Chapter 14: Newton's Laws of Motion

Isaac Newton (1642-1727) attended Trinity College, Cambridge, and obtained his degree in 1665. Although an excellent student, his early Cambridge years offered no indications of his future greatness. The plague of 1665-1666 forced Newton to return to his home in Woolsthorpe. The seclusion and lack of responsibilities over the next year and a half allowed Newton to work intensively by himself, with extraordinary results.

By 1666, at 24, he had made profound discoveries in mathematics (binomial theorem, calculus), optics (ray model), and mechanics. To this period also belongs the famous (though possibly untrue) incident of the falling apple, the realization that the force of gravity reached the "orb of the moon," and the discovery of the inverse second power law of gravitational attraction.

Returning to Cambridge, Newton continued his study of planetary motions as a problem in physics. Unfortunately, controversy with Huygens and

The subject matter of this chapter has been anticipated in several earlier sections of this text. In Section 3.4, we described inertia of motion and presented an operational definition of inertial mass. We introduced the force concept through an operational definition using a spring scale in Section 11.2. Finally, in Sections 13.2 and 13.3, the formal definitions of acceleration and momentum extended your ability to describe changes of velocity and to associate the effects of inertial mass with motion.

You are familiar with many examples of moving objects that interact with other nearby or remote objects. A few instances are the stream of water erupting from Old Faithful in Yellowstone Park, the arrow launched by the archer's bow, a sailboat racing across the wind, a stunt car rounding a curve on two wheels, and the entire earth orbiting around the sun. In Section 3.4 (as well as 11.2 and 13.2), you learned to interpret changes in motion as evidence of interaction. Both the eruption of the water of the geyser and its return to the ground are evidence of interaction between the water and something else. A body moving with a constant velocity shows no evidence of interaction by its motion and is in mechanical equilibrium. It may be completely free of interaction or it may be subject to compensating forces, like the skier who moves steadily uphill while interacting with the rope, the snow, and the earth (via the gravitational field).

14.1 Background

Galileo showed the essential similarity between a stationary object and an object moving at constant speed in a straight line, and he identified inertia as an key concept. Galileo systematically applied the concepts of velocity and acceleration to the study of moving bodies and thereby began a transformation (some would say revolution) in human thought that was completed by Newton two generations later. In the intervening years, Descartes introduced the rectangular coordinate frame so useful for a mathematical description of relative position and motion (Sections 2.1 and 13.1), and Huygens investigated the forces acting on a pendulum in circular motion. Newton, finally, formulated the science of classical mechanics, a deductive system of definitions and assumptions (called "Newton's Laws of Motion"), which replaced the Aristotelian system in use until Galileo's time. The concepts of force, mass, momentum, and acceleration were integrated into one powerful theoretical framework (summarized in Table 14.1) that is still being refined and used for the prediction of motion in macro- and cosmic-domain phenomena. Only for various forms of radiation, as we explained in Chapters 7 and 8, was Newton's theory found inadequate and replaced by a wave theory in the nineteenth and twentieth centuries. The justification for Newton's three laws of motion rests in their broad and amply-verified predictive power.

Newton's laws lead to mathematical models for the relation among force, inertial mass, acceleration, and momentum. These models will enable you to predict the motion of bodies that are subject to known forces and to make inferences about the forces from observed motion.

Hooke discouraged Newton, an extraordinarily sensitive personality, from publishing his results. Other scientists were also working on the same problems but had hit a variety of dead ends. In particular, no one had been able to demonstrate a rigorous mathematical connection between the observed elliptical shape of the planets' orbits and the dependence of the strength of gravitational attraction (the force) on the distance between the planet and the sun.

The geometrical properties of the ellipse had been well known since ancient times. As a planet moves along an elliptical path, the distance to the sun (located at one focus) changes, and the actual shape of the orbit seemed to have a close connection with the variation in the force on the planet as the distance to the sun changes. However, there were enormous mathematical difficulties involved in demonstrating this connection; the best mathematicians and "natural philosophers" (as physicists were called), despite intense effort, could not work out a convincing mathematical proof.

In 1684 Edmond Halley, an astronomer studying the motions of the planets, mentioned this problem to Newton. According to Newton's later account (as told to Demoivre), Newton immediately replied that he had solved it but couldn't find the paper and agreed to write it out again. Over the next two years, with Halley's unstinting encouragement and support, Newton wrote his monumental Principia*, containing not only the promised proof but also Newton's complete theory of motion and its sweeping explanation of the existing celestial observations. The appearance of the* Principia *in 1687 established Newton's reputation for all time.*

TABLE 14.1 DEFINITIONS AND LAWS IN NEWTON'S THEORY

Concept	*As stated by Newton*	*As stated in this text*
gravitational mass	*The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.*	*Gravitational mass of an object is measured by the number of standard units of mass that are required to balance the desired object on an equal-arm balance.*
velocity	*not stated.*	*Average velocity is the ratio of the displacement divided by the time interval required for the displacement.*
momentum	*The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.*	*Momentum is the product of inertial mass multiplied by instantaneous velocity.*
inertia	*The inertia, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*	*Inertial mass is measured by the number of standard units of mass that are required to give the same rate of oscillation of an inertial balance.*
force	*An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line*	*Force is measured by a standard spring scale. The scale reading indicates the magnitude of the force. The direction of the force is related to the direction of the spring scale.*
Law 1	*Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*	*Every particle persists in its state of rest or of uniform, unaccelerated motion in a straight line unless it is compelled to change that state by the application of an external net force.*

TABLE 14.1, CONTINUED

Concept	*As stated by Newton*	*As stated in this text*
Law 2	*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*	*The change of momentum of a particle subject to a net force during a time interval is equal to the average net force times the duration of the time interval.* $\boldsymbol{F}_{av}\,\Delta t = \Delta\boldsymbol{\mathcal{M}}$
Law 3	*To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*	*The interaction of two bodies is described by two equal and opposite forces*

The center-of-mass model. There are several ways in which Newton's theory introduces assumptions, in addition to the laws, that enabled him to reduce complicated phenomena to simpler ones. In Section 11.2 we already mentioned the assumption that the net force acting on a body is the sum of the partial forces arising within the two-body subsystems of the original system. A second assumption, used in conjunction with the first and second laws, is that a moving body can be described as one particle located at a central point called the center of mass of the body. That is, the inertial mass, position, velocity, and acceleration of the entire body are the mass, position, velocity, and acceleration of the particle located at the center of mass.

If the body is quite small (such as a drop of water or even a baseball), then the one-particle model usually permits an adequate description of its motion. If the body is large or has a complicated shape (such as a skier), then the one-particle model may be adequate for its overall motion, but a more complicated working model has to be adopted for a more complete description of its interaction, energy storage, and so on. For example, a boxer might be seen as three particles, two fists and a torso; the medium for wave propagation was represented by a model consisting of many oscillators (Section 6.1); we used the MIP model to represent an elastic body (Section 11.7); in Chapter 16, we will introduce a model of many non-interacting particles to represent a gas. Newton's third law makes possible the analysis of such systems in terms of the first and second laws. However, systems requiring a complicated description (for example, rotating rigid bodies such as bicycle wheels or gyroscopes) or working models consisting of more than one or two interacting particles will not be discussed in detail in this chapter.

Interaction mechanisms. In Newton's simplified approach to moving bodies, the various mechanisms by which they interact are not described in detail. The forces may be elastic, frictional, gravitational, electric, or magnetic. You may even find the MIP model helpful in understanding how the chair on which you sit is an elastic system that can exert a force to balance the force of gravity acting on you. Since Newton described an MIP model for matter, it is likely that he, too, used it to visualize interaction mechanisms. His theory of moving bodies, however, does not make direct reference to this model and is not logically dependent on it.

". . . It seems probable to me that God in the beginning form'd matter in solid, massy, hard, impenetrable, movable particles, of such sizes and figures, and with such other properties, and in such proportion to space, as most conduced to the end for which he form'd them; and that these primitive particles being solids, are incomparably harder than any porous bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary power being able to divide what God himself made one in the first creation ..."

Isaac Newton
Opticks, 1704

14.2 Newton's first law of motion

Newton's first law of motion, sometimes called the *law of inertia,* is as follows: *Every particle persists in its state of rest or of uniform, unaccelerated motion in a straight line unless it is compelled to change that state by the application of a net force.*

Comparison of Newtonian and Aristotelian views. The assumption expressed in this statement was contrary to that of Aristotelian scholarship, according to which force was the cause of uniform motion and only the state of rest was an equilibrium state for a non-interacting body. Newton's First Law, in contrast, asserts that motion in a straight line at constant speed is *also* an equilibrium state and, therefore, does not require a net force. As an example, consider the two opposing interpretations of a skier going downhill (Fig. 14.1 and 14.2).

Aristotle (384-322 B.C.) Aristotle's father was the physician to Philip of Macedon, and Aristotle became tutor to the son of Philip, the young Alexander the Great. Aristotle studied for many years at Plato's Academy and then founded his own school, the Lyceum, in Athens. Aristotle exerted a profound influence on the development of science. His works were so comprehensive that until the Renaissance no systematic survey comparable to his was produced in the west. Aristotle remained the authority in Western academic circles for almost 2000 years.

The Newtonian observer would hold that the skier is subject to the force of gravity exerted by the earth and forces of support and friction by the snow. When these partial forces combine to give a zero net force, the skier remains at rest or skis at a uniform rate (Fig. 14.1). When the friction decreases (on a patch of ice, for instance), the skier accelerates (Fig. 14.1d); when the slope levels out so that gravity and support together are less than friction, the skier slows down.

The Aristotelian would insist that a force is always acting on the skier while he is moving, presumably larger or smaller depending on the skier's speed (Fig. 14.2). Only when the skier stops completely would the Aristotelian hold that there is no unbalanced force. As we have remarked in Chapter 3, some of our intuition about motion parallels the Aristotelian view because friction is so pervasive in terrestrial phenomena. We therefore tend to overlook friction and think of the partial forces that actually balance friction during steady motion as adding up to a net force.

A second and possibly even more revolutionary implication of Newton's first law is that changes in the state of motion of a body must always be ascribed to an external agency. This again is contrary to Aristotelian and commonsense views, in which motion of a person or an animal usually has an internal cause. We will discuss the skier and a

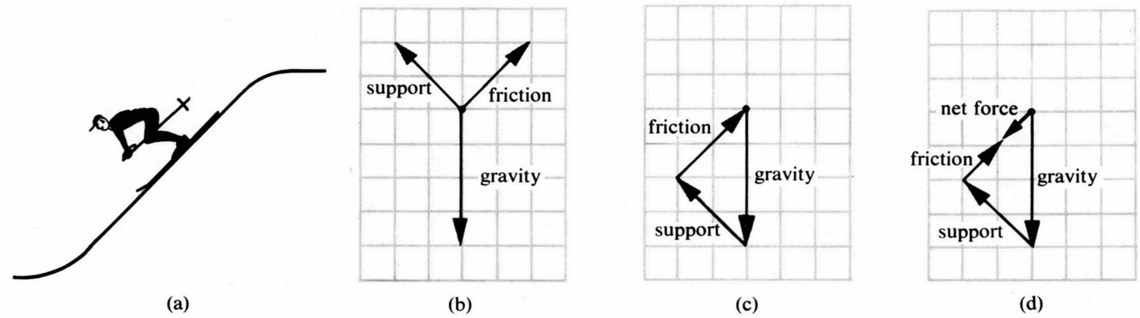


Figure 14.1 Newtonian view of a skier sliding downhill, subject to the three partial forces of gravity, friction, and support.
(a) Gravity is directed vertically downward, friction is along the gliding surface, and support is at right angles to the gliding surface.
(b, c) Force diagrams: three partial forces add to zero net force.
(d) Force diagram (reduced friction): three partial forces add to a nonzero net force and the skier speeds up.

"I call absolutely light that whose nature is to move always upward, and heavy whose nature is to move always downward, if there is no interference."

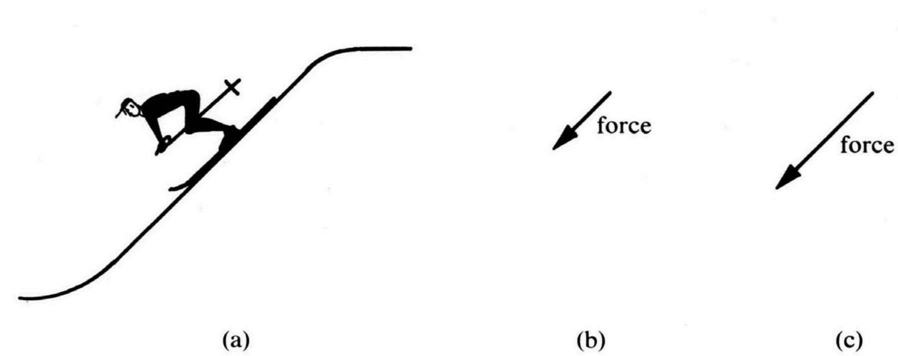
Aristotle
On the Heavens,
4th century, B.C.

man walking uphill as two examples. The modem scientist would agree that the skier accelerates downward because of the force of gravity. According to Aristotle, however, the skier moves downward because all bodies have a tendency to reach their "natural place" of repose, which is downward for solid and liquid materials.

Aristotle would ascribe motion of the man walking uphill to the man's desire to walk. The cause of motion is internal: the man's desire and his muscular energy. Where is Newton's external net force?

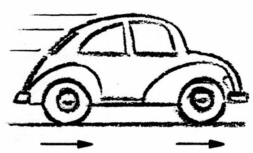
To answer this question, you must realize that the Newtonian theory distinguishes the energy source that is necessary for motion from the force that causes the motion. In the case of the skier going

Figure 14.2 Aristotelian view of a skier sliding downhill.
(a) A skier slides downhill.
(b) A force is acting to maintain motion.
(c) If the slope is steeper, or smoother, a larger force acts and causes a larger speed.



downhill, the gravitational field transmitted the force and acted as a source of the skier's kinetic energy, so the distinction was not apparent. When the man walks uphill, however, his muscles are the energy source, while now the frictional interaction between his feet and the ground supplies the force that enables him to move forward with each step. In the absence of friction (on an iceberg or on a path covered with very loose gravel), the net force on him might be inadequate for him to progress upward and he might even slide back down. Think of a skier walking uphill, for example.

Does the energy source exert a force? A few other examples will further illustrate the distinction between the energy source and the objects or systems that exert the force that sets a body in motion. When the archer shoots his arrow, the string actually exerts a net force on the arrow and is the coupling element to the bow, which originally stored the energy. When a car accelerates, the fuel in the engine is the energy source, but the force that sets the car into motion is exerted by the road through its frictional interaction with the tires. If the car is on an ice patch and the force of friction is small, then the engine turns, the wheels spin, energy is transferred, but the car does not move. When a glass marble bounces on a steel plate (Section 11.7), both the deformed glass and steel act as energy sources, but it is the upward force of the steel plate on the marble that reverses the motion of the marble. When a child rides on a horse on a merry-go-round, the motion is in a circle (not a straight line), hence there is a net force acting on the child. In this example, the energy source is the fuel in the engine, but the force on the child is exerted by the horse on which he or she rides.



Inertial reference frames. Before we conclude this section, we would like to point out one difficulty with Newton's first law. Even though it refers to the motion of a particle, it does not specify the reference frame relative to which this motion is to be observed. A reference frame in which Newton's first law applies is called an *inertial reference frame.* So far we have been using reference frames attached to the surface of the earth, and we have found reasonable agreement between theory and observation. Hence they were (nearly) inertial reference frames. In a reference frame attached to a merry-go-round, however, a stone on the ground and the rest of the earth move in a circle around the merry-go-round! To treat the merry-go-round as an inertial reference frame, you would conclude that starting the motor sets the entire earth in motion and that stopping the motor stops the earth again. Within the Newtonian theory, you then have to invent the necessary forces exerted by external objects on the massive earth to set it in motion. If you are willing to do this, you may go ahead with Newton's theory. If the invention of "fictitious" forces seems too big a price to pay, you may classify the merry-go-round as a non-inertial reference frame in which Newton's first law cannot be applied. The choice is yours, but you must stick to your decision consistently once you have made it.

Moving automobile reference frame. The same considerations apply to the reference frame attached to a moving automobile (Fig. 14.3a). As long as the speedometer reading is a constant 50 miles per hour, the

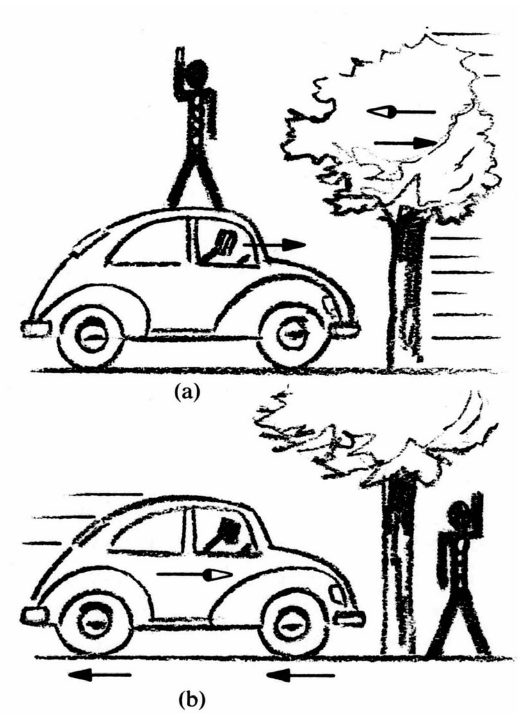


Figure 14.3 A car's brakes are applied.
(a) The reference frame is attached to the car. What object exerts the forces that cause the tree to stop and the driver to lurch forward?
(b) The reference frame is attached to the road. The force of friction exerted by the road on the tires changes the velocity of the car.

landscape moves past at a steady rate. The driver inside is at rest. Neither the landscape nor the driver, therefore, is subject to a net force, a fact compatible with your common sense. Now the driver steps on the brakes. Quickly the landscape comes to a stop, and he lurches forward in his seat. To apply Newton's theory in this reference frame, you must invent a force that causes the landscape to stop and another force that causes the driver to lurch forward. If such forces act, the stopping car is an inertial reference frame. If the invention of such forces seems too farfetched, you may classify the stopping car as a non-inertial reference frame in which Newton's first law does not apply. Again, the choice is yours. When you have ridden in an automobile, you undoubtedly have sensed a "force" that caused you to lurch forward when the car's brakes were applied. You may therefore decide to adopt this reference frame in spite of the problems with the moving landscape.

Road reference frame. Does the automobile problem become simpler in a reference frame attached to the road (Fig. 14.3b)? Such a reference frame is an inertial one. At the beginning, the landscape is at rest (zero net force), while the automobile and the driver inside are both moving steadily (zero net force). Now the driver steps on the brakes. Quickly the car comes to a stop (net frictional force exerted by the road on the car via the tires). The driver continues in motion until a net force acts to bring him to a stop; this may be exerted by the car floor on his feet or by the seat belts on his body. Relative to the car, therefore, the driver lurches forward. Relative to the road, however, he merely comes to a stop a little later than the car does, when the restraint of the

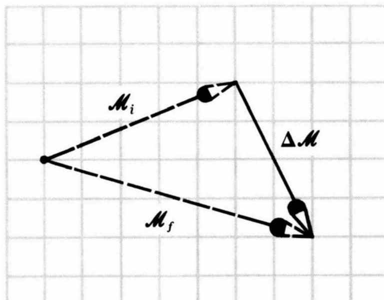
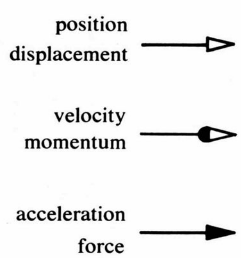


Figure 14.4 Arrows $\mathbf{M}_i$ *and* $\mathbf{M}_f$ *represent the momenta before and after the action* of the force. The change of mo-*mentum is* $\Delta \mathbf{M} = \mathbf{M}_f - \mathbf{M}_i$

seat belt becomes effective. In the road reference frame it is not necessary to invent interactions that provide the needed forces; the familiar interactions are sufficient. Since physicists are reluctant to invent special solutions to each problem, such as were required when the merry-go-round or the car were treated as inertial reference frames, they prefer to apply Newton's theory in the earth-fixed reference frames for which this step is unnecessary.

Selection of inertial reference frames. As a general rule, a reference frame attached to the most massive object in the system is an inertial frame. For practical purposes this means the earth for terrestrial phenomena, the sun for the solar system, and the galaxy during interstellar travel. If a system includes several objects of comparable mass, such as several stars, then an inertial frame is more difficult to find.

Arrows have the following significance:



Equation 14·1

average net force (newtons) $\mathbf{F}_{av}$
time interval (sec) Δt
change of momentum (kg-m/sec) $\Delta\mathcal{M}$

$$\mathbf{F}_{av}\Delta t = \Delta\mathcal{M} \qquad \text{(a)}$$

$$\mathbf{F}_{av} = \frac{\Delta\mathcal{M}}{\Delta t} \qquad \text{(b)}$$

14.3 Newton's second law of motion

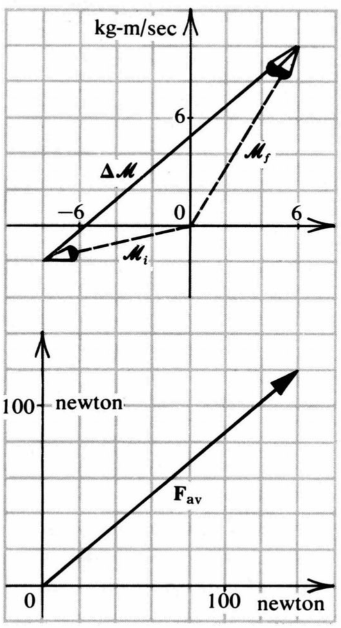
Statement of Newton's second law. We have explained that, according to the first law, a change in the state of motion of a particle is accompanied by a net force acting on the particle. The question answered by the second law is just how the unbalanced force and the change in motion are related.

In Chapter 13, velocity and momentum (inertial mass times velocity) were introduced to describe the motion of a particle. During uniform motion, each of these quantities remains constant, that is, retains its magnitude and its direction. Change of motion may be described by a change of either or both quantities. The mathematical model proposed by Newton in his second law relates the net force acting during a time interval to the change in momentum that is caused by the force, as follows.

The average net force times the time interval during which it acts is equal to the change of momentum of the particle on which the force acts (Eq. 14.1a).

The meaning of a change of momentum is illustrated in Fig. 14.4. The change of momentum is the difference between the momentum of the particle after and before the action of the net force. Note that momentum and force have direction and magnitude. The change of momentum therefore also has direction and magnitude (Examples 14.1 and 14.2).

The momentum of the particle after the action of the net force depends on the relative direction of the change of momentum and the initial momentum, as shown in the various parts of Fig. 14.5.



EXAMPLE 14·1. **A batter changes the momentum of a baseball from [−8, −2] kilogram-meters per second to [6, 10] kilogram-meters per second by hitting it with his bat. Find the average force under the assumption that ball and bat were in contact for 0.10 second.**

Data:

$\mathcal{M}_i = [-8, -2]$ kg-m/sec, $\mathcal{M}_f = [6, 10]$ kg-m/sec
$\Delta t = 0.10$ sec

Solution:

$$\Delta \mathcal{M} = \mathcal{M}_f - \mathcal{M}_i = [6, 10] \text{ kg-m/sec} - [-8, -2] \text{ kg-m/sec} = [14, 12] \text{ kg-m/sec}$$

$$\mathbf{F}_{av} = \frac{\Delta \mathcal{M}}{\Delta t} = \frac{[14, 12] \text{ kg-m/sec}}{0.10 \text{ sec}} = [140, 120] \text{ newtons}$$

EXAMPLE 14·2. **A car traveling at 60 miles per hour crashes into a tree and comes to a "stop" in 1 second. How large a force is necessary to reduce the driver's momentum so that he does not hit the windshield?**

Data:

driver's mass (estimate)	$M_I \approx 75$ kg
driver's speed	$v = 60$ mph ≈ 27 m/sec
time to stop	$\Delta t = 1$ sec

Solution:

driver's initial momentum:

$$\mathcal{M}_i = M_I \mathbf{v} \approx 75 \text{ kg} \times 27 \text{ m/sec} \approx 2000 \text{ kg-m/sec, in the direction of the car's motion}$$

driver's final momentum:

$$\mathcal{M}_f = 0$$

momentum change:

$$\Delta \mathcal{M} = \mathcal{M}_f - \mathcal{M}_i \approx -2000 \text{ kg-m/sec in the direction of motion}$$

$$\mathbf{F}_{av} = \frac{\Delta \mathcal{M}}{\Delta t} \approx \frac{-2000 \text{ kg-m/sec}}{1 \text{ sec}} = -2000 \text{ newtons in the direction of motion}$$

The minus sign means that the force is opposite to the direction of motion. Its magnitude is 2000 newtons or about 450 pounds. This is much more than the force of friction between the driver's pants and the seat. Seat belts can provide the necessary force.

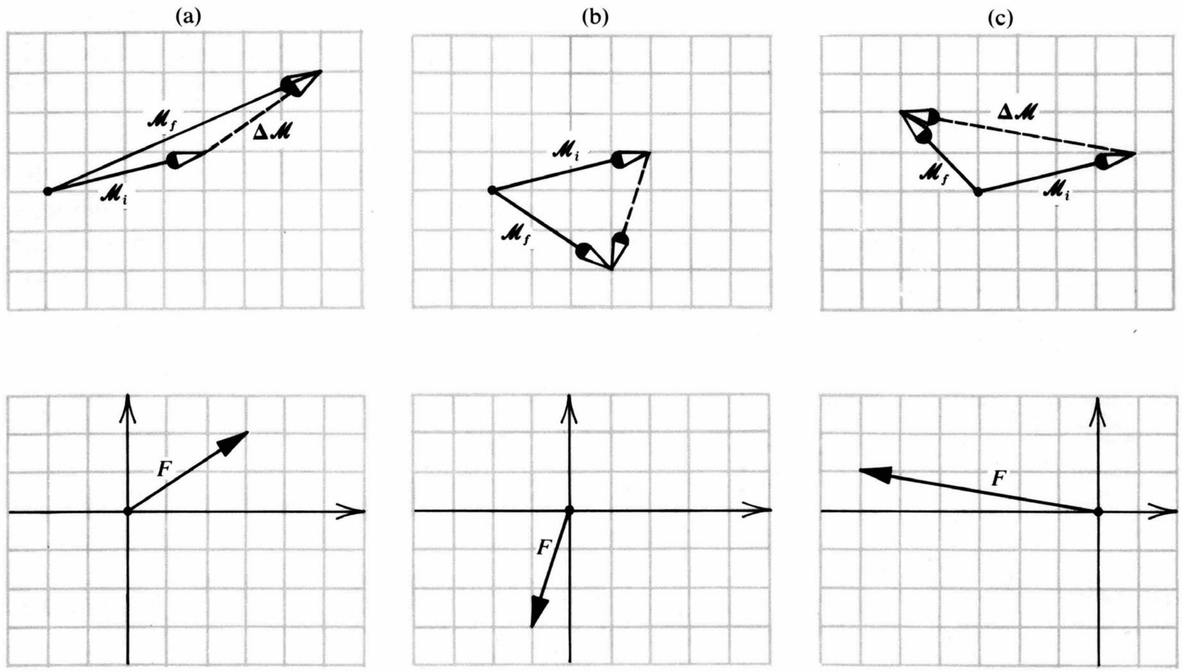


Figure 14.5 Three examples to illustrate how the final momentum of the particle after the action of the force ($\mathcal{M}_\mathrm{f}$) depends on the change of momentum ($\Delta\mathcal{M}$, shown as a dashed arrow), which is in the direction of the force. The same initial momentum ($\mathcal{M}_\mathrm{i}$) is used in each example. In all diagrams: $\mathcal{M}_\mathrm{f} = \mathcal{M}_\mathrm{i} + \Delta\mathcal{M}$.

Equation 14.2
$F_{av} = 0$ *implies* $\Delta\mathcal{M} = 0$

Equation 14.3
$\Delta t = 0$ *implies* $\Delta\mathcal{M} = 0$

Immediate consequences of the second law. Several consequences follow immediately from Newton's model. First, the change of momentum is zero if the average net force is zero (Eq. 14.2). This implication is a restatement of the first law, because the change of a particle's momentum is zero only if its velocity is constant.

Second, the change of momentum is zero if the time interval Δt is zero (Eq. 14.3). A nonzero change of momentum can be achieved only by the action of a force during a nonzero time interval. In other words, there are no instantaneous jumps of momentum. Even the bouncing marble (Section 11.7) is brought to rest and reaccelerated in a finite, though extremely short, time interval. This consequence of the theory reflects the inertia of the particles on which the force acts.

Third, the *change* of momentum is in the direction of the net force. A particle initially at rest (zero momentum) acquires a momentum in the direction of the force acting on it. A particle already in motion may acquire an increased momentum, a decreased momentum, or a deflected momentum due to the action of the force (Fig. 14.5). The *final* momentum and, therefore, the final velocity after the action of the force is often *not* in the direction of the force. This is one of the most difficult consequences of the Newtonian theory to accept, because it is contrary to a deeply ingrained notion: that objects move in the same direction as the applied force. This ingrained notion, however, disregards the inertia of particles. Because particles have inertia, *both* their previous momentum *and* the force acting at any instant determine what they will do next, that is, their motion in the immediate future.

Acceleration. To use the mathematical model in Newton's second law for describing moving particles whose position and velocity are observed, we will rephrase it so that it refers directly to acceleration instead of to momentum. The definitions of acceleration and momentum given in Sections 13.2 and 13.3 are repeated here: acceleration is the change of velocity divided by the time interval (Eq. 13.9); momentum is the product of inertial mass times velocity (Eq. 13.10). You may wish to refer to Section 13.2 for an explanation of the acceleration concept in physics and how it differs from everyday usage of the word.

Equation 13·9

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Equation 13·10

$$\mathcal{M} = M_I \mathbf{v}$$

The momentum of a particle may change because its inertial mass changes, because its velocity changes, or because both mass and velocity change. We will pursue only the theory for particles whose velocity may change, but whose inertial mass remains constant. In this theory, the change of momentum of a particle is equal to its inertial mass multiplied by the change of velocity (Eq. 14.4). When this mathematical model for momentum change is used in Newton's second law, the latter can be put into the form that the average net force equals the inertial mass times the average acceleration (Eq. 14.5). By choosing sufficiently short time intervals, the average force and average acceleration can both be made approximately equal to the instantaneous force and acceleration as in Eqs. 2.3 and 13.2 for the speed and the velocity. The resulting form of Newton's second law, net force equals mass times acceleration (Eq. 14.6), is the one most frequently quoted.

Equation 14·4

$$\Delta \mathcal{M} = M_I \Delta \mathbf{v}$$

Equation 14·5

$$\mathbf{F}_{av} = M_I \frac{\Delta \mathbf{v}}{\Delta t} = M_I \mathbf{a}_{av}$$

Equation 14·6

instantaneous force **F**
instantaneous acceleration **a**

$$\mathbf{F} = M_I \mathbf{a}$$

Predictions from Newton's second law. With the help of the formulation of Newton's second law in Eq. 14.6, you can compare some of its predictions with experimental observations to determine how satisfactory the theory really is. A few consequences of the model can be inferred directly from Eq. 14.6:

1. Constant net force produces a constant acceleration.
2. Zero net force produces a zero acceleration.
3. Net force is directly proportional to acceleration (doubling the force doubles the acceleration).
4. The force and the acceleration are in the same direction.
5. Two different particles experience equal accelerations if the net forces on them are in the same direction and the more massive particle is subject to a larger force in proportion to its larger inertial mass.

Some of these predictions, such as 2, 4, and 5, can be tested experimentally by direct observation and without measurements of the magnitude of the acceleration. Predictions 1 and 3 require more extensive measurement of acceleration. How the acceleration can be calculated approximately from successive observation of the position of a moving object is explained in the next paragraphs.

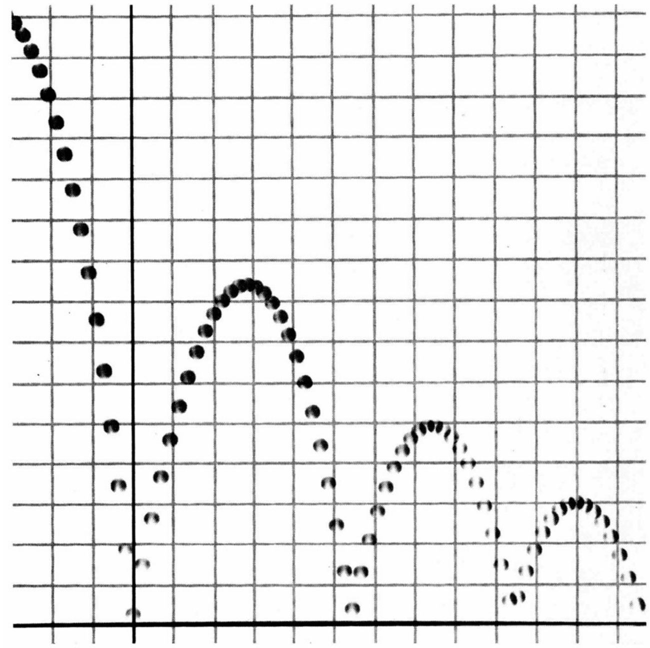


Figure 14.6 Multiflash photograph of a bouncing golf ball. The grid lines, whose spacing represents 0.11 meter, indicate the distance scale. The time intervals are 0.033 seconds.

Multiflash photographs. A record of the successive positions of the moving object is made conveniently through the technique of multiflash photography, described in Section 13.1. By flashing a light on the photographic subject repeatedly without changing the film, you obtain a multiple exposure on which the positions of the object at the times of the flashes can be seen. Eq. 13.1 relates the time interval between flashes, the average velocity, and the displacement. A multiflash photograph of a bouncing golf ball is shown in Fig. 14.6. The multiflash photograph records motion relative to the camera.

Equation 13·1

average velocity	$\mathbf{v}_{av}$
time interval between flashes	Δt
displacement between flashes	$\Delta\mathbf{s}$

$$\mathbf{v}_{av} = \frac{\Delta\mathbf{s}}{\Delta t}$$

"Frictionless" motion. To conduct laboratory experiments with controlled forces, we must eliminate the effects of gravity and friction. The former can be accomplished by restricting the motion to a horizontal surface, such as a table, that exerts a balancing upward force on the moving object (imagine an MIP model for the rigid table) and thereby prevents motion in the vertical direction. Friction cannot be eliminated completely, but it can be made very slight, as we explained in Section 3.4, by maintaining a thin layer of gas between the moving object and its supporting surface. Then the net force acting on the object is equal to the sum of the horizontal partial forces acting on it.

In one technique to reduce friction, a table or track is provided with many holes through which jets of air escape. As shown in Fig. 3.4, if a smooth object is placed on such a surface, the jets lift the object so it can "float" on a thin cushion of air and thus can glide almost without friction. Another device consists of a disk or "puck" that carries a container filled with dry ice (frozen carbon dioxide) (Fig. 14.7). As the carbon dioxide vaporizes, it escapes through a small hole in the center of the bottom of the disk (Fig. 14.8) and forms a thin layer of carbon dioxide gas between the disk and the glass surface on which it rests.

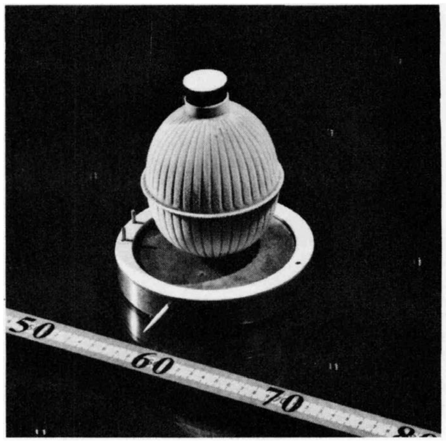


Figure 14.7 A "frictionless" dry ice puck.

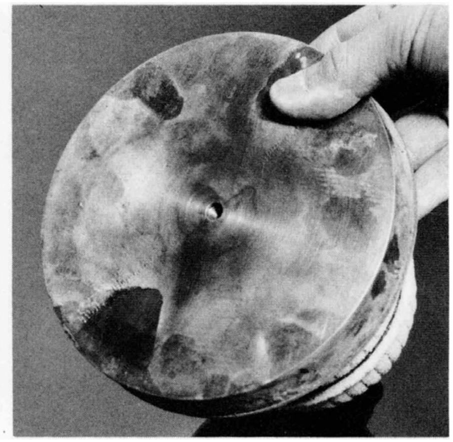


Figure 14.8 Carbon dioxide gas escapes through the hole in the puck base to form a thin film on which the "frictionless" puck glides.

This gas layer reduces the frictional force between the two surfaces to a very small value, so that we can study the motion of the disk without being distracted by the usual presence of friction.

Figure 14.9 shows a multiflash photograph of a dry ice puck moving from left to right alongside a meter stick. The light was flashed at the rate of 24 times in 10 seconds, or at intervals of 0.42 second. The speed of the puck can be calculated from a measurement of its displacement during each time interval (Table 14.2). Since the images are equally spaced, the displacements during equal times are equal, and the puck in this experiment approaches the ideal of constant velocity.

What would be the conclusion if the velocity were found to change? You could then conclude that there was an interaction influencing

Figure 14.9 A dry ice puck moving parallel to the meter stick was photographed at intervals of 0.42 second. Data from this experiment are analyzed in Table 14.2.

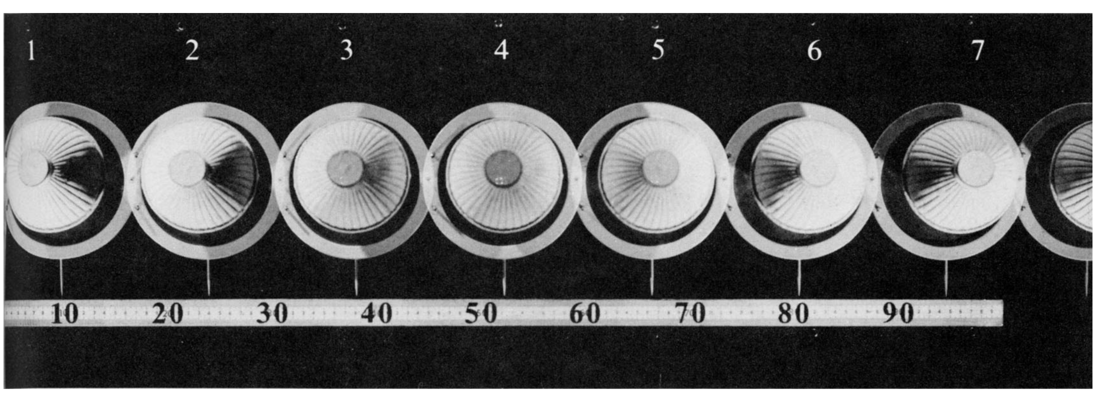


TABLE 14·2 MOTION OF THE FRICTIONLESS PUCK IN FIG. 14·9

Pictures defining interval	*Time interval (sec)*	*Displacement (m)*	*Velocity (m/sec)*
1–2	*0.42*	*[0.14, 0.00]**	*[0.33, 0.00]*
4–5	*0.42*	*[0.14, 0.00]*	*[0.33, 0.00]*
6–7	*0.42*	*[0.14, 0.00]*	*[0.33, 0.00]*

* Displacement components are measured to the right and up on Fig. 14·9.

the puck—perhaps friction, perhaps a slight tilt in the glass surface, or perhaps an air current. In other words, this experiment does not "prove" that free bodies move with constant velocity; instead it provides evidence that the puck is a very nearly free body according to Newton's first law.

Experimental test of Newton's second law. To observe the motion of a puck subject to a constant force, we attach a string and a circular spring (Fig. 14.10), which is a convenient elastic object for monitoring the constancy of the applied force. When the spring is deformed into an ellipse of predetermined standard shape, it transmits one unit of force, which we will call an *su* (for spring unit). How this force is related to the newton we do not know.

In Fig. 14.11, two forces, each 1 su in strength, are acting on the puck in opposite directions. The net force here is zero. In Fig. 14.12, two equal magnitude forces slightly larger than 1 su are acting at an angle; the net force acts in the direction bisecting the angle between the two forces.

Figure 14.10 (below). The circular spring is attached to the dry ice puck to measure the applied force.

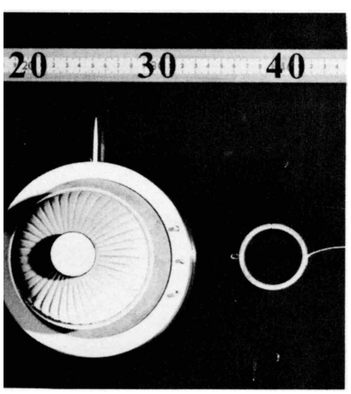
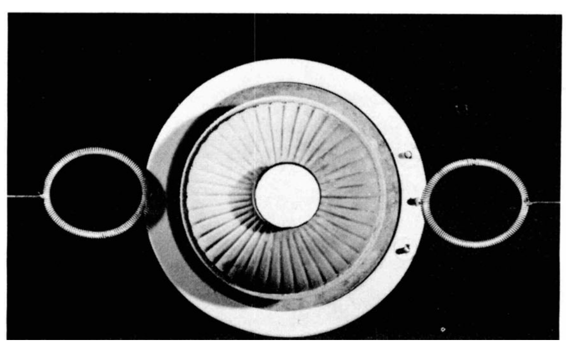
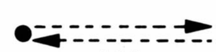


Figure 14.11 (below). Two equal-magnitude but oppositely directed forces act on the puck.
(a) The puck with deformed springs attached is in mechanical equilibrium,
(b) Force diagram, showing the combination of the two partial forces to give a zero net force.


(a)



(b)

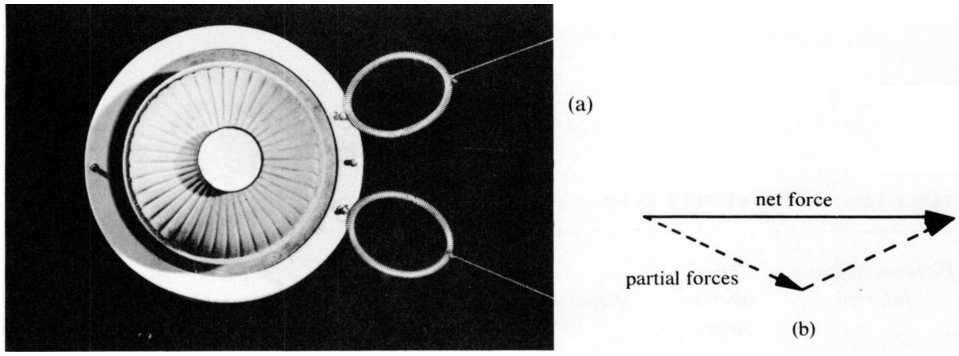


Figure 14.12 Two equal-magnitude forces at a slight angle act on the puck. (a) The puck with deformed springs attached is not in mechanical equilibrium, (b) Force diagram showing the combination of the two partial forces to give a nonzero net force.

Comparison of two forces. The effects on the puck of a force of 1 su and of a force of 2 su are shown in the multiflash photographs in Fig. 14.13 and Fig. 14.14, respectively. You can see that the pucks move in a straight line (as defined by the meter stick) in the direction of the force, and that the speed is not constant but increases. According

Figure 14.13 The dry ice puck is accelerated to the right by the action of a net force of 1 su,

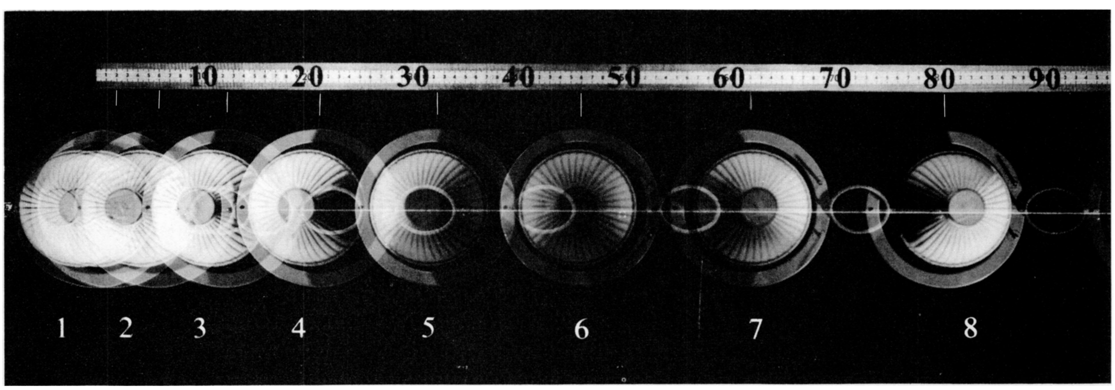


Figure 14.14 The dry ice puck is accelerated to the right by the action of a net force of 2 su.

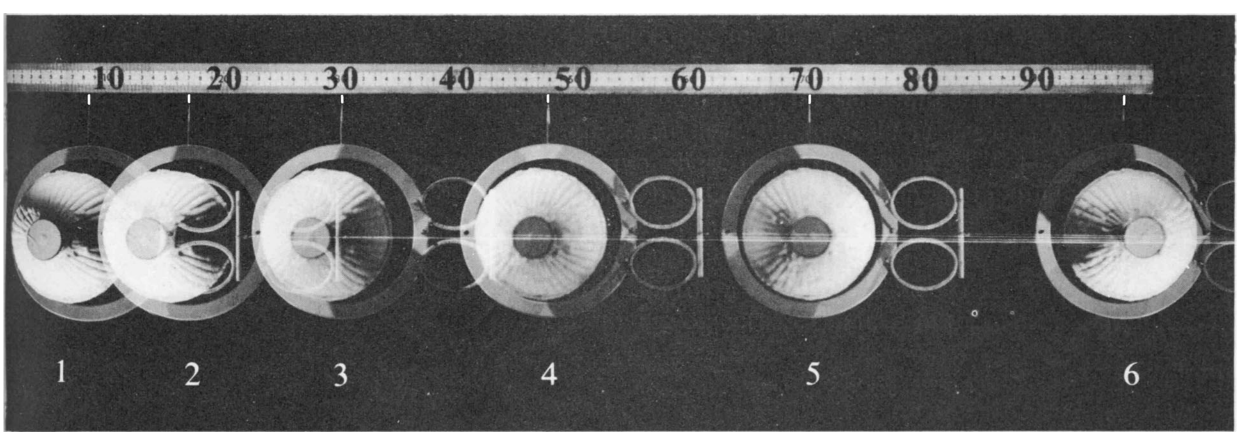


TABLE 14·3 MOTION OF PUCK IN FIG. 14·13

Pictures defining interval	*Time interval (sec)*	*Displacement (m)*	*Average velocity (m/sec)*	*Change of average velocity (m/sec)*	*"Acceleration" (m/sec/sec)*
1–2	0.42	[0.041, 0.000]	[0.10, 0.00]	[0.05, 0.00]	[0.12, 0.00]
2–3	0.42	[0.063, 0.000]	[0.15, 0.00]		
4–5	0.42	[0.122, 0.000]	[0.27, 0.00]	[0.05, 0.00]	[0.12, 0.00]
5–6	0.42	[0.135, 0.000]	[0.32, 0.00]		

to the Newtonian theory, the constant net force acting on the puck in each experiment should result in a constant acceleration. The force of 2 su acting in Fig. 14.14 should produce twice the acceleration as the force of 1 su in Fig. 14.13. The acceleration calculated in Tables 14.3 and 14.4 gives evidence that the Newtonian theory can be applied successfully to the motion of the dry ice puck.

Calculation of acceleration. As a matter of fact, we are unable to calculate the acceleration as defined in Eq. 13.9, because the multi-flash photographs do not enable us to determine the puck's instantaneous velocity. We have therefore computed an approximate "acceleration" in the last columns of Tables 14.3 and 14.4. You can see that the data in these two columns are in the ratio of two-to-one, just as the forces are in the ratio of two-to-one. In fact, you can see that all corresponding entries for the two pucks in Tables 14.3 and 14.4 are in this ratio. Because of this fact, you can imagine the outcome of a thought experiment in which the flashes occur at very short time intervals. The calculated average velocities will then be very close to the instantaneous velocities, the accelerations can be found from Eq. 13.9, and all numerical values will still be in the ratio two-to-one.

From here on we will merely write speed, velocity, acceleration, momentum*, and* force *to refer to the instantaneous quantities. The average quantities will be designated as such so you may identify them properly (*average speed, average velocity*, . . .).*

Other applications. With this positive outcome, we invite you to use the Newtonian theory to describe objects that are moving subject to the gravitational interaction with the earth. These investigations will lead to further evidence of the theory's usefulness. In Chapter 15, we will derive a theory of periodic motion and apply it to the planets in the solar system as well as to pendulums and other oscillators on earth.

14.4 Motion near the surface of the earth

All objects near the surface of the earth are subject to the force of gravity. They may also be subject to other forces, such as the elastic force exerted by the floor on a bouncing golf ball, the force of friction

TABLE 14·4 MOTION OF PUCK IN FIG. 14·14

Pictures defining interval	*Time interval (sec)*	*Displacement (m)*	*Average velocity (m/sec)*	*Change of average velocity (m/sec)*	*"Acceleration" (m/sec/sec)*
1–2	0.42	[0.083, 0.000]	[0.20, 0.00]	[0.10, 0.00]	[0.24, 0.00]
2–3	0.42	[0.128, 0.000]	[0.30, 0.00]		
4–5	0.42	[0.226, 0.000]	[0.54, 0.00]	[0.10, 0.00]	[0.24, 0.00]
5–6	0.42	[0.270, 0.000]	[0.64, 0.00]		

exerted by the road on a skidding car, or the force of friction exerted by the air on a flying airplane, but all objects near the earth are, at all times, certainly subject to the downward force of gravity.

The flat earth model. As we have described in Section 11.3, the gravitational intensity near the surface of the earth has the constant magnitude of 10 newtons per kilogram and a direction toward the center of the earth. Since the earth is a sphere that is very large compared to the macro-domain size of everyday phenomena, we will make a working model in which the earth is a flat horizontal surface and the direction of the gravitational intensity is vertical, at right angles to the horizontal plane (Fig. 14.15). The force of gravity on an object near the surface of the earth is therefore a constant force, determined by the object's gravitational mass, but independent of the object's position (Eq. 14.7). Many objects remain at rest even though they are subject to the force of gravity, because they are supported in such a way that the net force acting on them is zero.

Free fall. When all supports are removed, however, an object begins to fall and picks up speed, as shown in the multiflash photograph

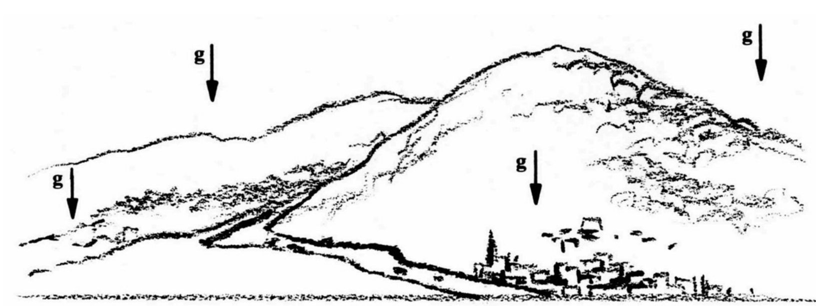
Equation 14.7 (same as Eq. 11.4)

force of gravity (newtons) = $\mathbf{F}_G$
gravitational mass (kg) = M_G
gravitational intensity (newtons/kg) = $\mathbf{g}$

$$\mathbf{F}_G = \mathbf{g}M_G$$

(at surface of earth $\mathbf{g}$ *= 10 newtons/kg, downward.*

Figure 14.15 The flat earth model. The gravitational intensity is vertical everywhere and has the same magnitude everywhere.



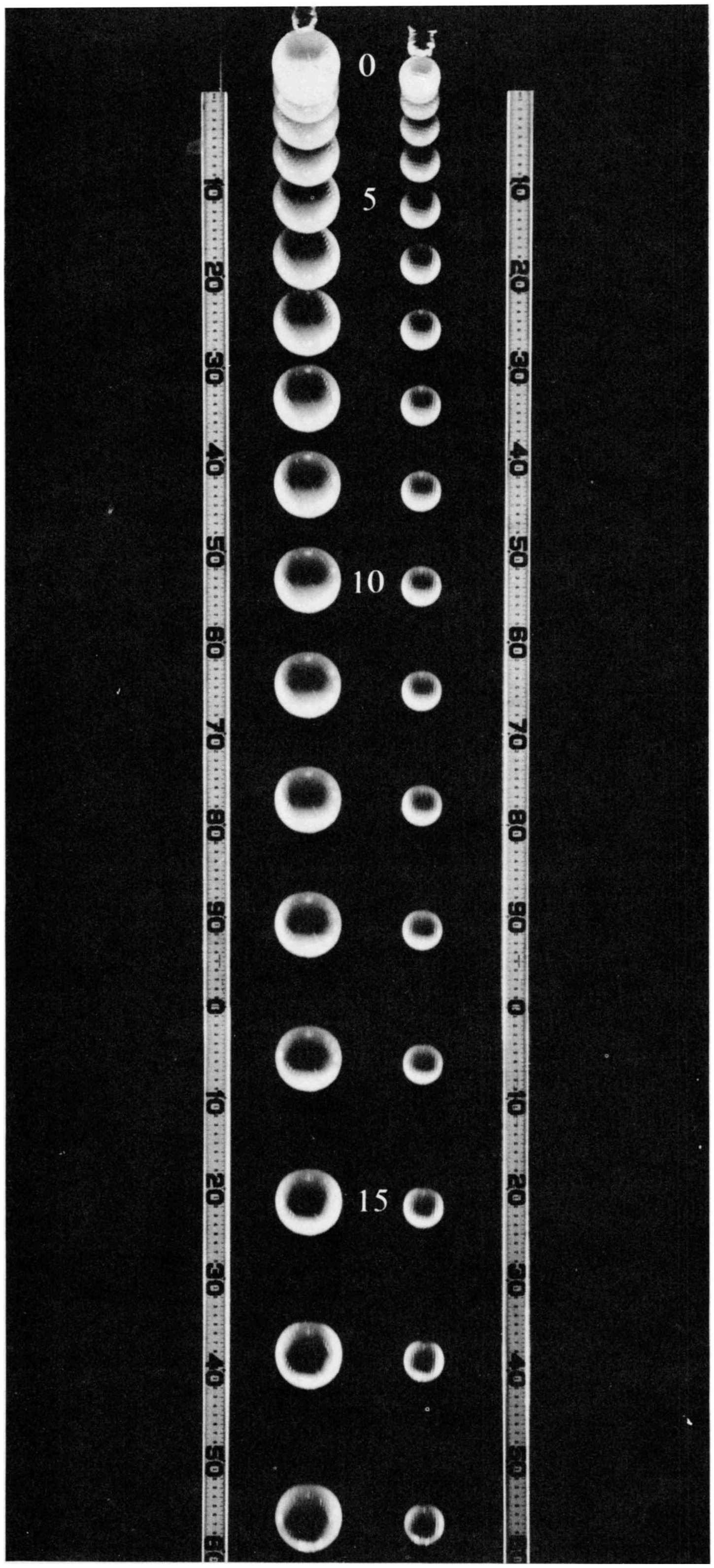
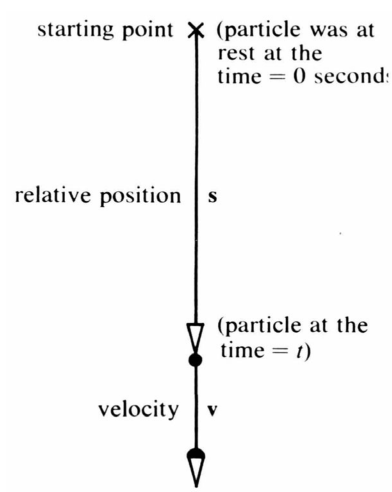


Figure 14.16 A baseball and a golf ball were released from rest at the same time. The time intervals between photographs were Δt= 0.033 second. Note the increasing displacement between successive images of each ball. In the text, we use measurements on this photo to show that the distance covered is proportional to the elapsed time to the second power.

We can also use the photo to answer a more straightforward question: does the heavier object fall faster? As you can verify in the photo, the displacements of the balls, and therefore their velocities, are equal *throughout the motion.* Conclusion*: the velocities of two objects with different masses dropped simultaneously from rest increase* at the same rate *in free fall. This may seem contrary to common sense, but common sense is wrong in this case! By measuring the displacements carefully on the meter sticks, can you verify Eq. 14.7 (acceleration =* **g** *= 10 m/sec/sec, downward)?*

Figure 14.17 (below) A particle is falling under the influence of gravity. It started at the coordinate origin with zero velocity. Its velocity and position increase as the first and second powers of the elapsed time, respectively. These math models are derived in Eqs. 14.9-14.12.



Equation 14.8

$$\mathbf{a} = \frac{\mathbf{F}_G}{M_I} = \mathbf{g}\frac{M_G}{M_I} = \mathbf{a}_{av}$$

Equation 14.9

actual velocity $= \mathbf{v}$

average velocity $= \mathbf{v}_{av}$

$$\mathbf{v}_{av} = \frac{1}{2}\mathbf{v}$$

Equation 14.10

elapsed time of falling $= t$

position relative to starting point $= s$

$$\mathbf{s} = \mathbf{v}_{av}t = \frac{1}{2}\mathbf{v}t$$

Equation 14.11

$$\mathbf{v} = \mathbf{a}_{av}t$$

Equation 14.12

$$\mathbf{s} = \frac{1}{2}\mathbf{v}t = \frac{1}{2}\mathbf{a}_{av}t^2 = \frac{1}{2}\mathbf{g}\frac{M_G}{M_I}t^2$$

Equation 14.13

$$\frac{M_G}{M_I} = 1 \text{ or } M_G = M_I$$

of a falling baseball and a falling golf ball in Fig. 14.16. Since each ball is subject to a constant net force, the Newtonian theory (Eq. 14.6, $\mathbf{F} = M_I\mathbf{a}$, combined with Eq. 14.7, $\mathbf{F}_G = \mathbf{g}M_G$) predicts that it should fall with *constant* acceleration equal to the gravitational intensity times the ratio of gravitational to inertial masses [$\mathbf{a} = \mathbf{g}(M_G/M_I)$]. The acceleration must be constant because $\mathbf{g}$, M_G, and M_I are all constant; in addition, a constant acceleration is always equal to the average acceleration (Eq. 14.8).

Velocity and position during free fall. Since the two balls start out at rest and suffer a constant acceleration, it is relatively easy to make a mathematical model for relating the distance they fall to the elapsed time. First, for a particle that starts from rest (zero initial velocity), the change of velocity is equal to the actual velocity and the displacement is equal to the position relative to the starting point (Fig. 14.17). Second, the average velocity of such a particle, whose speed builds up steadily from zero to its final value, is equal to one half of the actual velocity (Eq. 14.9). Third, the average velocity and the elapsed time can be multiplied, according to Eq. 13.5, to give the position of the particle relative to the starting point (Eq. 14.10). Fourth, the equation defining the average acceleration (Eq. 13.9, $\mathbf{a}_{av} = \Delta\mathbf{v}/\Delta t$) can then be rewritten to give the velocity of the falling particle, which is directly proportional to the time of falling (Eq. 14.11). Finally, this formula for the velocity and Eq. 14.8 are used to relate the distance of fall to the elapsed time (Eq. 14.12). The result is that the distance is proportional to the elapsed time raised to the second power.

In Table 14.5 this mathematical model is compared with the measurements made on the falling balls in Fig. 14.16. The prediction depends, as you might have expected, on the ratio of gravitational to inertial masses of each ball. You can see, however, that the prediction agrees well with the data if the ratio of masses is equal to one (Eq. 14.13).

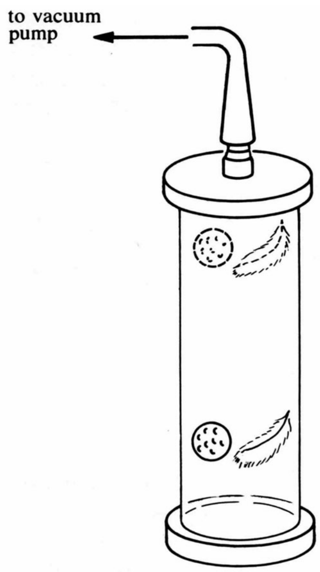
Comparison of gravitational and inertial masses. A close relation between the two masses of each ball can be inferred even without measurements. The photograph shows that the accelerations of the two balls are at least approximately equal because they appear to start out and remain side by side. According to Newton's theory, the net forces on them are in proportion to their inertial masses (prediction 5, page 375).

TABLE 14·5 DISTANCE OF FREE FALL FROM EXPERIMENT AND THEORY

Time (*sec*)	*Distance – experiment* (*Fig. 14·16*) (*m*)	*Distance – theory* (*Eq. 14·12*) (*m*)
0.000	0.00	0.00 (M_G/M_I)
0.167	0.15	0.14 (M_G/M_I)
0.333	0.56	0.55 (M_G/M_I)
0.500	1.25	1.25 (M_G/M_I)
0.567	1.59	1.60 (M_G/M_I)

"... just as the downward movement of a mass of gold or lead, or of any other body endowed with weight, is quicker in proportion to its size."

Aristotle
On the Heavens
4th century B.C.



Equation 14.14

free fall near earth's surface

a *=* ***g*** *=10 m/sec/sec, downward*

Equation 14.15

initial velocity $= v_i$
final velocity $= v_f$
average velocity $= v_{av}$
$v_{av} = \frac{1}{2}\,(v_i + v_f)$

But you know from Eq. 14.7 that the forces of gravity acting on them are proportional to their gravitational masses. It appears, therefore, that gravitational and inertial masses are proportional to one another! You can infer from the data in Table 14.5 that the two masses of each ball are numerically equal (Eq. 14.13).

As a matter of fact, it is found that all objects that fall freely in the earth's gravitational field experience the same acceleration of 10 meters per second per second. Even objects such as feathers, which ordinarily are affected greatly by the air, experience this acceleration in an air-free tube. This acceleration is numerically equal to the magnitude of the gravitational intensity (Eq. 11.3).

There are two physically equivalent but logically distinct views you may now take. One is that the two types of mass are *not* exactly equal and that the relative degree of equality must be established experimentally. As explained above, the data in Figure 14.16 and Table 14.13 show that $M_G = M_I$ to an accuracy of about 1% (1 part in 100). This issue is of great importance in physics, and experimenters (including Newton himself) have carried out a variety of ingenious and sensitive tests. The latest attack on this problem was by Adelberger in 1990, confirming that $M_G = M_I$ to an accuracy of 1 part in 10^{12}! A new experiment is now planned which would measure with great accuracy any differences in acceleration of two weights in "free fall" while orbiting the earth; the experimenters expect an accuracy of 1 part in 10^{18}!

The second view is to modify Newton's theory by simply adding the assumption that M_G and M_I are equal. This agrees with common sense; the modified theory then predicts that the acceleration of free fall is exactly equal to the gravitational intensity (Eq. 14.14, from Eqs. 14.8 and 14.13). Newton took this view, as you can verify in Table 14.1. He did not state a definition for inertial mass, but based his definitions of momentum and inertia on the gravitational mass concept he had defined earlier. Einstein also adopted this assumption (called the Principle of Equivalence) in his general theory of relativity. We will take the same view and assume for the rest of this text that $M_G = M_I$ (Eq. 14.13) holds exactly with no experimental uncertainties.

Acceleration of gravity. The gravitational intensity (**g**) is often called the *acceleration of gravity.* Its value in newtons per kilogram is equal to its value in meters per second per second. To achieve this equality, we based the definition of the joule (Sect. 9.2) and the definition of the newton (Sect. 11.2) on an object with mass of 0.10 kilogram. As you saw in Sect. 11.2, this choice for the newton determined the value of **g** (Eq. 11.3).

The acceleration of all objects falling freely near the surface of the earth is close to 10 meter/sec/sec in the vertical direction (Eq. 14.14). A very useful property of all uniformly accelerated objects is that their average velocity for any time interval is halfway between their initial and final velocities (Eq. 14.15). An application of this formula is worked out in Example 14.3.

You must remember that these results are based on the Newtonian one-particle theory and take into account only the force of gravity. If air resistance is important, if the object is partially supported by a sloping surface (like the skier on a hill), or if still other interactions must be considered, then a more complete theory within the Newtonian framework must be used.

EXAMPLE 14·3. A stone is dropped from the Leaning Tower of Pisa. (a) Find the speed at various times after release. (b) Find the distance fallen at various times after release.

Solution:

(a) Use Eqs. 14·11, 14·13, and 14·14 or $v = 10t$.

$t = 0$ sec: $v = 10 \times 0 = 0$ m/sec
$t = 1$ sec: $v = 10 \times 1 = 10$ m/sec
(and so on for $t = 2, 3, 4, \ldots$, see table).

(b) I. Use Eqs. 14·12, 14·13, and 14·14: $\mathbf{s} = \frac{1}{2}\mathbf{g}t^2$ or $\mathbf{s} = (5t^2$, downward), $|\mathbf{s}| = 5t^2 = s$.

$t = 0$ sec: $s = 5 \times 0 = 0$ m
$t = 1$ sec: $s = 5 \times 1^2 = 5$ m
$t = 2$ sec: $s = 5 \times 2^2 = 20$ m
(and so on for $t = 3, 4, \ldots$, see table).

II (alternate). Use Eq. 14·10 and solution to (a).

$$|\mathbf{s}| = \frac{1}{2}\,vt \text{ and } v = 10t$$

$t = 1$ sec, $v = 10$ m/sec:

$$|\mathbf{s}| = \tfrac{1}{2}vt = \tfrac{1}{2} \times 10 \text{ m/sec} \times 1 \text{ sec} = 5 \text{ m}$$

Table of speeds and distances of free fall

Time (sec)	Speed (m/sec)	Average speed (m/sec)	Distance (m)
0	0	0	0
1	10	5	5
2	20	10	20
3	30	15	45
4	40	20	80*

*The leaning tower is only 54 meters high. Hence the stone strikes the ground between 3 and 4 seconds after release and the mathematical models used in the calculation are no longer valid. To calculate the time before impact, use $|\mathbf{s}| = 5t^2$.

$|\mathbf{s}| = 54$ m, or $t = \sqrt{|\mathbf{s}|/5} = \sqrt{54/5} \approx \sqrt{10.8}$ sec ≈ 3.3 sec.

14.5 Newton's third law of motion

Even though the one-particle model for a moving body has been extremely valuable in leading to a successful description of the overall behavior of moving bodies, it clearly has limitations. In fact, these limitations are evident whenever you look more closely at a moving body and realize that it actually is a system of interacting parts, like the skier, who can maneuver his legs to make a turn; the stunt car, the wheels of which spin when it accelerates; and the interplanetary rocket, which leaves behind a trail of hot exhaust as it hurtles into space. For a more complete understanding of how these systems function, you must relate their motion to the operation and interaction of their various parts. The one-particle model, in which the entire system is represented by a single particle at the center of mass, is not adequate.

"The mutual actions of two bodies upon each other are always equal and oppositely directed."

Isaac Newton
*Principia, **I**, 1687*

Newton's third law supplements the first two in such a way that they can be applied to systems represented by complicated models consisting of many particles, ultimately even atoms or molecules if necessary. The most widely known statement of Newton's third law follows. *For every action, there is an equal and opposite reaction.*

Interpretations of the third law. The words "action" and "reaction," which are no longer part of the physics vocabulary, signify forces acting during a time interval. We may, therefore, interpret the third law as stating that the interaction of two bodies is described by two forces equal in magnitude and opposite in direction. This interpretation (see Section 11.2) applies directly to the two forces (Fig. 11.2). We have another option, however. Since a force acting during a time interval produces a change of momentum, two equal but opposite forces acting during the same time interval produce equal but opposite changes of momentum. We may, therefore, also interpret the third law as stating that the interaction of two bodies results in changes of their individual momenta that are of equal magnitude but oppositely directed.

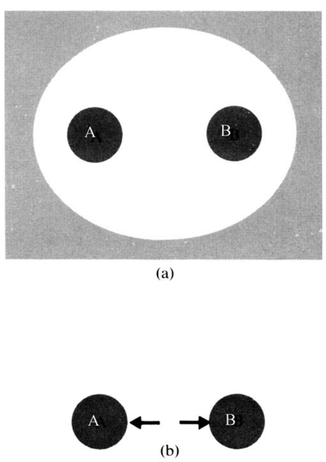


Figure 11.2
(a) Two interacting bodies
(b) The two forces of interaction. One force acts on each body.

Conservation of momentum. The second of these interpretations is especially significant, because it leads to the law of *conservation of momentum.* If the interaction of two bodies results in equal and opposite changes of momentum, the sum of the momenta of the two bodies is not changed by their interaction. The momentum gained by one body is lost by the other. There is transfer of momentum from one body to the other. If three or more bodies in a system interact at once, momentum may be transferred between the members of each interacting pair, with the result that the gains and losses of momentum balance out and the sum of all the momenta does not change. This is the law of conservation of momentum: The bodies in an isolated system can exchange momentum, but the total momentum of the system is conserved (constant).

The law of conservation of momentum, which we have here derived from Newton's theory, has turned out to be much more generally valid than Newton's theory itself. For instance, the law applies to radiation and to phenomena in the micro domain, for both of which

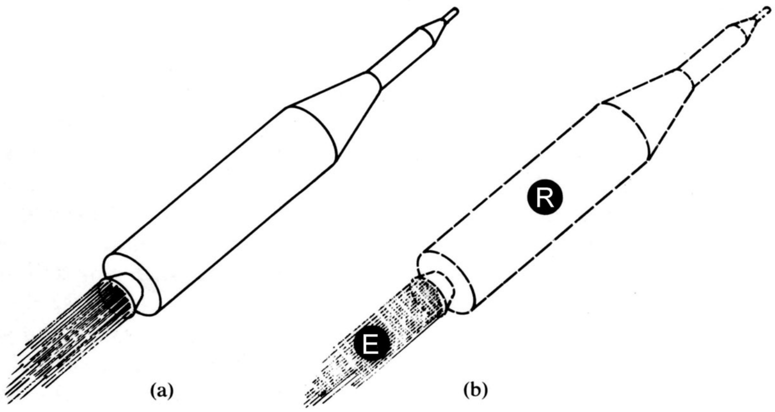
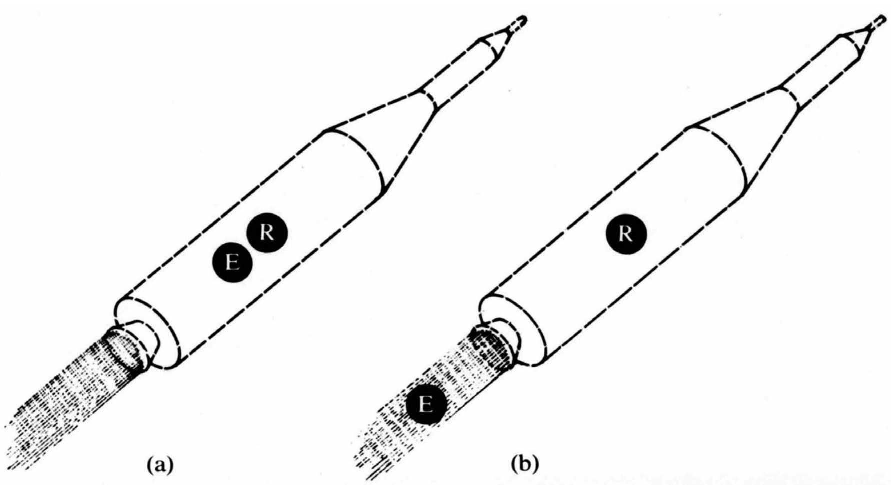


Figure 14.18 Two-particle model for a rocket,
(a) The rocket while engine is in operation,
(b) The rocket, with particle R representing the rocket-plus-fuel subsystem, and particle E representing the exhaust subsystem.

Newtonian mechanics is inadequate (see Chapters 7 and 8). This law, along with the law of conservation of energy (Section 4.1), is one of the cornerstones on which all current physical theories are built.

Rocket propulsion. As an example of how to apply Newton's theory, we present a two-particle theory of rocket propulsion (Fig. 14.18). A rocket is a system that ejects material through a nozzle at one end. Consider a particular 1-second interval while the rocket engine is burning fuel and ejecting the combustion products through nozzles at the rear of the rocket. The rocket-plus-remaining-fuel after the interval is one "particle" in the model, the material ejected during the 1-second interval is the other "particle." At the beginning of the 1-second interval, the two "particles" move at the same velocity (Fig. 14.19a). Each has a momentum determined by this velocity and its own mass. At the end of the 1-second interval, the ejected material is in the exhaust trail and no longer has the velocity it had before (Fig. 14.19b). It therefore has a

Figure 14.19 (to right). Conservation of momentum in rocket propulsion.
(a) Rocket (R) *and the fuel-oxygen subsystem about to burn* (E) *are traveling at the same speed* (v_i) *relative to the earth reference frame.*
(b) Fuel-oxygen subsystem, now exhaust (E)*, is ejected, at speed* $-v_{exhaust}$*, (relative to the rocket) so it loses momentum relative to the earth. This momentum is transferred to the rocket subsystem* (R)*, which then moves faster* (v_f) *relative to the earth reference frame.*



changed momentum. According to Newton's third law, the momentum is not lost from the two-particle system, but it is transferred to the rocket and the remaining fuel. The rocket "particle" therefore moves on with an increased velocity. The process is now repeated, with a new two-particle system being made out of the rocket and its fuel to describe the ejection of material during the next 1-second time interval. And so on.

The quantity of momentum transferred in 1 second is a measure of the strength of the interaction between the two "particles." It is determined by the rate of fuel consumption, temperature in the combustion chamber, and other construction details of the rocket engine. The momentum transferred in 1 second is called the *thrust* of the engine (Example 14.4) and is usually measured in newtons or pounds (1 pound of thrust is approximately 4.5 newtons). A rocket engine is sometimes called a *reaction motor* because its functioning depends on the action-reaction principle of Newton's third law.

EXAMPLE 14·4. A rocket motor burns 1 ton of fuel–oxygen mixture per second. The exhaust speed is 700 meters per second relative to the rocket. Find the thrust of the rocket. (1 ton = 1000 kilograms.)

Solution: **Calculate the thrust while the rocket is strapped down for a test firing.**

Data: for "exhaust particle,"
$M = 1000$ kg; $|\Delta \mathbf{v}| = 700$ m/sec; $\Delta t = 1$ sec

$$|\mathbf{F}| = \frac{M|\Delta \mathbf{v}|}{\Delta t} = \frac{1000 \text{ kg} \times 700 \text{ m/sec}}{1 \text{ sec}} = 700{,}000 \text{ newtons}$$

thrust = 700,000 newtons or 160,000 lbs

Question: **Does the thrust change while the rocket is moving? Repeat the calculation for the rocket moving at 1000 meters per second. Then the exhaust moves at 300 meters per second.**

14.6 Kinetic energy

If you have a driver's license, you very likely were taught that the distance required to stop a car increases fourfold when its speed doubles. Have you ever wondered why? When a bicycle rider approaches a hill, she usually pedals as fast as she can so that she will get to the top of the hill more easily. Just how far up will her speed carry her? In both these examples, there is a transfer of energy from kinetic energy to another type: thermal energy of the brakes, or gravitational field energy of the bicycle, rider, and earth system.

As we have said in Chapter 4, kinetic energy is the energy stored in moving objects. Thus, the kinetic energy of the car determines how far it will advance as the brakes bring it to a stop. The bicyclist maximizes her kinetic energy as she approaches the hill.

When a force acts on a particle, its velocity or momentum changes, and usually its energy changes also. In this section we will derive a mathematical model for the relation of kinetic energy to speed. We will show how this relation can be used in conjunction with the law of conservation of energy to predict the motion of objects under many circumstances, such as the car coming to a stop and the bicycle moving uphill.

Equation 14·16

kinetic energy	KE
work	W
net force	$\mathbf{F}$
displacement component along the force direction	Δs_F

$$\mathrm{KE} = W = |\mathbf{F}|\Delta s_F$$

Derivation. Instead of constructing the model in the light of experimental results, we will derive it from Newton's theory. Imagine a particle at rest (zero speed, zero kinetic energy) that is acted upon by a constant net force until it is moving with the velocity v. The kinetic energy of the particle is, according to the law of conservation of energy, equal to the work done by the net force (Eq. 14.16). To find the work, we have to calculate the distance through which the particle moved while it was being accelerated by the action of the force.

Equation 14·17

position relative to starting point	$\mathbf{s}$
velocity	$\mathbf{v}$
elapsed time	t

$$\mathbf{s} = \tfrac{1}{2}\mathbf{v}t$$

This problem is very similar to the problem of free fall solved in Section 14.4. There, too, a constant force speeded up a particle that was initially at rest. The principal differences between that and the present tasks are that now the force can be any force (not only the force of gravity), and the motion can occur in any direction (not only vertically). Still, the motion and the force are in the same direction, because the particle starts from rest (Fig. 14.20).

Equation 14·18

mass M

$$\mathbf{F} = M\frac{\mathbf{v}}{t}$$

The relative position of the particle is equal to one half of the velocity times the time (Eq. 14.17 from Eq. 14.10, $\mathbf{s} = \mathbf{v}_{av}t = (1/2)\mathbf{v}t$). The net force also can be related to the actual velocity (equal to the change of velocity) and to the elapsed time (Eq. 14.18 from Eq. 14.5, $\mathbf{F}_{av} = M_I\mathbf{a}_{av}$). Since the force, the velocity, and the relative position are all in the same direction, the component of the displacement along the force direction is equal to the magnitude of the relative position (Eq. 14.19). When the formulas are combined to calculate the work and therefore the kinetic energy, we obtain a mathematical model (Eq. 14.20), which has been simplified using the fact that the magnitude of the velocity is

Equation 14·19

speed v

$$\Delta s_F = |\mathbf{s}| = \tfrac{1}{2}vt$$

Equation 14·20

$$\mathrm{KE} = |\mathbf{F}|\Delta s_F = M\frac{|\mathbf{v}|}{t} \times \frac{1}{2}vt = \frac{1}{2}Mv^2$$

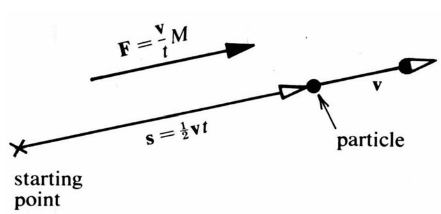


Figure 14.20 The kinetic energy of a particle is equal to the work done by a constant force that accelerates the particle from zero velocity to its actual velocity. The force required and the position relative to the starting point reached by the particle are related to the velocity by Eqs. 14.17 and 14.18.

equal to the speed (Eq. 13.8). Thus, the kinetic energy equals one half times the mass times the speed to the second power (Example 14.5).

Applications. This mathematical model for kinetic energy may be applied to moving bodies, such as automobiles, footballs, spaceships, hailstones, pendulums, and the earth in its orbit around the sun. However, this model does not apply directly to sound and light, since both of these phenomena are described better by a wave model than by

---

EXAMPLE 14·5. Find the kinetic energy of a car with mass of 1500 kilograms moving at a speed of 27 meters per second (60 miles per hour).

Solution:

$$\mathrm{KE} = \frac{1}{2} Mv^2 \approx \frac{1}{2} \times 1500 \text{ kg} \times (27 \text{ m/sec})^2$$

$$\approx 750 \times 730 \approx 550{,}000 \text{ joules}$$

EXAMPLE 14·6. Calculate the braking distance of a car. The force of friction on good roads is about one half the force of gravity. What is the braking distance at various speeds?

Data:

braking distance s, car speed v.

force of friction $|\mathbf{F}| \approx \frac{1}{2}|\mathbf{g}|M$, direction opposite to motion

work done by car $W = |\mathbf{F}|\,\Delta s_F \approx \frac{1}{2}|\mathbf{g}|\,Ms$

kinetic energy $\mathrm{KE} = \frac{1}{2}\,\mathrm{M}v^2$

Solution:

Kinetic energy is completely converted to thermal energy by friction:

$$\mathrm{KE} = W \quad \text{or} \quad \tfrac{1}{2} Mv^2 \approx \tfrac{1}{2}|\mathbf{g}|Ms$$

$$s = \frac{v^2}{|\mathbf{g}|} = \frac{1}{10}\,v^2$$

Table of results

Speed (m/sec)	Speed (mph)	Braking distance (m)	Braking distance (ft.)
0	0	0	0
10	23	10	33
20	45	40	130
30	68	90	295
40	90	160	525

a particle model. The mathematical model for kinetic energy (Eq. 14.20, KE = $\frac{1}{2}Mv^2$) *can* be used in conjunction with the MIP model for matter by applying the equation to each separate particle in the model system.

With the mathematical model for kinetic energy, you can apply the law of conservation of energy to processes involving moving particles, just as you used the mathematical model for thermal energy to predict temperature changes. As a first example, we return to the braking distance of a speeding automobile. When the brakes are applied, energy is transferred from the kinetic energy of the car to thermal energy of the brake linings by way of the frictional interaction of the brakes. Doubling the speed of the car increases its kinetic energy fourfold (KE depends on v^2, Eq. 14.20); therefore, the car must do four times the work, which requires traveling four times as far, so as to permit the transfer of all this energy to the brakes (Example 14.6 above). We would thus expect to find four times as much thermal energy in the brakes after the stop.

Summary

Isaac Newton formulated a theory of moving bodies that applies to all macro-domain phenomena except radiation and is still one of the foundations of physics as well as an outstanding model of a successful physical theory. The central quantitative concepts in Newton's theory are force, mass, velocity, momentum, and acceleration. To deal with instantaneous velocity and acceleration, Newton developed a new branch of mathematics now called the calculus.

Newton's approach was one of reducing complex phenomena to simple ones. Just as a house may be built of bricks, so it is possible to build theories of motion for complex systems (the entire solar system, a bicycle with many moving parts, and even fluids), out of Newton's laws of motion for one particle.

Mass, momentum, velocity, and acceleration are used to describe the motion of each particle. The influence on each particle of its interaction with all other particles is summarized in the net force concept. This influence is described in the first and second laws of motion.

Newton's first law: Every particle persists in its state of rest or of uniform, unaccelerated motion unless it is compelled to change that state by the application of an external net force.

Newton's second law: The change of momentum of a particle subject to a net force during a time interval is equal to the average net force times the duration of the time interval.

Since motion is defined only in relation to a reference frame, the applications of Newton's first and second laws require the prior selection of a suitable reference frame, which is called an inertial frame. Newton's first law may be used to select such a reference frame; those frames in which the first law is contradicted by observations are not inertial frames and must not be used. Once an inertial frame has been selected, the second law is used to make quantitative predictions of the motion of particles relative to this frame.

Equation 14.5

net force = $\boldsymbol{F}$
mass = M
acceleration = $\boldsymbol{a}$

$$\boldsymbol{F} = M\boldsymbol{a}$$

Equation 14.13

inertial mass = M_I
gravitational mass = M_G

$$M_I = M_G$$

Equation 14.20

kinetic energy = KE
mass = M
speed = v

$$KE = \tfrac{1}{2}mv^2$$

The most convenient mathematical statement of Newton's second law is given in Eq. 14.5. This formula may be applied with great effectiveness to the motion of particles near the surface of the earth, where they are subject to the force of gravity described in Section 11.3. One of the outcomes of these applications is support for the assumption that the gravitational mass of a particle equals its inertial mass (Eq. 14.13). Newton's theory of particle motion can also be applied to predict the overall motion of complex systems by means of a one-particle center-of-mass model.

To build up a more complete theory of complex systems, Newton introduced his third law of motion.

Newton's third law: The interaction of two bodies is described by two equal and opposite forces; or, the interaction of two bodies results in a transfer of momentum between them.

This law is used to relate the motion of one particle, controlled by the forces acting on it, to the forces it in turn exerts on the other particles. Newton's third law also leads to the law of conservation of momentum for complex systems.

A mathematical model for the kinetic energy of a particle moving with a certain speed can be derived from Newton's laws of motion. The kinetic energy is proportional to the speed to the second power (Eq. 14.20).

Additional examples

EXAMPLE 14.7. An arrow with a mass of 0.1 kilogram is shot 50 meters straight up into the air and then falls to the ground.

(a) How much elastic energy was stored in the bow?

(b) What was the arrow's initial upward speed?

(c) With what speed will the arrow strike the ground?

(d) How long will the arrow remain in the air?

Solution: We use the following model: the arrow is one particle; its interaction with the air is negligible. The only force acting on the flying arrow is the force of gravity; therefore, the net force is equal to the force of gravity.

(a) The energy stored in the bow must equal the gravitational field energy of the arrow-earth system when the arrow is 50 meters high. Use ground level as the reference level for gravitational field energy.

gravitational field energy:

$E_G = |\boldsymbol{g}|Mh$ = 10 newtons/kg x 0.1 kg x 50 m = 50 joules

elastic energy of bow = 50 joules

(b) The arrow's initial kinetic energy was equal to the elastic energy stored in the bow. This determines the speed of the arrow.

$KE = \tfrac{1}{2}Mv^2$ or $50 = \tfrac{1}{2} \times 0.1 \times v^2$, or $1000 = v^2$, or 32 m/sec = v.

Initial speed of arrow = 32 m/sec

(c) When the arrow returns to the ground, all the gravitational field energy has been transferred back to kinetic energy. Hence the final speed is equal to the initial speed.

final speed of arrow = 32 m/sec

(Note: Similar reasoning can be applied at any instant of the downward motion and leads to the conclusion that the downward motion is just the reverse of the upward motion.)

(d) The arrow's total time in the air is the time to rise plus the time to fall. The time to rise is determined by the distance and average speed.

distance:

s = 50 m, $v_{av} = ½v = ½ \times 32 = 16$ m/sec

time rising:

$$t = \frac{s}{v_{av}} = \frac{50}{16} = 3.1 \text{ sec}$$

The distance and the average speed during downward motion are the same as those during upward motion. Hence, the time the arrow takes to fall equals the time it takes to rise.

time falling:

t= 3.1 sec

total time:

3.1 sec + 3.1 sec = 6.2 sec

EXAMPLE 14.8. Two children on roller skates face each other and push one another apart. Child A has a mass of 30 kilograms, child B 60 kilograms. Child A rolls away with a speed of 2 meters per second relative to the ground.

(a) What is the speed of child B relative to the ground?
(b) What is the speed of child A relative to child B?
(c) What is the kinetic energy of the two-child system?

Solution: Use conservation of momentum of the two-child system, under the assumption that no net external force acts on this system as a whole. The partial force exerted by the ground on each child just cancels the force of gravity on each child.

(a) total momentum before push	$\mathcal{M}$ = [0, 0] kg-m/sec
total momentum after push	$\mathcal{M}$ = [0, 0] kg-m/sec
velocity of child A after push	$\mathbf{v}_A$ = [2, 0] m/sec
child A, momentum after push	$\mathcal{M}_A = M_A\mathbf{v}_A$ = 30 kg x [2, 0] m/sec
	= [60, 0] kg-m/sec
child B, momentum after push	$\mathcal{M}_B = M_B\mathbf{v}_B = 60\ \mathbf{v}_B$

$$\mathcal{M}_A + \mathcal{M}_B = \mathcal{M}$$
$$[60, 0] + 60\,\mathbf{v}_B = 0$$
$$\mathbf{v}_B = [-1, 0] \text{ m/sec}$$

The speed of child B is 1 meter per second, directed opposite to that of child A.

(b) The children are moving apart from their starting point at 2 meters per second and 1 meter per second. Hence the speed of A relative to B is 3 meters per second, away from B.

(c) Kinetic energy of child A:

$$KE = \tfrac{1}{2}Mv^2 = \tfrac{1}{2} \text{ x } 30 \text{ kg x } (2 \text{ m/sec})^2 = 60 \text{ joules}$$

Kinetic energy of child B:

$$KE = \tfrac{1}{2}Mv^2 = \tfrac{1}{2} \text{ x } 60 \text{ kg x } (1 \text{ m/sec})^2 = 30 \text{ joules}$$

total kinetic energy: 90 joules

List of new terms

center of mass
inertial reference frame
acceleration of gravity
free fall
momentum transfer

List of symbols

F	force
|F |	force magnitude
$\mathbf{F}_{av}$	average force
$\mathcal{M}$	momentum
$\Delta\mathcal{M}$	change of momentum
Δt	time interval
$\mathbf{a}_{av}$	average acceleration
$\mathbf{a}$	acceleration
M_I	inertial mass
$\mathbf{v}$	velocity
$\mathbf{v}_{av}$	average velocity
v	speed
$\Delta\mathbf{v}$	change of velocity
$\mathbf{F}_G$	force of gravity
M_G	gravitational mass
$\mathbf{g}$	gravitational intensity
$\mathbf{s}$	position
t	elapsed time
KE	kinetic energy
W	work
Δs_F	displacement component in force direction
$\|\mathbf{s}\|$	position magnitude

Problems

1. Identify the partial forces that act on: (a) an arrow being shot (immediately after the archer releases the string); (b) a sailboat in a race; (c) a stunt car rounding a curve on two wheels; (d) the earth; (e) a drop of water erupting from Old Faithful; (f) a raindrop.

2. Are any of the objects in Problem 1 in mechanical equilibrium? Explain your answer.

3. Compare Newton's and this text's formulations of Newton's theory (Table 14.1). Comment on the nature (operational or formal) of the definitions, undefined quantities, and hidden assumptions. (You may consult any reference you wish.)

4. Describe the causes of motion of two or three moving objects according to Newton's theory and compare them with your common-sense view.

5. Identify three or more bodies for which the center-of-mass model should be adequate and three or more for which it might be very misleading. Explain your reasons.

6. Give two examples from everyday life in which the source of kinetic energy for a moving body is *not* the system that exerts the force setting the body in motion. Explain your answer.

7. Describe two or more non-inertial reference frames. Explain why you believe they are non-inertial.

8. Describe two or more inertial reference frames. Explain why you believe they are inertial.

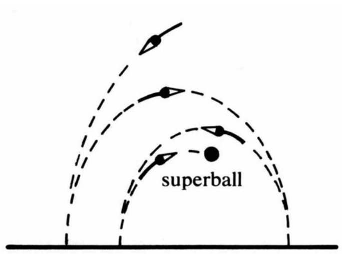
9. A long time exposure of the night sky, made by a camera fixed on the ground, shows star "trails" in the shape of circular arcs centered on the North Star. Use this evidence to discuss whether a reference frame attached to the earth is an inertial frame. If all the star "trails" are quarter circles, for how long was the film exposed?

10. Give two examples from everyday experience that are easier to explain in the Aristotelian theory than the Newtonian theory. Include both types of explanations for each example.

11. (a) Restate Newton's second law so it applies directly to acceleration instead of to momentum.
(b) Do you expect that this form of the law might be *more* or *less* general in its applicability? Explain.

12. Give two or more examples from everyday life in which the velocity of a moving body is *not* in the direction of the net force acting on it.

13. A 1500-kilogram car is advertised to accelerate from a standing start to 60 miles/hr in 10 seconds.
(a) What is the average horizontal force (in newtons) exerted by the road on the car? Explain why you do not need to consider the vertical force. Hint: convert mi/hr to m/sec using 1 mi = 1600 m and 1 hr = 3600 sec; thus 1 mi/hr = 1600 m/3600 sec = 0.44 m/sec.
(b) What is the average horizontal force exerted by the car on the road? (Use Newton's third law.)
(c) How far does the car travel in the 10 seconds?
(d) Calculate the work done by the force in you found in (a) acting through the distance you found in (c). Compare the work to the kinetic energy of the car (Example 14.5).



14. A ball (mass 0.1 kilogram) is thrown upward from ground level so it rises to a height of 30 meters (100 feet) and falls back. Find each of the following:
(a) the time it was in the air;
(b) the initial upward speed;
(c) the kinetic energy it had just after it was thrown;
(d) the gravitational field energy of the ball-earth system when the ball is at the top of its trajectory.

15. Obtain a "super ball" and carry out some experiments in which it bounces strangely. (See the diagram to left.) Explain your observations qualitatively in terms of Newton's theory.

16. Calculate the average acceleration of the golf ball in Fig. 14.6 from measurements made on the photograph, as follows.
(a) Select three time intervals during which the ball was *not* bouncing on the table. Comment on your result.
(b) Select two time intervals during which the ball *was* bouncing on the table. Comment on your result and compare with (a).

17. Suppose you did *not* know the time interval between flashes in Fig. 14.6. Devise a procedure and apply it to determine the time interval. You may assume that all time intervals are equal and that the gridlines in the figure are (as stated in the caption) 0.11 meter apart.

18. Figure 14.9 was used to verify that the dry ice puck was in mechanical equilibrium according to Newton's first law (constant speed, straight-line motion). What definitions of "speed" and "straight line" were used implicitly?

19. What is the minimum number of successive flash photographs that are needed for a determination of acceleration? Explain.

20. Explain in what way the acceleration listed in Tables 14.3 and 14.4 is an "approximate" rather than an average value. Describe conditions under which the approximation should be a good one and those when you expect it to be poor.

21. Describe one or more familiar phenomena for which the flat earth model (Section 14.4) is *not* adequate.

22. Compare the two possible points of view toward Eq. 14.13 and explain your preference.

23. Imagine the thought experiment in which a marble is dropped in a freely falling chamber.
(a) Describe the observations made in such an experiment, using the chamber as reference frame.
(b) Is the chamber an inertial reference frame while it is falling freely? Explain your answer.

24. You often read in the newspaper about the "weightlessness" of astronauts in an orbiting spacecraft.
    (a) Discuss this description from the Newtonian point of view.
    (b) Comment on the suitability of the spacecraft as an inertial reference frame before launching, during takeoff, during orbital motion, and during reentry.

25. Interview four or more children (ages 8-12) concerning their explanation of motion. (Suggestion: Select some moving objects and ask, "Why does . . . move?" or, "What makes . . . move?" or another form of the question that you find effective.) Interpret the children's answers in terms of Newtonian and Aristotelian theories.

Mother: "Johnny! Don't pull the cat's tail like that." Johnny: "I'm only holding it, Mom. The cat is pulling."

26. Interpret the conversation in the margin to the left in the context of the Newtonian theory. Identify the partial forces that act, the net force on the cat's tail, and the aptness of Johnny's remark.

27. Give two or more examples from everyday life of the conservation of momentum. If possible, explain any apparent loss of momentum.

28. Apply Newton's third law to identify the "reaction" forces in the following examples of forces.
    (a) A rocket exerts a force expelling the exhaust gases.
    (b) A car's tires exert a frictional force against the road.
    (c) The earth exerts a gravitational force on you.
    (d) A stretched rubber band exerts an elastic force on your fingers stretching it.
    (e) The sun exerts a gravitational force on the earth.

29. Because momentum is conserved, it makes sense to speak of a "momentum source" that exerts a force and thereby transfers momentum to a moving object. Identify the momentum source (which exerts force) and the energy source for the moving object in each of the following examples. In each case, the momentum source *may or may not* be the same as the energy source.
    (a) A bullet is fired by a gun.
    (b) A baseball is hit by a bat.
    (c) A bowling pin is hit by a bowling ball.
    (d) An automobile accelerates.
    (e) A broad jumper leaps.

30. Two children on ice skates face each other and push with their hands against one another. Assuming that the children have different masses, describe what will happen.

31. A 75-kilogram man stands in the stern (back) of a 50-kilogram canoe while the canoe is stationary in the water. He walks toward the bow (front) with a speed of 1 meter per second relative to the canoe.
    (a) Describe qualitatively what will happen.
    (b) Calculate the speed of the man relative to a shore-based reference frame. (Neglect friction between the boat and the water.)

32. A small rocket motor ejects 5 kilograms of exhaust gases per second, with an exhaust speed of 1700 m/sec (relative to the rocket). The rocket is moving at 600 meters per second relative to the earth and its mass is 200 kilograms at that time.
(a) Calculate the rocket motor's thrust.
(b) Calculate the speed change of the rocket in one second.
(c) Calculate the kinetic energy change of the rocket in one second.
(d) Calculate the kinetic energy change of the fuel-oxygen mixture burned in one second and exhausted.
(e) Four and a half kilograms of exhaust are produced by the combustion of 1 kilogram kerosene fuel. Calculate the chemical energy consumed during 1 second of operation (see Table 10.6). Discuss what happens to this energy from the viewpoint of energy conservation.

33. (a) How much kinetic energy does an 80-kilogram parachutist acquire when he jumps from a plane and falls 500 meters before the chute opens? (Assume that there is no friction with the air.)
(b) What happens to the kinetic energy when the chute opens and the man-chute system slows down suddenly?
(c) Enumerate the partial force (or forces) that bring about the change of velocity after the chute opens.

34. Make a mathematical model that relates the speed of a falling object to the distance it has fallen from rest. *(Hint:* The kinetic energy of a falling object is transferred from the gravitational field energy stored in the object-earth system.)

35. Romeo would like to throw flowers to Juliet, whose balcony is 6 meters above street level. What upward speed does Romeo have to impart to the flowers? *(Hint:* Kinetic energy is transferred to gravitational field energy.)

36. A pole vaulter wishes to clear a bar 5 meters high. Approximately what speed must he attain during his running start?

37. Write a critique of the Newtonian theory from *your* point of view. Identify those features (if any) that you find useful, intellectually satisfying, difficult, confusing, or contrary to your experience.

38. Give a qualitative analysis of the following situations in terms of Newtonian theory. Identify the partial forces acting on the italicized objects, point out a nonzero net force acting on any object, and mention any relationships between forces that are consequences of Newton's laws.
(a) A stalemated tug-of-war between *two children* using a *rope.*
(b) A *car* accelerating on a level *road.*
(c) A *pole vaulter* vaulting over a *bar* with a fiberglass *pole.* (Select two stages during the motion for analysis.)

Bibliography

D. Cassidy, G. Holton, J. Rutherford, *Understanding Physics*, Springer Verlag, 2002.

S. Chandrasekhar, *Newton's Principia for the Common Reader,* Oxford University Press, 1995 (hardback), 2003 (paperback). A version in relatively down-to-earth language with commentary by an outstanding theoretical physicist.

I. B. Cohen, *The Birth of a New Physics,* Doubleday, Garden City, New York, 1960.

L. N. Cooper, *An Introduction to the Meaning and Structure of Physics,* Harper and Row, New York, 1968. See the chapter "The Newtonian World."

S. Drake, *Galileo at Work: His Scientific Biography*, University of Chicago Press, Chicago, Illinois, 1978.

A. Eddington, *The Nature of the Physical World,* University of Michigan Press, Ann Arbor, Michigan, 1958.

A. Eddington, *Space, Time, and Gravitation,* Harper and Row, New York, 1959.

G. Galilei, *Dialogues Concerning Two New Sciences,* Northwestern University Press**,** Evanston, Illinois, 1950. See the sections Galileo called "First Day" and "Third Day."

M. B. Hesse, *Forces and Fields,* Littlefield, Totowa, New Jersey, 1961.

M. Jammer, *Concepts of Force: A Study in the Foundations of Dynamics,* Harper and Row, New York, 1957.

F. Manuel, *A Portrait of Isaac Newton*, DaCapo Press (reissue), 1990.

I. Newton, *The Principia, Mathematical Principles of Natural Philosophy,* translated by I. B. Cohen and A Whitman, University of California Press, 1999. This is the most recent translation and appears to be in the process of becoming the standard. There are many older translations; by far the best known, and the standard for many years, was the translation by Andrew Motte (done in 1729) as revised by Cajori (published as two volumes in 1962 by the University of California Press). [Editor's Note: all quotations from the *Principia* in this textbook are from the Motte/Cajori translation.]

R. E. Peierls, *The Laws of Nature*, Scribner's, New York, 1956.

R. S. Westfall, *Never at Rest*, *A Biography of Isaac Newton,* Cambridge University Press, 1983.