionality. Because the word "vector" is not in common use, we will not use it further in this text, and we will refer directly to magnitude and direction whenever these are important. Other physics books generally use the vector terminology.

13.1 Velocity

The two words "speed" and "velocity" are commonly used as synonyms to describe the rate of relative motion in everyday language. However, relative motion, if you wish to specify it adequately, includes two distinct ideas: 1) the numerical rate of speed and 2) the direction of motion. Therefore, in physics, the words “speed” and “velocity” have very specific meanings: we will use the word *speed* to describe *only* the numerical rate of relative motion (Section 2.2), as might be indicated on an automobile speedometer or an anemometer (a wind speed measuring device). On the other hand, the word *velocity* includes *both* the rate *and* the direction of relative motion, for example "The wind velocity is 30 miles per hour *from the northwest*." The phrase "from the north-west" in this statement indicates the wind direction, while the phrase "30 miles per hour" indicates the wind speed. Thus velocity includes *both* the simpler numerical idea of speed *and* the direction; we can say that the velocity concept is inclusive of and more complex than the speed concept.

TABLE 13·1 PHYSICAL QUANTITIES WITH MAGNITUDE AND DIRECTION IN SPACE

Physical quantity	*Algebraic symbol*	*Diagramatic symbol*	*Magnitude*	*Definition*
position	**s**		|**s**|	*Section 2·3*
displacement	**Δs**		|**Δs**|	*Section 2·3*
force	**F**		|**F**|	*Section 11·2*
velocity	**v**		|**v**|	*Section 13·1*
change of velocity	**Δv**		|**Δv**|	*Section 13·1*
acceleration	**a**		|**a**|	*Section 13·2*
momentum	$\mathcal{M}$		|$\mathcal{M}$|	*Section 13·3*

There are good reasons that the distinction between velocity and speed has not become part of everyday language. The movement of persons, animals, and vehicles on the ground is usually confined to roads or paths that are easily identified. Once the path of motion is determined, its direction is known and only the speed of motion and its sense (forward or backward) need to be communicated.

However, most objects (for example, children in a playground, boats, airplanes...) can move in a wide range of directions; therefore, *both* the speed and the direction of motion relative to the earth (or some other reference frame) are important. Therefore, to describe the motion of such objects, we must use the more complex concept of velocity rather than the simpler numerical notion of speed.

Consider, for instance, the problem faced by the navigator of a ship, who can observe the velocity (speed and direction) of the ship relative to the ocean water and who knows that the ship is in an ocean current whose velocity (speed and direction) relative to the earth's land masses has been charted. To reach his destination, he has to use the available information to compute the velocity (speed and direction) of the ship relative to the land masses. You face a similar problem when you try to

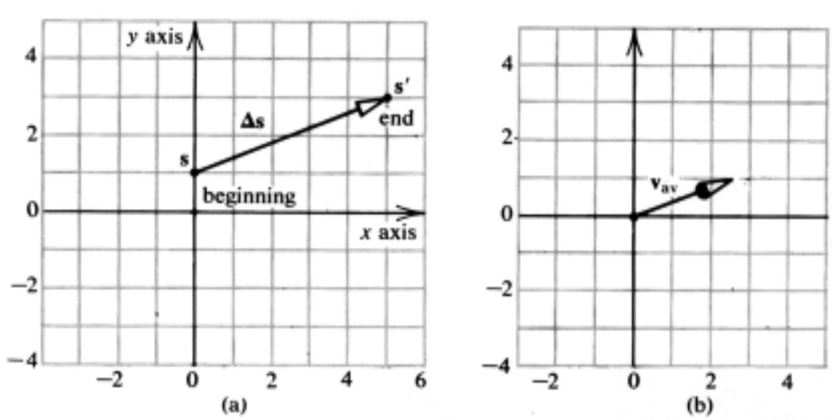

Figure 13.1 *Relation of position, displacement and average velocity.*

(*a*) *Displacement* **s** to **s'**, **Δs** = [5.2] *centimeters in a time* Δt = 2 sec*onds*.

(*b*) *Average velocity*:

$$\mathbf{v}_{av} = \frac{\Delta\mathbf{s}}{\Delta t} = \frac{[5,\ 2]\ cm}{2\ \sec}$$

$$= [2.5,\ 1]\ cm/\sec$$

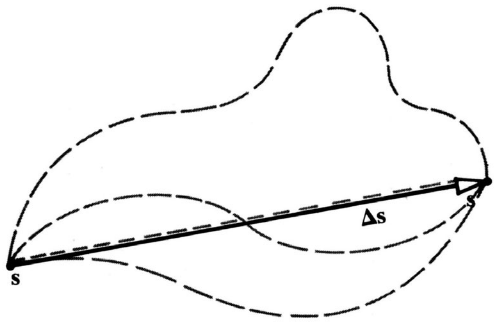

Figure 13.2 Alternate paths may result in the same displacement even though different distances are traversed. The magnitude of the displacement is equal to the actual distance traversed <u>*only*</u> *if the actual path is along the straight line.*

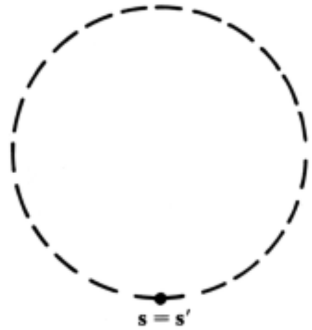

Figure 13.3 Displacement in a complete circle is the same as zero displacement. Δs=[0,0]. Of course, the total distance traveled is not zero.

paddle a canoe past a boulder in a swiftly moving stream. Only the velocity (speed and direction) of the canoe relative to the water is subject to your control; the velocity (speed and direction) of the water relative to the boulder is outside your control. Yet whether or not you hit the boulder depends only on the canoe's velocity (speed and direction) relative to the boulder, and this velocity is the result of adding together the other two velocities.

FORMAL DEFINITION
The average velocity is the ratio of the displacement divided by the time interval required for the displacement.

Average velocity. We will now present a formal definition of the average velocity of a moving object during a time interval. The *average velocity* ($\mathbf{v}_{av}$) is defined in terms of the duration of the time interval and the moving object's displacement during the time interval (Eq. 13.1). The definition is similar to that of speed (Eq. 2.1), but involves the displacement instead of the total distance traveled.

Equation 13.1

average velocity (*m*/*sec*) $= \mathbf{v}_{av}$
displacement (*m*) $= \mathbf{\Delta s}$
time interval (sec) $= \Delta t$

$$\mathbf{v}_{av} = \frac{\mathbf{\Delta s}}{\Delta t}$$

The average velocity describes only what is known about the motion of an object from two "snapshots" giving its position at the beginning and the end of the time interval Δt (Fig. 13.1). The snapshots give no information about what happened during the time interval: they do not reveal whether the object moved on a straight line between the two positions, or whether it made a larger detour (Fig. 13.2). An extreme case is that of a child on a merry-go-round, which may appear in the same position at the beginning and end of the interval, yet have traveled in a large circle on the merry-go-round (Fig. 13.3). According to the definition, its average velocity during the entire interval was zero because its displacement was zero. The average speed, however, was not equal to zero, since the child traveled all around a circular path of perhaps 100 feet in length.

The average velocity therefore may provide an inaccurate account of an object’s motion. You can avoid this problem by using a sufficiently short time interval and defining a new quantity: the *instantaneous velocity.*

Instantaneous velocity. To have a better description of the motion of the object, you must take snapshots or otherwise record its position more frequently. In fact, the position should be recorded so frequently

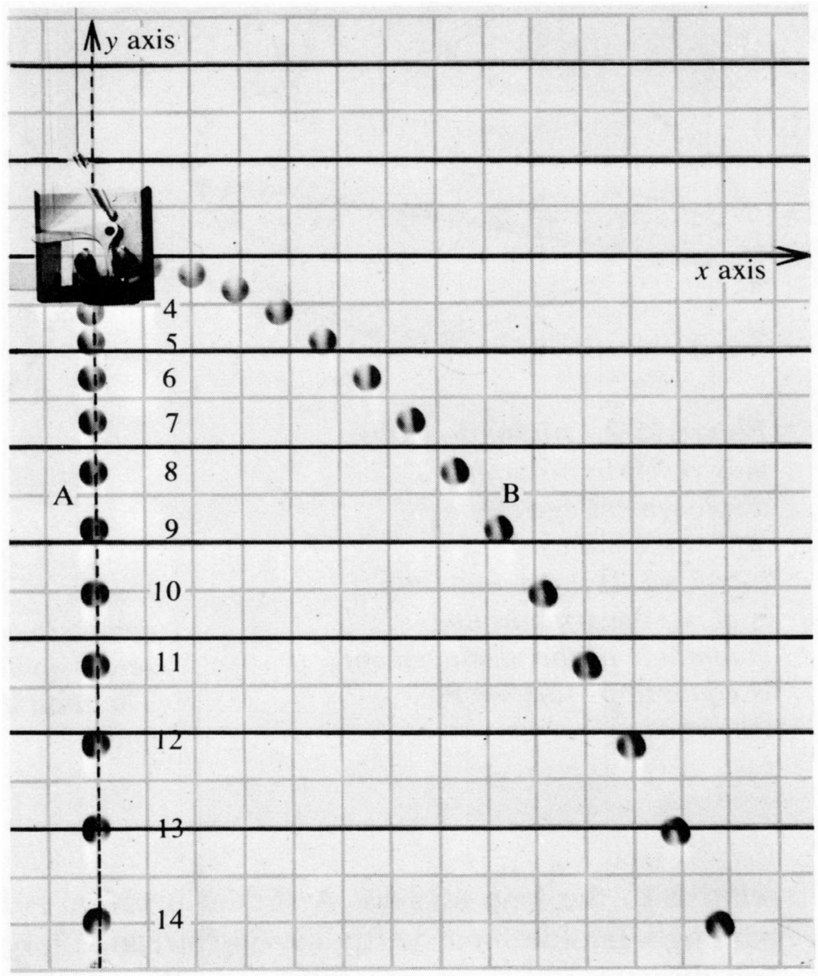

Figure 13.4 Two golf balls were released at the same time and then photographed at intervals of 1/30 second.

Scale: The black horizontal lines are 15 cm, or 0.15 m, apart. Thus each square in the grid is 0.075 m on a side and the actual distances between the positions of the balls can be found by simply estimating the position of each image in relation to the grid. For example, the bottom of ball image A9 is at about [0.0, -6.0] squares and the bottom of ball image A10 is at about [0.0, -7.4] squares. Therefore, the displacement of ball A in interval 9-10 is
Δs = [0.0, -1.4] squares
= [0.0, -1.4 sq.](0.075 m/sq)
= [0.0, -0.105] m.
This agrees quite well with the vertical (y) displacement of ball B in interval 9-10 found in Table 13.2.

that you can be sure no excursions or other irregular behavior had occurred between the recorded positions. Then the average velocity in each time interval, as defined by Eq. 13.1, would describe the actual motion of the object very accurately. When Eq. 13.1 is used with such a very short (infinitesimal) time interval, it defines the *instantaneous velocity* **v** (Eq. 13.2); the word "instantaneous," meaning over a "vanishingly small" or "infinitesimal" interval, emphasizes the distinction with the "average" velocity. Instantaneous velocity is a concept that is extremely useful for making mathematical models, but only the average

TABLE 13.2 ANALYSIS OF GOLF BALL MOTION, FIGURE 13.4

Pictures defining interval	Δt (sec)	$\Delta\mathbf{s}$ (m)	$\mathbf{v}_{av}$ (m/sec)
4-5	0.033	[0.067, -0.048]	v_5 = [2.00, -1.4]
9-10	0.033	[0.067, -0.106]	v_{10} = [2.00, -3.2]
13-14	0.033	[0.067, -0.15]	v_{14} = [2.00, -4.5]

Equation 13.2

$$\mathbf{v} = \mathbf{v}_{av} = \frac{\Delta \mathbf{s}}{\Delta t}$$

(Δt infinitesimal)

Infinitesimal means exceedingly small or vanishingly small. But how do we know when the interval is short enough? In practical terms, we always want to know the velocity to a certain accuracy set by our measuring instruments. Therefore, to decide when the time interval is short enough to be considered infinitesimal, we can continue to take position measurements at shorter time intervals until the value of the velocity calculated from the formula stays constant to within the desired accuracy.

René Descartes (1596-1650) was able to combine geometrical and arithmetical reasoning by identifying velocity components. He wrote: [the vertical component is] ... "that part which would make the ball move from above downward ...." [and the horizontal component is] "... the tendency which made it move toward the right."

velocity can be found experimentally from measurements of displacements and time intervals. As we have mentioned, you can find as close an approximation as you wish to the instantaneous velocity by measuring the average velocity over successively shorter time intervals. Using a very short time interval also means that the difference between the magnitude of the displacement and the distance traveled (Fig. 3.2) can be made as small as necessary; therefore, the magnitude of the velocity and the speed will also converge to the same value.

An example. We will analyze the familiar and important motion of a freely falling object (Fig. 13.4). We will show that the horizontal component of the velocity stays the same and only the vertical component changes, a result that we will refer to again in Chapter 14. Two techniques for determining the positions of a moving object at closely spaced time intervals were described in Section 2.2: motion pictures and multiple exposure photographs. Fig. 13.4 is a multiple exposure of two golf balls released at the same time from the mechanism at the top. Ball A was allowed to fall straight down, and ball B was shot out horizontally with an initial speed of 2.00 meters per second. The camera shutter remained open while short flashes of light illuminated the balls at intervals of 1/30 second. The black lines are horizontal strings placed 15 centimeters apart. A rectangular coordinate grid (as introduced by Descartes, Chapter 2) is shown by which you can actually measure the rectangular components of the displacement of each ball for each interval. You can then use this information to find the average velocity of each ball for each interval between pictures.

Three sample calculations of the average velocities of ball B are presented in Table 13.2. The average velocity in the fifth interval is called v_5, the average velocity in the tenth interval is v_{10}, and so on. You see that the x-component (horizontal) of the velocity does not appear to change, but that the y-component (vertical) changes greatly.

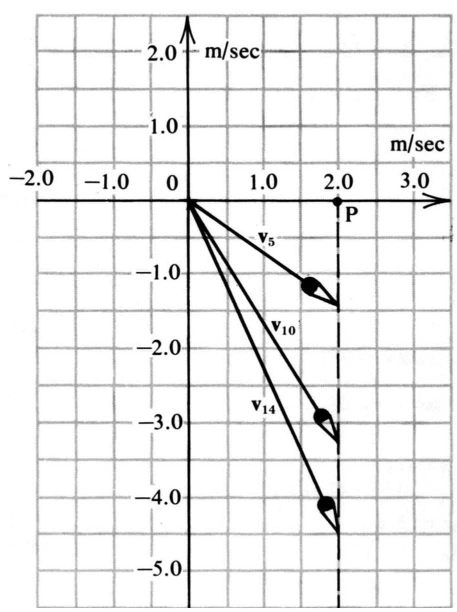

Figure 13.5 Average velocities v_5, v_{10}, and v_{14} of falling golf ball B in Fig. 13.4.

Figure 13.5 is a coordinate frame diagram to show the average velocities calculated in Table 13.2. The three arrows represent the velocities of ball B in the three intervals tabulated. This diagram gives a picture of the variation of the ball's average velocity with time. The velocity becomes progressively directed more and more strongly in the downward direction. Since the golf ball did not execute irregular motion, you may imagine that the instantaneous velocity also changed smoothly and became progressively directed downward. The instantaneous velocity at the very beginning was 2.00 meters per second to the right, as described in the conditions of the experiment. A good mathematical model for the golf ball motion is that the head of the arrow representing the instantaneous velocity begins at the point P and gradually rotates from the horizontal direction toward the vertical direction, with its tip always on the dashed line.

Change of velocity. You can carry out arithmetic operations with velocities just as you did with displacements. You must, however, remember the physical significance of these operations when they are applied as part of a mathematical model. We said above, for example, that the average velocity of the falling golf ball was changing. How much did it change between the fifth interval and the tenth interval (Fig. 13.4)? It changed by the arrow shown as $\mathbf{\Delta v}$ in Fig. 13.6. This must be added to the velocity $\mathbf{v_5}$ in the earlier interval to result in the velocity $\mathbf{v_{10}}$ in the later interval. In other words, the change of velocity $\mathbf{\Delta v}$ is the difference between the two velocities. Note that the change of velocity has a magnitude and a direction and is represented by an arrow in the diagrams. In Fig. 13.6 the change of velocity is directed vertically downward.

Equation 13.3

$$\mathbf{v}_{10} = \mathbf{v}_5 + \mathbf{\Delta v}$$

Equation 13.4

$$\mathbf{\Delta v} = \mathbf{v}_{10} - \mathbf{v}_5$$

The relation of the arrows in Fig. 13.6 can be stated also in the form of an equation. You can think of $\mathbf{v_{10}}$ as equal to the sum of $\mathbf{v_5}$ plus $\mathbf{\Delta v}$ (Eq. 13.3), or you can think of $\mathbf{\Delta v}$ as the difference between $\mathbf{v_{10}}$ and $\mathbf{v_5}$ (Eq. 13.4). We pointed out in Chapter 3 that changes in motion are evidence of interaction of the moving object with another object. Therefore, the calculation of changes of velocity is important because it reveals the presence of interaction and helps us to identify the other objects that are the source of that interaction. For instance, the change of velocity of the falling golf ball is directed vertically downward because the force of gravity is directed vertically downward. The relation between change of velocity and force will be examined further in Chapter 14.

Equation 13.5

$$\mathbf{\Delta s} = \mathbf{v}_{av}\Delta t$$

EXAMPLE:

$\mathbf{v}_{av} = [2, 15]$ m/sec

$\Delta t = 3$ sec

$\mathbf{\Delta s} = \mathbf{v}_{av}\Delta t$

$= [2, 15]$ m/sec $\times$ 3 sec

$= [6, 45]$ m

Finding the displacement. The relationship of average velocity, displacement, and time interval (Eq. 13.1) can be used in two different ways. So far, we have used it to find the average velocity from data of the displacement and the time interval. In addition, the relationship can be reversed to permit calculation of the displacement, which is equal to the average velocity multiplied by the time interval (Eq. 13.5).

This new relationship, a mathematical model for the displacement, is useful only while the average velocity is not changing greatly or when

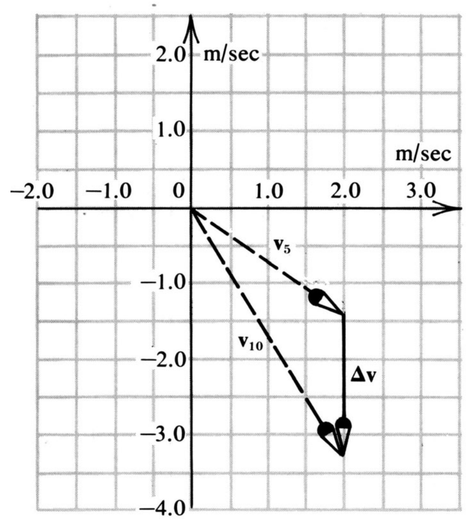

Figure 13.6 Change of the average velocity of falling golf ball B in Fig. 13.4.

Equation 2.1 (average speed)

speed (m/sec) = v_{av}
distance (m) = Δs
time interval (sec) = Δt

$$v_{av} = \frac{\Delta s}{\Delta t}$$

Equation 13.6 (displacement and distance)

$$|\Delta \mathbf{s}| = \Delta s$$

Equation 13.7 (average velocity and average speed)

$$|\Delta \mathbf{v}_{av}| = \Delta v_{av}$$

Equation 2.2 (instantaneous speed and average speed)

$$v = v_{av} = \frac{\Delta s}{\Delta t}$$

(Δt must be infinitesimal)

Equation 13.8 (instantaneous velocity and instantaneous speed)

$$|\mathbf{v}| = v$$

the time interval is infinitesimal. It is similar to the example of the automobile trip described in Figure 1.8 except that now the direction of motion as well as the speed must be unchanging.

Velocity versus speed. At the beginning of this section we pointed out that the velocity concept describes motion more completely than does the speed concept. Speed is the rate at which distance is traversed without regard to direction. Speed and distance, therefore, are not represented by boldface symbols but by ordinary letters (Eq. 2.1). The average speed of the child on the merry-go-round (Fig. 13.3) is substantial because of the large distance traversed during one revolution, but the average velocity during one revolution is nevertheless zero. Whenever an object changes its direction of motion and moves along a curved or zigzag path, the average speed and the average velocity are *not* related simply to one another. Whenever motion is along a straight line and its direction does not change, the magnitude of the displacement is equal to the distance traversed (Eq. 13.6; see Fig. 13.2); hence, the magnitude of the average velocity is equal to the average speed (Eq. 13.7).

This same relation of speed and velocity *also* holds in the limit of infinitesimal time intervals. In this limit we speak of the instantaneous speed (Eq. 2.2) and the instantaneous velocity (Eq. 13.2). The magnitude of the instantaneous velocity is equal to the instantaneous speed (Eq. 13.8). Basically, the reason behind this relationship is that the complications arising from changes in the direction of motion do not operate during an infinitesimal time interval. The automobile speedometer, for instance, is designed to measure the instantaneous speed of a car. When you combine the speedometer reading with the direction of motion of your car (perhaps indicated on a magnetic compass mounted under the windshield), you obtain your car's instantaneous velocity (speed and direction).

Before concluding this section, we should point out an example that clearly reveals the key difference between the concepts of speed and velocity. A car travels along a road at 60 miles per hour, slows

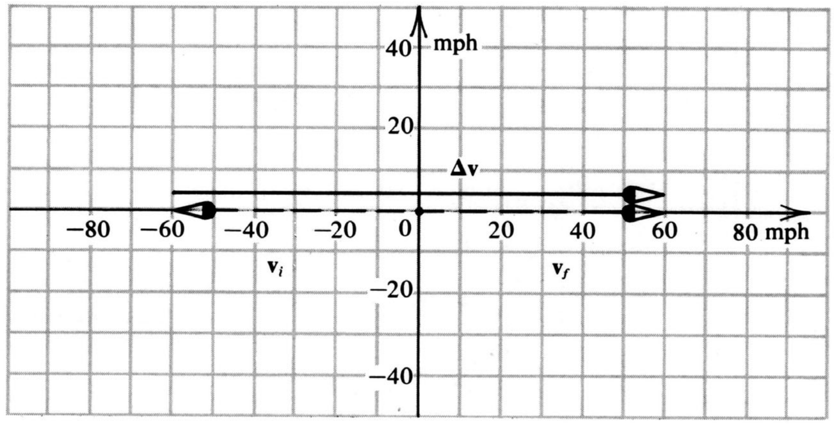

Figure 13.7 A car reverses its direction of motion from the left to the right. The change of velocity is 120 miles per hour to the right.

down and makes a U-turn, then accelerates to 60 miles per hour again. Its speed at the end is the same as at the beginning, 60 miles per hour. However, while its final velocity has the same magnitude as the initial velocity, they are pointing in opposite directions. Therefore, the *change in velocity,* illustrated in Fig. 13.7, has a magnitude of 120 miles per hour and points in the same direction as the final velocity!

13.2 Acceleration

Everyone who has been in a rapidly accelerating car or in a jet airplane has experienced the sensation of being pushed back into the seat during takeoff. When a high-speed elevator starts or stops, the passengers sometimes experience feelings of nausea. The passengers in roller coasters and other rides at the amusement park are pressed into their seats or pushed from side to side. In all these happenings, the human body experiences changes of velocity (of speed or direction of motion, or of both) that affect the internal organs and may cause "seasickness."

Acceleration is the quantity used to describe changes of velocity. The most common example is the speeding up of an automobile when the driver steps on the accelerator. Acceleration is therefore usually associated with an increase of speed. Newton's very successful theory, however, relates all changes of speed and direction of motion of objects to their interactions with other objects. As a result, we must formulate a definition of acceleration that is more general than everyday usage and that includes decrease of speed and changes in direction of motion as well as increase of speed.

Definition of acceleration. It is immediately clear that the change in velocity is by itself not directly related to the strength of interaction. Compare, for instance, the car that accelerates to 60 miles per hour

FORMAL DEFINITION
The average acceleration is the ratio of the change of instantaneous velocity to the time interval required to effect the change.

Equation 13.9 (definition of acceleration)

change in instantaneous velocity (m/sec) = $\mathbf{\Delta v}$
time interval (sec) = Δt

$$\mathbf{a}_{av} = \frac{\mathbf{\Delta v}}{\Delta t}$$

Units of acceleration:

1 m/sec/sec = 1 meter per second per second

1 mph/sec = 1 mile per hour per second
= 0.45 meter per second per second

(Can you verify the above value, assuming that 1 mile $\approx$ 1600 m?)

from a standing start in 10 seconds (with a roar of the engine and flying gravel), and the car that does the same in 1 minute with hardly any noise or notice by the passengers inside. Though the velocity change is the same (0 to 60 miles per hour), the interaction between the tires and the road (or the passengers and their seats) is much larger in the first case than in the second. The time interval during which the velocity change is accomplished appears to be significant and is included in the definition of acceleration. To pin down the time interval and the exact velocity change, the definition refers to the instantaneous and not the average velocity (Eq. 13.9).

Examples. The definition of acceleration can be applied to golf ball B in Fig. 13.4 only in approximate fashion, because instantaneous velocities have not been determined. Nevertheless, the acceleration of golf ball B will serve as an illustration in Example 13.1 below.

In Example 13.2 below we calculate the actual accelerations of the two automobiles mentioned above. The example of the car making the U-turn (Fig. 13.7) could be used to calculate an acceleration if the time interval Δt required for the U-turn were known. If the U-turn is accomplished in a short time interval, the acceleration must be large, according to Eq. 13.9. Physically, this is possible only on a road with good traction. On an icy pavement, the time interval would have to be long and the acceleration small. This shows that the maximum possible acceleration is closely related to the interaction between tires and road.

Another example of acceleration occurs when a car traveling west makes a left turn and goes south (Fig. 13.8 and Example 13.3). This situation may seem different from the ones above, because only the direction of motion changes, not the speed. Nevertheless, we can still find the change in velocity by considering the x- and y-components. The change in velocity needed is the velocity we must *add* to the initial velocity to end up with the final velocity. Therefore, the change in velocity must point both east (to cancel the initial velocity) and south (to end up with the proper final velocity). As shown in Fig. 13.8b, the change of velocity, and therefore the acceleration, are at a 45-degree angle to the initial and final directions, pointing *toward the inside* of the curve. This conclusion is generally true: *whenever an object travels in a curve, the acceleration points toward the inside of the curve.*

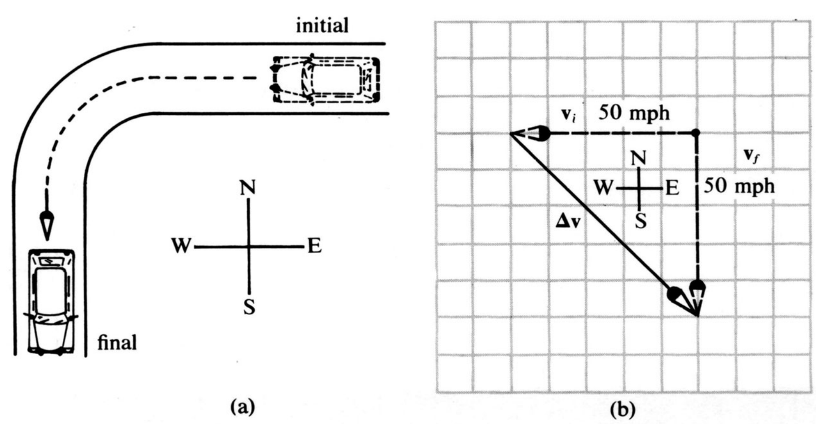

Figure 13.8 A car drives around a curve, (a) Top view of road, with car turning from westward to southward direction at 50 miles per hour.
(b) Velocity diagram, showing initial velocity $\mathbf{v_i}$*, final velocity* $\mathbf{v_f}$*, and the velocity change* $\mathbf{\Delta v}$ *of 70 miles per hour to the southeast.*

EXAMPLE 13·1. **Acceleration of golf ball B, Fig. 13·4.**

Data:

$\mathbf{v}_5 = [2.0, -1.4]$ m/sec; $\mathbf{v}_{10} = [2.0, -3.2]$ m/sec; $\Delta t = 0.165$ sec

Solution:

$$\Delta\mathbf{v} = \mathbf{v}_{10} - \mathbf{v}_5 = [0, -1.8] \text{ m/sec}$$

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{[0, -1.8] \text{ m/sec}}{0.165 \text{ sec}} \approx [0, -11] \text{ m/sec/sec}$$

EXAMPLE 13·2. **(a) Acceleration of an automobile from 0 miles per hour to 60 miles per hour in 10 seconds.**

Data:

$\mathbf{v}_i = [0, 0]$ mph; $\mathbf{v}_f = [60, 0]$ mph; $\Delta t = 10$ sec

Solution:

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = [60, 0] \text{ mph}$$

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{[60, 0] \text{ mph}}{10 \text{ sec}} = [6, 0] \text{ mph/sec} \approx [2.7, 0] \text{ m/sec/sec}$$

Some useful conversions:

a) 1 mile = 1600 meters (approx.)

b) 1 mile/hr (mph) = 0.44 m/sec

c) 1 m/sec = 2.25 mph

Can you verify b) and c) by starting with a)?

(b) Acceleration of an automobile from 0 miles per hour to 60 miles per hour in 1 minute.

Data:

$\mathbf{v}_i = [0, 0]$ mph; $\mathbf{v}_f = [60, 0]$ mph; $\Delta t = 60$ sec

Solution:

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = [60, 0] \text{ mph}$$

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{[60, 0] \text{ mph}}{60 \text{ sec}} = [1, 0] \text{ mph/sec} \approx [0.45, 0] \text{ m/sec/sec}$$

EXAMPLE 13·3. **Find the acceleration of the car in Fig. 13·8. It passes the curve in 3 seconds.**

Data:

$\mathbf{v}_i = [-50, 0]$ mph; $\mathbf{v}_f = [0, -50]$ mph; $\Delta t = 3$ sec

Solution:

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = [50, -50] \text{ mph}$$

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{[50, -50] \text{ mph}}{3 \text{ sec}} \approx [16, -16] \text{ mph/sec} \approx [7, -7] \text{ m/sec/sec}$$

EXAMPLE 13·4. **A car slows down from 60 miles per hour to 40 miles per hour in 20 seconds. Find the acceleration.**

Data:

$\mathbf{v}_i = [60, 0]$ mph; $\mathbf{v}_f = [40, 0]$ mph; $\Delta t = 20$ sec

Solution:

$$\mathbf{\Delta v} = \mathbf{v}_f - \mathbf{v}_i = [-20, 0] \text{ mph}$$

$$\mathbf{a}_{av} = \frac{\mathbf{\Delta v}}{\Delta t} = \frac{[-20, 0] \text{ mph}}{20 \text{ sec}} = [-1, 0] \text{ mph/sec} \approx [-0.45, 0] \text{ m/sec/sec}$$

Finally, consider the example of a car slowing down. A car traveling initially at 60 miles per hour coasts gradually, in 20 seconds, to a speed of 40 miles per hour on a straight road (as in Example 13.4, above). What is the acceleration? You may respond that the car is not accelerated at all, that it is decelerated. This is true, in everyday language. The scientific definition of acceleration, however, refers to a velocity change and does not depend on whether the change represents an increase or a decrease in speed. The acceleration of the coasting car is calculated in Example 13.4, above, and it comes out with a negative x-component; this indicates that the acceleration is, in this case, directed in the *opposite* direction from the velocity, as expected for a decrease in speed.

It is clear from all these examples that the scientific meaning of the word "acceleration" frequently does not correspond to the intuitive meaning of the word. For interpreting changes in velocity, the definition we have given (Equation 13.9) is more useful than the everyday meaning. These examples also show how to apply the scientific definition of acceleration for all the possible ways that the velocity can change.

13.3 Momentum

FORMAL DEFINITION
Momentum is the product of inertial mass multiplied by instantaneous velocity.

Changes in motion brought about by interaction during a rear-end collision are a painful part of many people's experience. Imagine the cautious driver waiting at a red light, who glances into his rearview mirror and sees a trailer-truck bearing down on his car from behind! Even though the truck is advancing quite slowly, our hero may grip his steering wheel in grim anticipation. By contrast, a light motorcycle or bicycle seen in the same rearview mirror would hardly be a cause for alarm unless it was approaching at very high speed.

Equation 13.10

inertial mass (*kg*) = M_I
velocity (m/sec) = $\mathbf{v}$
momentum = $\mathcal{M}$

$$\mathcal{M} = M_I \mathbf{v}$$

Units of momentum:
kg-m/sec = kilogram-meters per second

This example illustrates that *both* speed and mass of a moving object influence its interaction with other objects upon collision. In Section 3.4, we defined the inertial mass as a measure of a body's resistance to change in motion and the momentum as an important property of a moving body, equal to the product of the inertial mass and the speed (Eq. 3.1). We now redefine the momentum to assign it a direction in space as well as a magnitude by relating it not to the speed but to the velocity (Eq. 13.10). The direction of the momentum is defined to be exactly the same as the direction of the velocity: the momentum arrow points in exactly the same direction as the velocity arrow. However, the two arrows are a different length because the momentum and the velocity have different magnitudes. In fact, as you would expect, the magnitude of the momentum is equal to the mass times the speed (the magnitude of the velocity). The momentum concept will be used to formulate Newton's theory of moving bodies in Chapter 14.

Summary

Equation 13.1 (average velocity)

$$\mathbf{v}_{av} = \frac{\Delta \mathbf{s}}{\Delta t}$$

Equation 13.9 (definition of acceleration)

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Equation 13.10 (definition of momentum)

$$\mathcal{M} = M_I \mathbf{v}$$

Relative position, displacement, velocity, acceleration, and momentum are physical quantities that are described by a magnitude and a direction in space. You must distinguish between the average and the instantaneous velocity of a moving body. The average velocity, measured from observations of the path of the moving object, is equal to the displacement divided by the time interval (Eq. 13.1). The instantaneous velocity is an extrapolation of the average velocity to a time interval that is so short (infinitesimal) that the velocity does not change. The velocity must also be distinguished from the speed, which is equal to the actual distance traveled divided by the time interval.

The average acceleration is equal to the change of instantaneous velocity divided by the time interval (Eq. 13.9). The momentum of a body is equal to the velocity multiplied by the inertial mass of the body (Eq. 13.10). We will use the concepts of acceleration and momentum in the next chapter (Ch. 14) to formulate Newton's theory of moving bodies, which shows how the motion of an object is related to the forces on that object.

List of new terms

average velocity
instantaneous velocity
infinitesimal time interval
average acceleration
momentum

List of symbols

$\mathbf{s}$	position	$\mathbf{v}$	instantaneous velocity
$\Delta\mathbf{s}$	displacement	v	instantaneous speed
Δs	distance	$\Delta\mathbf{v}$	change of velocity
Δt	time interval	$\mathbf{a}_{av}$	average acceleration
$\mathbf{v}_{av}$	average velocity	$\mathcal{M}$	momentum

Note: Boldface symbols, such as **s** and **v**, represent quantities with *both* magnitude and direction. Such quantities are represented as arrows with a length (magnitude) pointing in a particular direction. Such quantities cannot be represented with a single number and must be expressed mathematically with rectangular (x, y) or polar (r, θ) coordinates. Rectangular coordinates are often more convenient, especially in calculations where quantities such as displacements or velocities must be added and subtracted.

Problems

1. Formulate an operational definition for velocity. Compare your definition with the formal definition in Section 13.1. Discuss advantages and disadvantages of the formal and the operational definitions.

2. Consider the child on the merry-go-round (Fig. 13.3) and suppose it moves in a circle with a radius of 6 meters. The merry-go-round takes 12 seconds to make one revolution.
(a) Calculate the distance the child travels during one revolution.
(b) Calculate the average speed of the child (ratio of actual distance traveled to time interval) during one full revolution.
(c) Calculate the average velocity of the child during one half of a revolution. Note: find the magnitude and direction (using polar or rectangular coordinates) of the velocity in a coordinate frame with origin at the child's starting position.
(d) Calculate the average velocity during: one quarter of a revolution; one sixth of a revolution; one twelfth of a revolution. (See note in (c).)
(e) Plot a graph of the magnitude of the average velocity versus the time of the interval Δt you used in questions (c) and (d).
(f) Plot a graph of the direction (angle) of the average velocity versus the time of the interval Δt you used in questions (c) and (d).
(g) Estimate the instantaneous velocity of the child by extrapolating to an infinitesimal time interval ($\Delta t = 0$) in questions (e) and (f).

3. Refer to Fig. 13.4 for this question.
(a) Find the average velocity of golf ball A during the intervals between pictures 4 and 5, 9 and 10, and 13 and 14. Compare these average velocities with those of golf ball B recorded in Table 13.2.
(b) Find the (approximate) average acceleration of golf ball A between the fifth and tenth intervals, and between the tenth and fourteenth intervals.
(c) Find the (approximate) average acceleration of golf ball B between the tenth and fourteenth intervals. Compare the accelerations you found in (b), (c), and in Example 13.1.
(d) Discuss and estimate the extent to which the instantaneous velocities of golf balls A and B in this experiment differ from the average velocities determined in part (a) and in Table 13.2.

4. (a) Describe examples (beyond those included in Section 13.2) of situations where the human body easily senses changes of velocity.
(b) Describe the body's response (if any) to motion at high velocity that is constant (does not change). Examples of this type of motion are an airplane or train traveling at constant speed in a straight line.

5. Calculate the highest forward (speeding up) acceleration of which existing automobiles are capable. Include all the data on which you base your work. (Consult magazines, automobile dealers.)

6. Estimate the highest acceleration of which automobiles are capable

when they come to an emergency stop (with the brakes slammed on). (Note: As explained in Section 13.2, acceleration is used to denote both increases and decreases in speed; the only condition is that the velocity changes.) State your estimate numerically.

7. Estimate the acceleration of other vehicles (for example, jet aircraft on takeoff and landing, motorcycles, speedboats). Include all the data on which you base your work. State your estimates numerically.

8. The velocity of a child on a merry-go-round (Fig. 13.3) is changing continually in direction. Make a rough quantitative estimate of the acceleration involved. If you did problem 2 above, you can use the results you found there for parts (d) through (g).

9. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

Bibliography

L. Cooper, *An Introduction to the Meaning and Structure of Physics*, Harper and Row, New York, 1968. See the part "On the Problem of Motion."

G. Galilei, *Dialogues Concerning Two New Sciences*, Northwestern University Press, Evanston, Illinois, 1950. Look at the section called "Fourth Day."

D. A. Greenberg, *Mathematics for Science Courses*, W. A. Benjamin, New York, 1964.

E. R. Huggins, *Physics 1*, W. A. Benjamin, New York, 1968. The author has an extensive discussion of multiple exposure photographs and their use in the analysis of motion.