

Chapter 16:

Heat and Motion

The gasoline engine in automobiles, motorcycles, lawnmowers, and other similar appliances is one of the major conveniences of modern technology, but it is also, because of the air pollution it causes, one of the major threats to civilization. The gasoline engine, diesel engine, and gas turbine jet engine are modern counterparts of the coal-burning steam engine, whose development by James Watt (1736-1819) led to the industrial revolution in the nineteenth century. All of them transform heat released by burning fuel into useful work for manufacturing, construction, and transportation; they are special types of what are called *heat engines*.

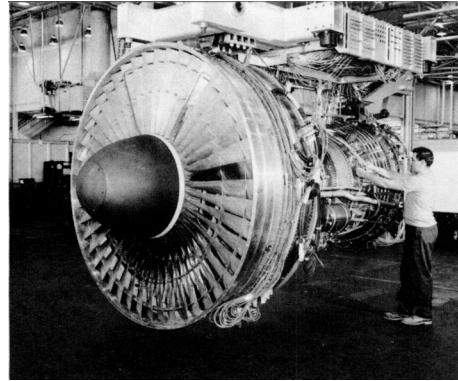
Before the industrial revolution, the major sources of energy transfer in the form of work were human beings and animals, which served as coupling elements to convert chemical energy of food-oxygen systems into kinetic energy (transportation), gravitational field energy (construction of buildings), and so on. The energy of fuel-oxygen systems was released as heat for cooking and warmth, but was not otherwise applied. Since the industrial revolution, the heat engines we enumerated above are used as coupling elements to convert the chemical energy of fuel-oxygen systems to useful work. In industrial nations, in fact, human and animal labor are fast becoming surplus commodities.

The conversion of heat to work in a heat engine is the reverse of the conversion of work to heat by friction. A very simple device to accomplish this was Hero's steam engine, invented in ancient times. Hero's engine operates like a whirling lawn sprinkler in that jets of steam emerge from a boiler through bent tubes and force the boiler to spin in the opposite direction. In modern engines, the steam or another hot gas at high pressure pushes against a piston in a cylinder or against the fan-like blades of a turbine (Fig. 16.1). In all heat engines, a gas is the essential coupling element that accomplishes the conversion of heat to work.

The engineering development of heat engines in the last 200 years illustrates the continued interplay of science and technology. The invention of the steam engine and the need for its further improvement focused attention on the nature of heat and properties of gases. As a result, the caloric theory of heat was replaced by the kinetic theory (Section 10.5), and the many-interacting-particles (MIP) model for matter became firmly established. This scientific progress, in turn, led to the invention of gasoline and turbine engines, which have now replaced the steam engine wherever high power and low mass are important.

Since gases play a central role in the operation of heat engines, the physics of the industrial revolution is linked closely to the physics of gases. We therefore begin this chapter with a description of the properties of gases, such as air, oxygen, nitrogen, and water vapor (gaseous water). The concept of gas pressure is related to the force exerted by a gas on its container. After an examination of the energy transfer to and from a gas sample, we explain the operation of a working model for a heat engine. Finally, we trace the introduction of the kinetic theory of gases and outline its successes.

Figure 16.1 This turbo fan aircraft engine is capable of producing a thrust of more than 200,000 newtons and has a mass of 4 tons.



16.1 Properties of gases

Air is such a tenuous material that most people take it for granted and are hardly aware of its existence. Nevertheless, the atmosphere of air surrounding the planet earth fulfills many functions that are essential for the existence of life: it absorbs ultraviolet radiation from the sun; it absorbs cosmic radiation; it stores thermal energy from the sun; and it contains oxygen, carbon dioxide, and gaseous water, which participate in metabolic processes of animals and plants. In this section, however, we would like you to ignore these functions temporarily and to think of air as an example of the kind of material called gases. Air possesses the physical properties that most gases exhibit: fluidity, low density, ability to fill its container completely and to exert pressure on the container walls, compressibility, and a large thermal expansion compared to solid and liquid materials.

We briefly discussed gases in Section 4.5, where we used the MIP model for matter to explain some of the properties that distinguish the gas phase from the solid and liquid phases. First, gases have much lower density than solids and liquids. Whereas 1 liter of water has a mass of 1 kilogram, 1 liter of atmospheric air has a mass of only about 0.0012 kilogram (Fig. 16.2a). Conversely, 1 kilogram of water occupies a space of 1 liter, but 1 kilogram of atmospheric air fills more than 800 liters (Fig. 16.2b). Other gases are generally similar to air in that their density is very low compared to that of solids or liquids.

A second distinguishing characteristic of the gas phase is its compressibility. Even with the application of only moderate forces, the volume filled by a sample of air can be greatly reduced or increased compared to the volume it filled as part of the atmosphere (Fig. 16.3). A good example of this is when you pump up a bicycle tire. As the volume is reduced, however, larger forces are required to reduce it still further. Solid and liquid materials hardly change in volume unless extremely large forces are applied.

A third special aspect of gases is their thermal expansion. Gases expand or contract much more for a certain temperature change than do solid or liquid materials. In Section 10.1 we referred to Galileo's thermometer, in which the expansion and contraction of a sample of air in a glass bulb served as a temperature indicator. We nevertheless

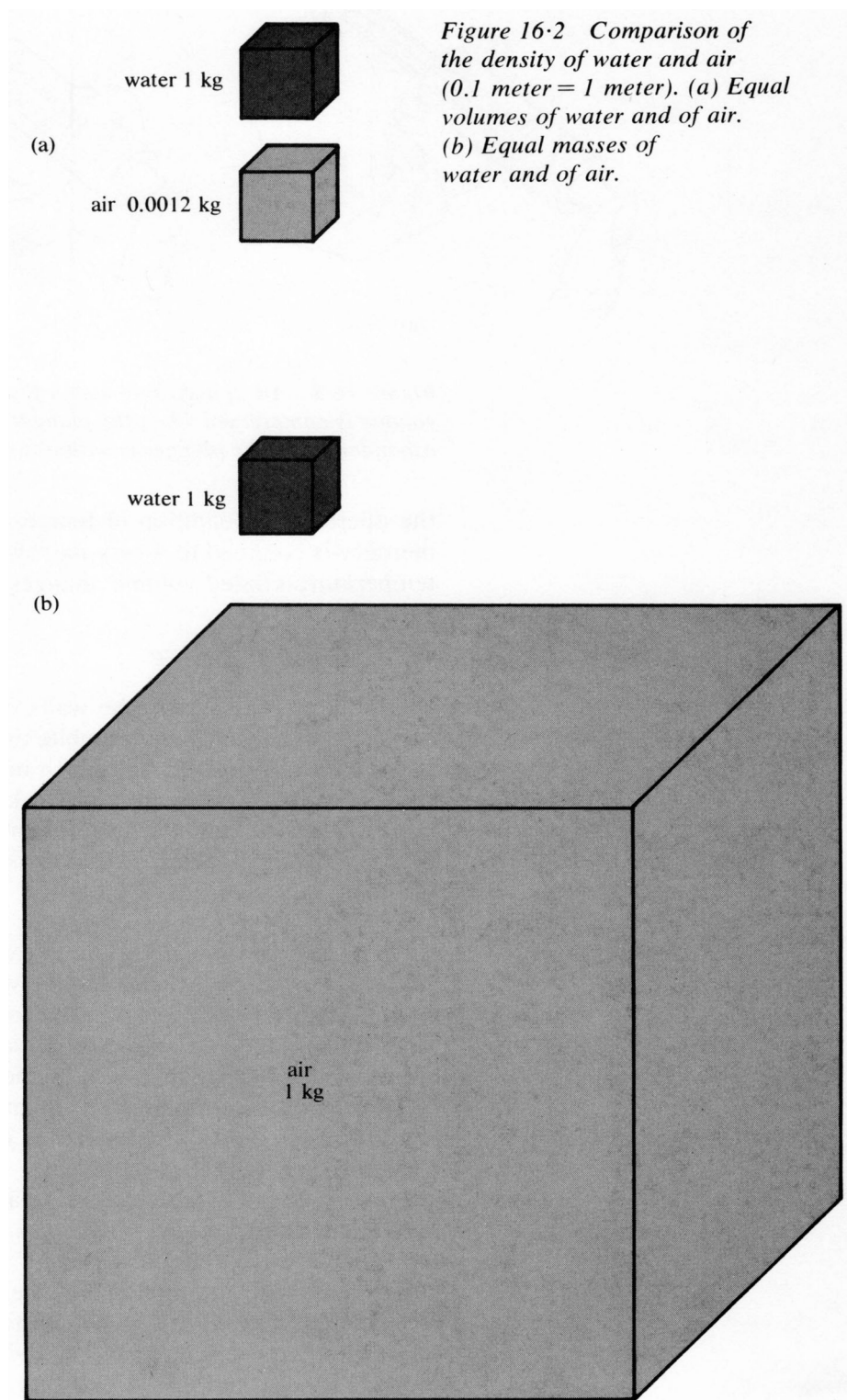
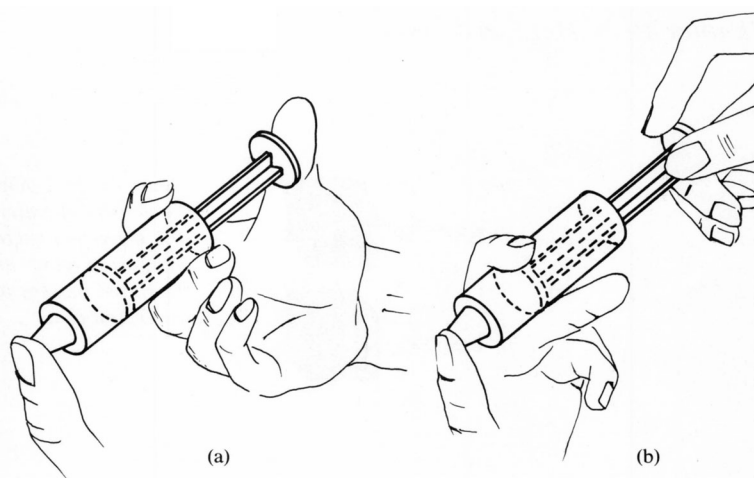


Figure 16.3 Air in a syringe with a finger over the open end.

(a) The air volume is compressed when the plunger is pushed in.

(b) The air volume is expanded when the plunger is withdrawn.



selected the mercury thermometer instead of an air thermometer for the operational definition of temperature. As pointed out in Section 10.1, the mercury must be confined to a narrow glass tube so as to make the small temperature-related volume changes visible (Fig. 10.3).

16.2 Gas pressure

Gases exert pressure on the walls of the container in which they are confined. Tennis balls, automobile tires, and children's balloons all contain compressed air, which makes these objects relatively stiff. If the container walls are too weak to withstand the pressure, they burst. In the experiment shown to the left, air is pumped out of a sealed container, which reduces the pressure inside, as a result, the pressure of the outside atmosphere crushes the container.



Force and pressure. Pressure (rather than force) is the measure of interaction strength that we use when a confined gas interacts with its container wall. The reason the force concept is not directly useful for gases is that a gas cannot be described by a single-particle center-of-mass model (Section 14.1). In the center-of-mass model, the force of interaction is concentrated on one particle, but the interaction between a gas and its container is distributed over the entire surface of the container wall (Fig. 16.4).

Pressure can nevertheless be related to force with the help of the following working model for the gas. Imagine the outside layer of gas represented by a layer of "gasbags" (Fig. 16.5) that press on the container wall. You can represent the interaction between each bag and the wall with two forces, one exerted by the gasbag on the container, and the other exerted by the container on the gasbag. According to Newton's third law, these two sets of forces are equal in magnitude and oppositely directed.

You can see from Fig. 16.5 that the forces produced by the gas pressure are directed at right angles to the boundary surface. You can also recognize that the magnitude of the forces depends on the size of the contact area between one gasbag and the boundary surface. If, for example, you were to divide one bag in Fig. 16.5 into smaller ones, then each smaller bag would exert a smaller partial force in proportion to its area (Fig. 16.6).

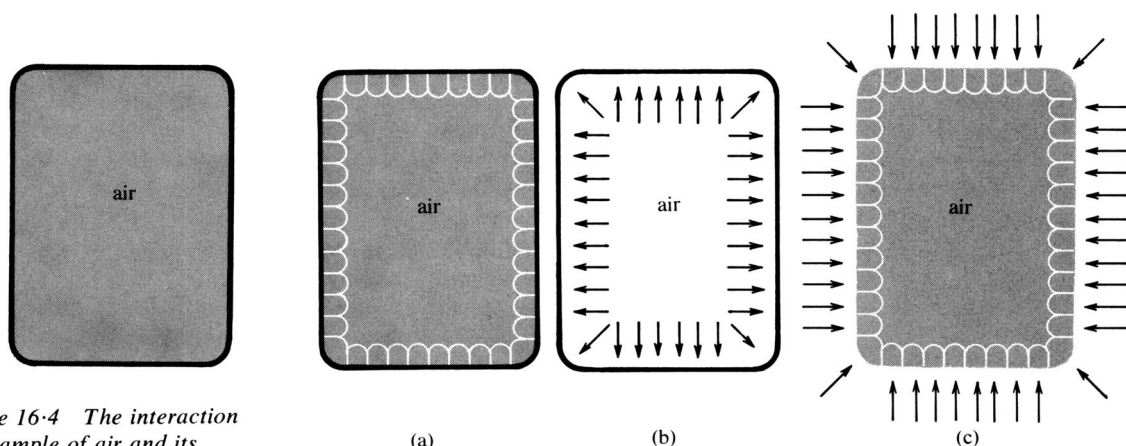


Figure 16-4 The interaction of a sample of air and its container is distributed all over the boundary surface between the two materials.

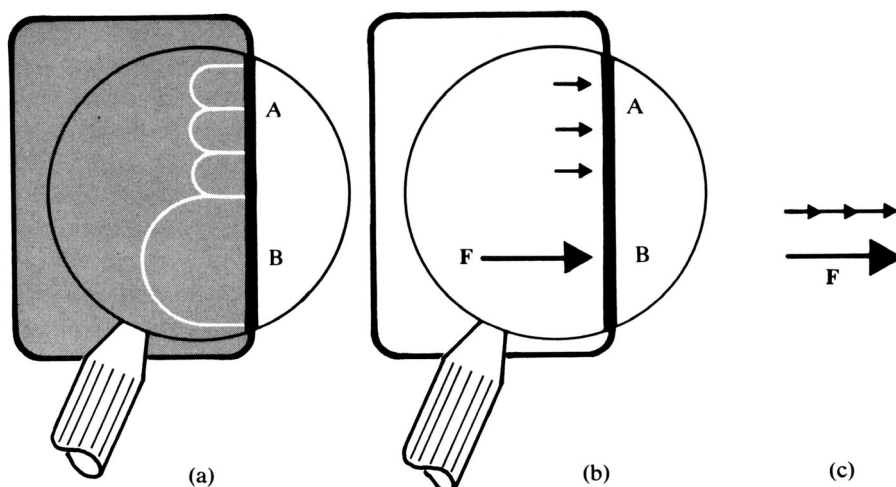
Figure 16-5 The interaction of a sample of gas with its container wall.
 (a) "Gasbag" layer model for the gas.
 (b) Forces exerted on the container by the gasbags.
 (c) Gasbags and forces exerted on them by the container.

Gas pressure gauges. Gauges for measuring gas pressure can be understood with the help of the "gasbag" working model pictured in Fig. 16.5. In effect, a gauge replaces the section of boundary surface with a device that registers the force exerted by the gasbag. You are undoubtedly familiar with several different types of pressure gauges and pressure readings. Air pressure in automobile and bicycle tires is measured in "pounds" (more accurately "pounds per square inch"), and usually ranges from 20 pounds (automobiles) to more than 100 pounds (racing bicycles). Atmospheric pressure as reported by the weather bureau is measured in "inches" of mercury and is usually about 30 inches. Other commonly used units of pressure are the "millimeter" of mercury and the "atmosphere." Since gas pressure results in the action of forces, it is also possible to relate the pressure to the newton.

Discovery of atmospheric pressure and the barometer. Evangelista Torricelli carried out a series of key experiments that showed that the atmospheric air could exert pressure and thus support a column of mercury about 0.76 meter (30 inches) high. He filled a long glass tube closed at one end with mercury and then inverted the tube, as shown in

Figure 16.6 Interaction of a gas and its container (detail of Fig. 16-5).

(a) One gasbag is divided into three equal smaller bags that press individually on container wall section A.
 (b) Forces exerted on container sections A and B.
 (c) Addition of the partial forces acting on container section A gives a force equal to that acting on section B.



Before Torricelli, the theory that "nature abhors a vacuum" was used to explain why liquids could not be poured out of containers with only one small hole.

FORMAL DEFINITION

A gas exerts a force on a surface of area A . Then the "**pressure**" of the gas is equal to the magnitude of the force divided by the area.

Equation 16-1

force	\mathbf{F}
magnitude of the force	$ \mathbf{F} $
area	A
pressure	P

$$P = \frac{|\mathbf{F}|}{A}$$

unit of pressure:
newtons per square meter
= newtons/m²

EXAMPLE

$$\begin{aligned} |\mathbf{F}| &= 1000 \text{ newtons} \\ &= 10^3 \text{ newtons} \\ A &= .01 \text{ m}^2 = 10^{-2} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} P &= \frac{|\mathbf{F}|}{A} \\ &= \frac{10^3 \text{ newtons}}{10^{-2} \text{ m}^2} \\ &= 10^5 \text{ newtons/m}^2 \end{aligned}$$

Fig. 16.7, with the other end of tube open to the atmosphere. Some of the mercury ran out and left an empty space at the top of the tube; the difference in height between the higher end of the mercury column and the lower end was always about 30 inches (0.76 m). Torricelli and his successors reasoned that the mercury remaining in the tube was in mechanical equilibrium (zero net force) and thus subject to two equal and opposed forces: the force of gravity (downward) and the force exerted by the atmosphere on the lower end of the mercury column. Torricelli reasoned that the downward force exerted by the atmosphere was transmitted from the open end of the tube by the mercury in the curved section (Fig. 16.7) and thus applied an upward force on the vertical column just large enough to support the observed height of 0.76 m.

Torricelli encountered substantial resistance from traditionalists, who refused to accept the idea that an empty space (or vacuum) existed at the top of the tube and who also found the idea that the weight of the atmosphere pushed the mercury *up* to be contrary to common sense.

Torricelli responded to such objections in the manner of a true scientist: with an experimental test. Torricelli reasoned that the atmospheric pressure should vary with height above (or below) sea level, and he tested this idea by carrying his experimental apparatus up a mountain and measuring the length of the mercury column at various points on the way up to the summit. The experimental results confirmed Torricelli's prediction. These and other experiments convinced most scientists of the validity of the concept of atmospheric pressure.

Torricelli's inverted mercury-filled tube is now called a *mercury barometer* and used as a *pressure gauge*. The length of the mercury column is an excellent measure of (and operational definition for) the atmospheric pressure and is measured in millimeters, inches, or any other unit of length. This is the barometric pressure scale used in weather reports. Water or oil is sometimes used as the barometric liquid instead of mercury. Can you identify the advantages and disadvantages of using water or oil rather than mercury?

Spring scale gauge. A second type of pressure gauge makes use of a suitably designed spring scale that can be attached to the gas container (Fig. 16.8). At a low gas pressure, the force reading is small; at a high pressure, it is large. Tire pressure is usually measured by this type of pressure gauge; hence the "pound" (1 pound equals 4.5 newtons) is used to describe tire pressure. The *aneroid barometer* is a meteorological instrument that incorporates a spring scale for measuring atmospheric pressure (Fig. 16.9); its dial, however, is usually calibrated in inches or millimeters of mercury, because those are the pressure units commonly used in weather reports.

Formal definition of pressure. Because we will be concerned with energy transfer from an expanding gas, and therefore with the force it exerts, we will introduce a formal definition for pressure as the force per unit area (Eq. 16.1). Whenever the force exerted by a gas is proportional to the area (and this is generally the case), the numerical value of the pressure does not depend on the size of the area chosen.

Evangelista Torricelli (1608-1647) was an Italian noble who came to Rome in 1628 to study mathematics. Upon becoming acquainted with Galileo's work on motion, Torricelli was overcome with admiration and wrote Galileo on the subject. Galileo was impressed and invited Torricelli to become his assistant. After his master's death, Torricelli became his successor at the Academy of Florence. He is best known as the inventor of the barometer, which created a sensation because Torricelli and his colleagues argued that the space above the inverted mercury column was essentially empty. This was a realization of tremendous significance, because Aristotle had held that it was impossible to create a vacuum. Like his mentor Galileo, Torricelli criticized Aristotelian physics and helped lay the groundwork of the new physics.

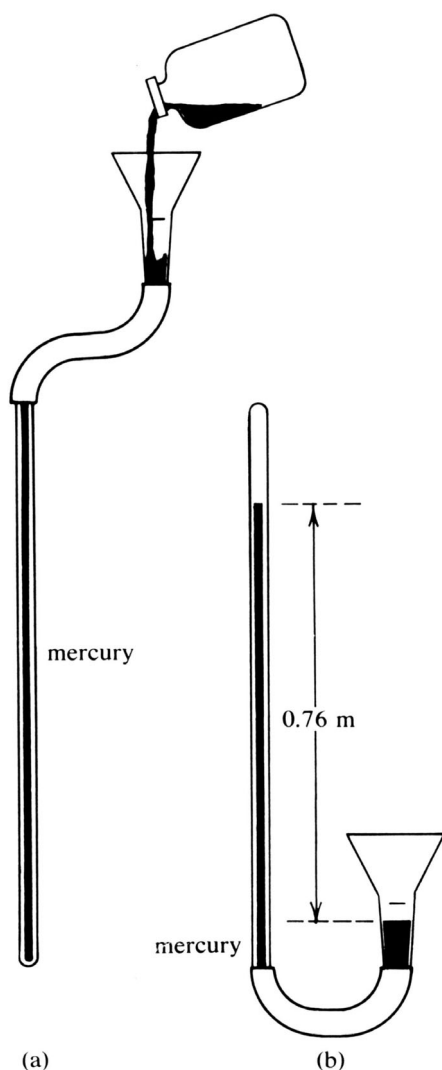


Figure 16.7 (to left) Torricelli's barometer and his experiment revealing the pressure of the atmosphere. (a) A long glass tube (more than 30 inches or 0.76 m) is filled with mercury. (b) The glass tube is turned upside-down. A gap appears at the top of the tube above the mercury column. Torricelli argued that the gap was empty, or a vacuum. He showed that the length of the gap (and the height of the mercury column) depends upon the atmospheric pressure. Torricelli interpreted this to mean that the weight of the 0.76 m mercury column was being supported by the weight, or pressure, of the atmosphere on the mercury surface in the open funnel.

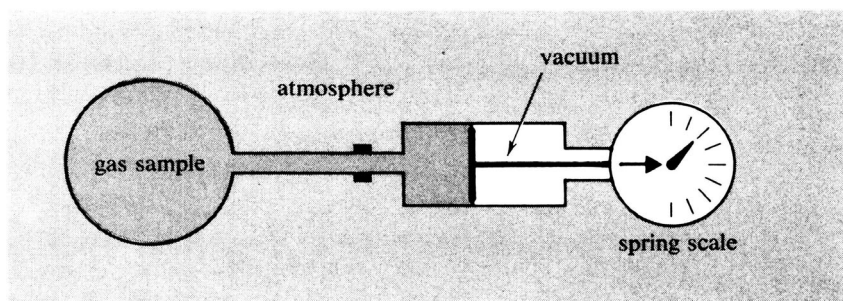
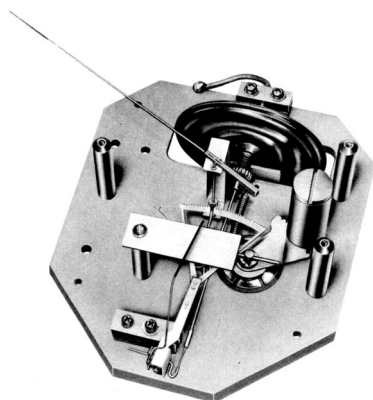


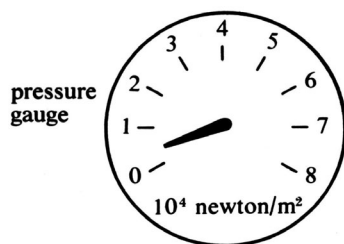
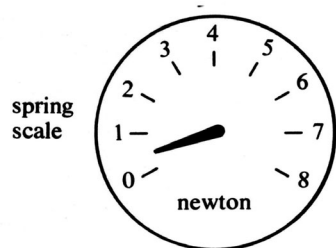
Figure 16.9 (to left) An aneroid barometer contains a thin metal membrane on a sealed, evacuated cylinder (at the back). The membrane flexes in and out in response to the outside air pressure. The long needle is connected by a sensitive gear to the membrane and thus registers the air pressure on a scale (not shown).

TABLE 16.1 RELATION OF UNITS FOR PRESSURE MEASUREMENT

Unit	Equivalent (newtons/m ²)	Equivalent (atm)	Use of unit
1 newton per square meter	1.0	1.0×10^{-5}	physics
1 millimeter of mercury	130.	$\frac{1}{760} = 1.3 \times 10^{-3}$	meteorology, physics
1 inch of mercury	3.4×10^3	$\frac{1}{30} = 3.4 \times 10^{-2}$	meteorology
1 pound per square inch	6.8×10^3	$\frac{1}{14.7} = 6.8 \times 10^{-2}$	technology
1 atmosphere	1.0×10^5	1.0	high-pressure measurements

A pressure gauge based on the formal definition of pressure can be made from the standard spring scale by suitably marking its dial in relation to the area on which the gas exerts pressure (Example 16.1). We will use such a pressure gauge for illustrative purposes when describing experiments.

Units for measuring pressure. We have mentioned a large number of different standard units of pressure in this section (Table 16.1). Some of them are used only under special conditions, as indicated. In the remainder of this chapter we will use only two of these units. One of them is the *newton per square meter*, which is indicated by the standard pressure gauge (Example 16.1). The other unit is the *atmosphere*. One atmosphere is the average pressure of the air in the earth's atmosphere



EXAMPLE 16.1. We wish to calibrate a spring scale pressure gauge with a plunger area of 1 square centimeter. The spring scale is to be calibrated in newtons.

$$A = 1 \text{ cm}^2 = (0.01 \text{ m})^2 = 10^{-4} \text{ m}^2$$

(a) For a dial reading $|\mathbf{F}| = 1$ newton.

$$\text{Pressure } P = \frac{|\mathbf{F}|}{A} = \frac{1 \text{ newton}}{10^{-4} \text{ m}^2} = 1 \times 10^4 \text{ newtons/m}^2$$

(b) For dial reading $|\mathbf{F}| = 2$ newtons.

$$\text{Pressure } P = \frac{|\mathbf{F}|}{A} = \frac{2 \text{ newtons}}{10^{-4} \text{ m}^2} = 2 \times 10^4 \text{ newtons/m}^2$$

Therefore, the spring scale dial can be used as a pressure gauge dial if each newton is interpreted as 10^4 newtons per square meter.

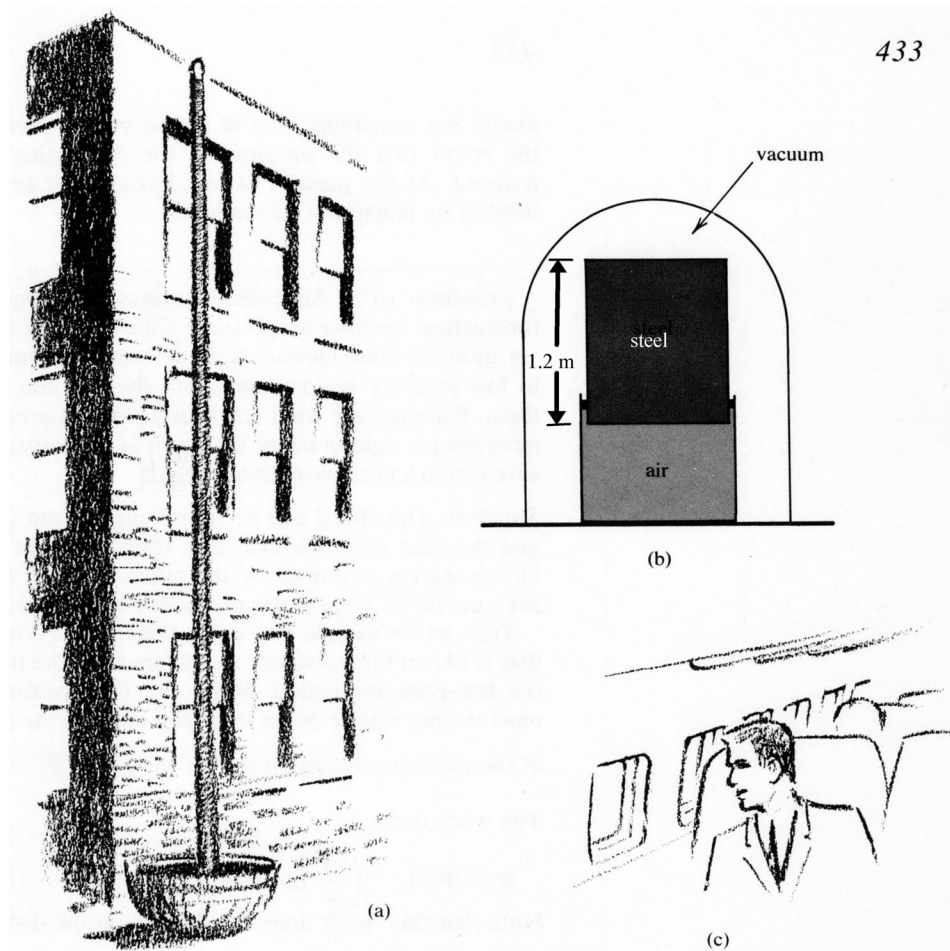


Figure 16.10 Some consequences of atmospheric pressure. The pressure of 1 atmosphere may be described in the following terms: 10 meters of water, 1.20 meters of steel, 0.76 meter (30 inches) of mercury, 10^5 newtons per square meter, and 14.7 pounds per square inch.

(a) The atmosphere pressing on the water surface supports a 10-meter-high column of water. On this basis, Galileo explained why, in his day, water could not be pumped from wells deeper than 10 meters.

(b) The trapped air at a pressure of 1 atmosphere supports a 10-ton steel block.

(c) Have you wondered why jet plane windows are so small? The net pressure force on an 8-by-8-inch window can be 90 pounds (8 in. \times 8 in. \times 14.7 lbs/in²). Is this net force directed inward or outward?

at sea level. This is very slightly more than 100,000 newtons per square meter. The atmosphere (unit) is therefore a much larger unit of pressure than the newton/m². Table 16.1 shows the relation of the newton per square meter and of the atmosphere to the other units that are in common use.

Because the earth's atmosphere is so important, we illustrate some aspects of atmospheric pressure in Fig. 16.10 and implications for the weather in Example 16.2. Since Torricelli's day, atmospheric pressure has been ascribed to the fact that the air near the ground supports the layers of air above. As you proceed upward through the atmosphere,

the remaining load of air on you becomes less and less, with the result that the pressure of the supporting air also is gradually reduced. At the peak of Mount Everest the air pressure is only one third of its magnitude at sea level.

EXAMPLE 16.2. Air pressure plays an important role in weather forecasting, because air pressure differences give rise to net forces that set air masses into motion. Air masses move from high-pressure regions to low-pressure regions and carry thermal energy and moisture with them. Estimate the wind speed created by a pressure difference of 10^3 newtons per square meter (0.3 inch of mercury) between a high-pressure region and a low-pressure region.

Solution: The speed can be found either from the net pressure force and the time over which it acts (Newton's second law) or from the kinetic energy acquired by the air as work is done on it by the net pressure force. We will use the kinetic energy method.

Take an air sample of 1 cubic meter, with a mass of 1.2 kilograms, that is blown for a distance Δs meters from the high-pressure region to the low-pressure region. Since the total pressure difference of 10^3 newtons per square meter is spread out over a distance of Δs meters, the magnitude of the net force acting on the air sample is $|\mathbf{F}| = (10^3/\Delta s)$ newtons.

The work done:

$$W = |\mathbf{F}| \Delta s = [(10^3 / \Delta s) \text{ newtons}] \times \Delta s \text{ meters} = 10^3 \text{ joules.}$$

Note that this result does not depend on the distance Δs .

The kinetic energy:

$$KE = \frac{1}{2} Mv^2 = \frac{1}{2} \times 1.2 \times v^2 = 0.6v^2$$

The kinetic energy equals the work:

$$0.6v^2 = 10^3 \text{ joules}$$

$$v^2 = (1.0/0.6) \times 10^3 = 17 \times 10^2$$

$$v = \sqrt{17 \times 10^2} = 40 \text{ m/sec}$$

Answer: 40 meters per second or $(40 \text{ m/s})[(2.25 \text{ mi/hr})/(\text{m/s})] = 90 \text{ mi/hr.}$

16.3 Energy storage by gases

A cylinder with a movable piston, such as in an automobile engine, is a convenient container for studying energy transfer to or from a gas sample. The gas may be heated as it interacts with a system at higher temperature than itself (for example, a flame), or the gas may be compressed by a force acting on the piston and doing work on the gas in

Robert Boyle (1627-1691) was born in Ireland. After education at Eton and on the continent, Boyle returned home in 1644 when his father died. Following 10 years of seclusion in Ireland, Boyle moved to Oxford and began to take active part in English scientific life. It was Boyle's celebrated experiments with gases, described in his *New Experiments Physicomechanical, Touching the Spring of the Air* (1662), that showed how changes in the volume of a gas were related to the gas pressure. Not the least of Boyle's achievements was the important part he played in establishing The Royal Society in 1662.

Equation 16.2 (Boyle's Law)

$$\begin{aligned} \text{gas volume (m}^3\text{)} &= V \\ \text{gas pressure} & \\ \text{(newtons / m}^2\text{)} &= P \\ \text{proportionality sign} &\propto \\ V &\propto \frac{1}{P}, \\ \text{or } VP &= \text{constant} \\ &\text{at fixed temperature} \end{aligned}$$

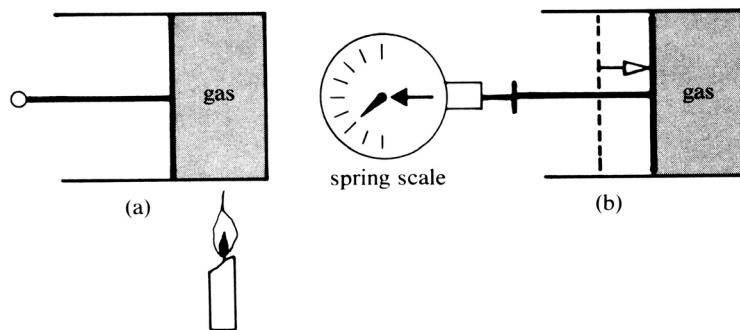


Figure 16.11 Energy transfer to a gas sample,
(a) Heat is absorbed by the gas.
(b) Work is done on the gas.

displacing the piston (Figure 16.11). Similarly, there are two ways in which the gas can release energy: it may be cooled by interacting with a system at lower temperature than itself or it may expand and push the piston out, thereby doing work. The heating and cooling processes, which we described in their applications to liquid and solid materials in Chapter 10, lead to the concept of specific heat and thermal energy storage for gases. The compression and expansion processes, which we described in their application to elastic systems in Section 11.6, lead to the concept of "elastic" energy storage by gases. As you will see in this section, experiments show that these two forms of energy storage by a gas are almost identical. Gases store thermal energy, which may be released either as heat (when the gas cools down at constant volume) or as work (when the gas expands against a piston).

"Elasticity" of gases. Everyone is familiar with the "spring" of air in an automobile tire. Air in a pump or syringe (with stopped-up opening) can be squeezed into a smaller volume by a plunger, but the plunger is forced back out when the hand is taken off (as illustrated in Fig. 16.3).

Boyle's law. In 1662, Robert Boyle published a description of "Two New Experiments Touching the Measure of the Force of the Spring of Air Compressed and Dilated." Boyle concluded from his observations that the volume of a gas sample is inversely proportional to its pressure, a relationship known as Boyle's law (Eq. 16.2). A sequence of Boyle's experiments is illustrated schematically in Fig. 16.12. You may recognize that gases are analogous to springs even though the mathematical models of Boyle's and Hooke's laws are quite different (Table 16.2). In fact, Hooke was a contemporary and occasional collaborator of Robert Boyle.

Charles' law. About 100 years after Boyle's publication, Jacques-Alexandre Charles (1746-1823) and Joseph Gay-Lussac (1778-1850) independently discovered a relationship between the volume and the

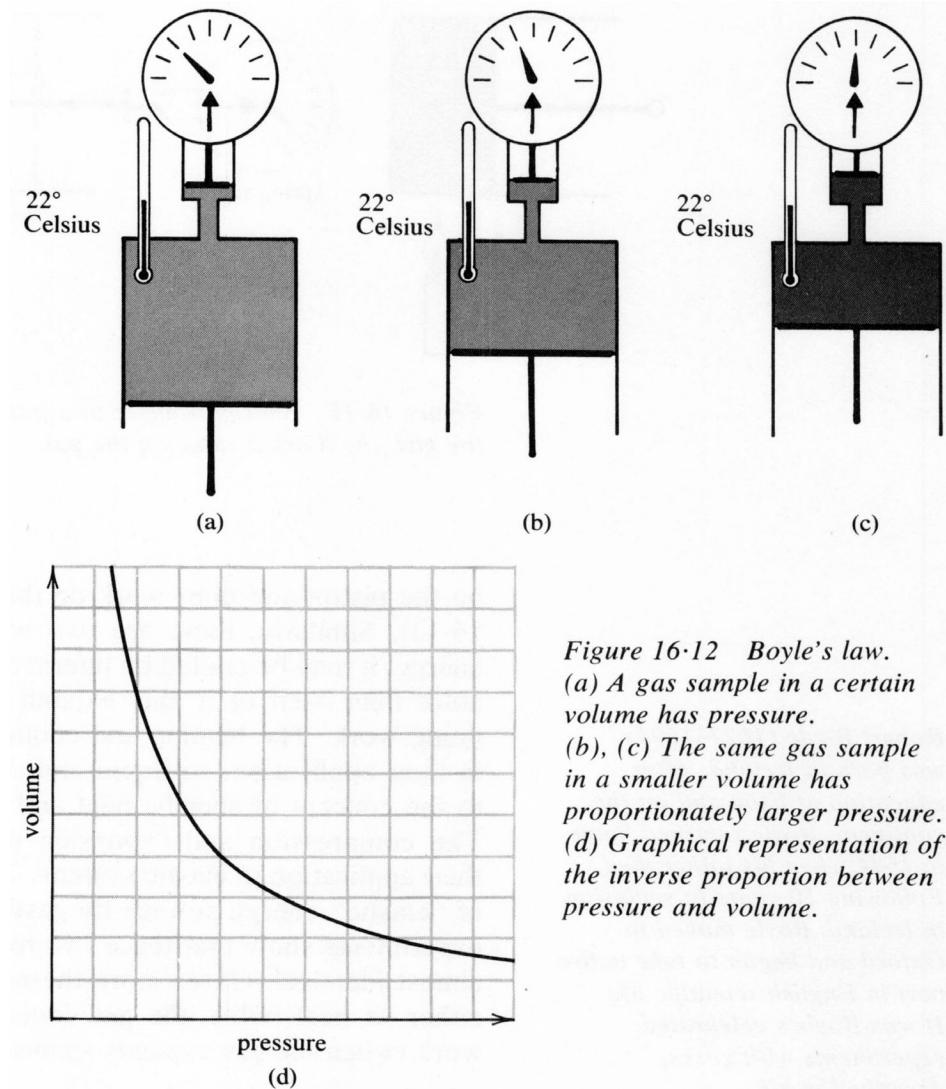


Figure 16.12 Boyle's law.
 (a) A gas sample in a certain volume has pressure.
 (b), (c) The same gas sample in a smaller volume has proportionately larger pressure.
 (d) Graphical representation of the inverse proportion between pressure and volume.

TABLE 16.2 ANALOGY BETWEEN GASES AND ELASTIC SYSTEMS

<i>elastic system (such as a spring)</i>	<i>Gas</i>
<i>elastic force</i>	<i>pressure</i>
<i>elastic deformation</i>	<i>volume change</i>
<i>Hooke's law</i>	<i>Boyle's law</i>
<i>elastic energy</i>	<i>"elastic" energy</i>
<i>isolated spring in mechanical equilibrium</i>	<i>(isolated gas in mechanical equilibrium)*</i>

*The analogy breaks down because an isolated gas, not interacting with any other system, spreads throughout space and is not in mechanical equilibrium. Therefore, a gas has no natural equilibrium state in which its "elastic" energy is zero.

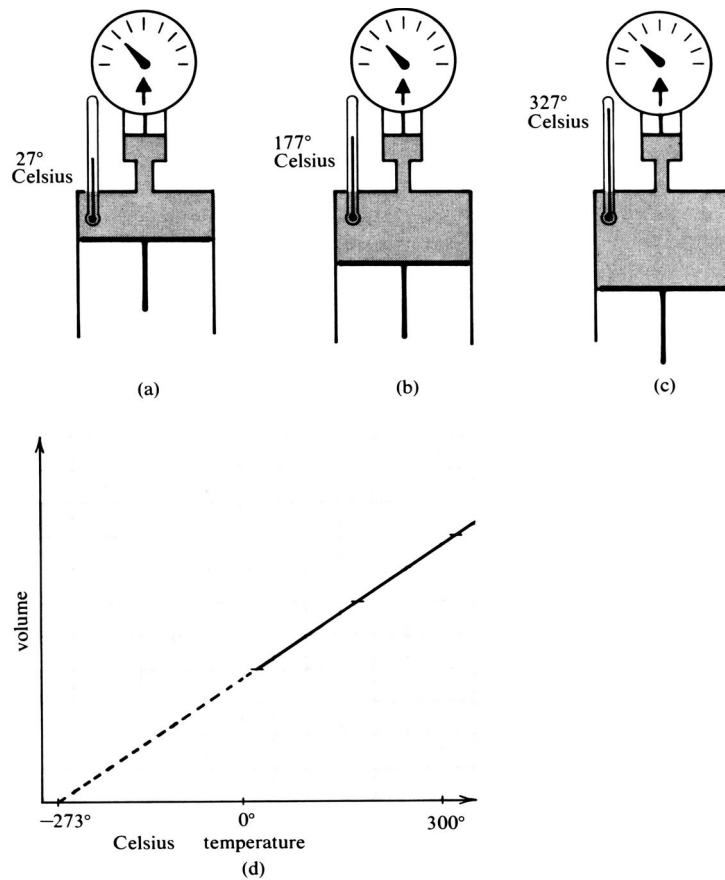


Figure 16.13 Charles' law.

(a) A gas sample at a certain temperature and pressure occupies a certain volume.
 (b), (c) The same gas sample at higher temperatures but the same pressure.
 (d) Graphical representation of the proportional increases in temperature and volume.

Equation 16.3
(Charles' Law)

temperature
 (deg Celsius)
 $= T$

$$V \propto T + 273$$

temperature of gas samples confined in a container at constant pressure (Fig. 16.13). The relationship that the volume is proportional to the "temperature plus 273 degrees" is known as Charles' law (Eq. 16.3). Basically, Charles' law provides a quantitative basis for the operation of Galileo's thermoscope (Fig. 10.1). The one condition, not recognized by Galileo, that must be kept constant is the pressure of the gas sample; otherwise there are volume changes derived from a gas's compressibility (Boyle's law) in addition to those associated with the temperature change.

Ideal gas model. You will now realize that the quantitative description of gas samples is made by means of four variable factors: the mass, the pressure, the temperature, and the volume. These four factors cannot be changed arbitrarily and independently, but are related by Boyle's law

TABLE 16.3 IDEAL GAS MODELS

<i>Gas</i>	<i>Mathematical model</i>
<i>carbon dioxide</i>	$PV = 190M(T + 273)$
<i>helium</i>	$PV = 2100M(T + 273)$
<i>hydrogen</i>	$PV = 4100M(T + 273)$
<i>nitrogen</i>	$PV = 290M(T + 273)$
<i>oxygen</i>	$PV = 260M(T + 273)$

Equation 16.4 (Ideal gas law for air)

$$\begin{aligned}
 \text{pressure (newtons/m}^2\text{)} &= P \\
 \text{volume (m}^3\text{)} &= V \\
 \text{mass of gas (kg)} &= M \\
 \text{temperature (deg Celsius)} &= T
 \end{aligned}$$

$$PV = 280M(T + 273)$$

(for air)

and Charles' law. A mathematical model that quite accurately describes the interdependence of the four variable factors for air is given in Eq. 16.4. This model is called an *ideal gas model*. You can see that it includes Boyle's law (when the mass and temperature are kept constant) and Charles' law (when the mass and pressure are kept constant). According to the model, you can arbitrarily choose values for any three of the variable factors listed above; the value of the fourth one is then predicted accurately by the model and can be calculated from Eq. 16.4.

An investigation of many different gases reveals that each is described very closely by a similar ideal gas model (Table 16.3). The only difference between the models for various gases is in the specific value of the numerical factor. All these gases and air, therefore, behave generally like the gas helium examined in Example 16.3, below.

Gases increase in volume without limit when their pressure is reduced. In this respect they differ greatly from solid and liquid materials, whose volume hardly changes when the pressure is reduced except that they might evaporate. If the pressure of a gas is increased and its volume reduced, the ideal gas model eventually ceases to be applicable, especially when conditions for liquefaction of the gas are approached.

Absolute zero. The factor $(T + 273)$ in the ideal gas model is particularly curious. According to this factor, the pressure or the volume of every gas becomes zero if the gas temperature is brought to -273° Celsius (Fig. 16.13d). This prediction applies to all the gases regardless of the initial state from which the gas is cooled to -273° Celsius. At this temperature the gas has lost all its "spring" and does not exert any pressure on its container. This does not happen in reality, of course, because the ideal gas model is mainly accurate near and above room temperature. It fails at lower temperatures, as it does at very high pressure, when the conditions for liquefaction are approached. Nevertheless, the temperature of -273° Celsius, which is called the *absolute zero* of temperature, has come to play a very important role in the theory of thermal phenomena.

Work done on or by a gas. The analogy between gases and elastic systems (Table 16.2) can be pursued to calculate the work done on a

Equation 11.5
(Definition of Work)work (joules) = W

$$W = |\mathbf{F}| \Delta s_F$$

Equation 16.5
(from Definition of Pressure, Eq. 16.1)

pressure
(newtons/m²) = P
area (m²) = A

$$|\mathbf{F}| = PA$$

Equation 16.6

volume change
(m³) = ΔV

$$\Delta s_F = \frac{\Delta V}{A}$$

Equation 16.7

$$\begin{aligned} W &= |\mathbf{F}| \Delta s_F \\ &= PA \times \frac{\Delta V}{A} \\ &= P \Delta V \end{aligned}$$

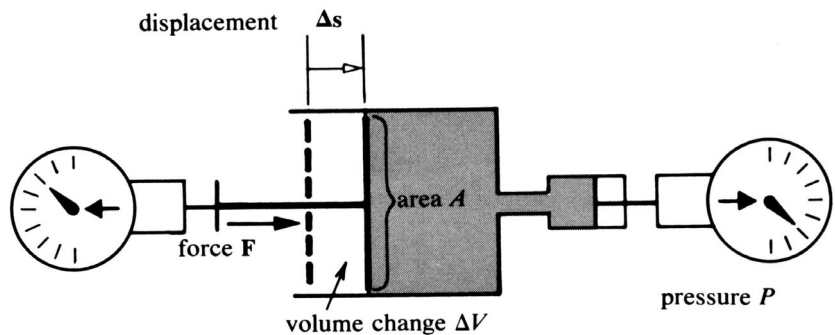


Figure 16.14 A gas in a cylinder is compressed. The force, displacement, pressure, area, and volume change are indicated. Note that the displacement is parallel to the direction of the force.

gas while it is being compressed. Work is done because a force acts to accomplish the compression, and a part of the container wall (a piston, for instance) is displaced to reduce the gas's volume (Fig. 16.14). The definition of work is given in Eq. 11.5. You can adapt the definition for use with gases by expressing the force in terms of the gas pressure (Eq. 16.5) and the displacement in terms of the volume change (Eq. 16.6). You then find that the work is equal to the gas pressure times the volume change (Eq. 16.7). The same calculation leads to the same formula for the work done by a gas while it is expanding.

To find the numerical value of the work in a particular experiment from Eq. 16.7, you must estimate an average value of the pressure, which changes according to Boyle's law during a compression or expansion process (Eq. 16.2). This can be done, but it requires mathematical techniques beyond the level of this text. Hence you may use Eq. 16.7 only for small fractional volume changes, for which the pressure remains approximately constant (Example 16.4).

EXAMPLE 16.3. A weather balloon contains 2 kilograms of helium. How much volume does it occupy at an altitude where the temperature is -50° Celsius and the pressure is 0.1 atmosphere?

Data:

$M = 2$ kg; $T = -50^\circ$ Celsius; $P = 0.1 \times 10^5$ newtons/m²

Solution: From Table 16.3,

$$\begin{aligned} V &= \frac{2100M(T+273)}{P} = \frac{2100 \times 2 \times (-50 + 273)}{0.1 \times 10^5} \\ &\approx \frac{2.1 \times 10^3 \times 2 \times 2.2 \times 10^2}{10^4} \approx 90 \text{ m}^3 \end{aligned}$$

EXAMPLE 16.4. How much work is done when 1 kilogram of air is compressed from atmospheric pressure into a volume of 0.70 cubic meter at 22° Celsius?

Data:

$T = 22^\circ \text{ Celsius}$; $M = 1 \text{ kg}$; $P = 10^5 \text{ newtons/m}^2$;
final volume = 0.70 m^3

Solution: (a) Find the initial volume so the volume change can be determined. (b) Check the pressure variation to make sure it is "small."
From Eq. 16-4,

(a) initial volume:

$$V = \frac{280M(T + 273)}{P} = \frac{2.8 \times 10^2 \times 1 \times 3.0 \times 10^2}{1 \times 10^5} \\ \approx 0.84 \text{ m}^3$$

volume change:

$$\Delta V = 0.84 \text{ m}^3 - 0.70 \text{ m}^3 = 0.14 \text{ m}^3$$

work:

$$W = P\Delta V = 1 \times 10^5 \text{ newtons/m}^2 \times 0.14 \text{ m}^3 \\ = 1.4 \times 10^4 \text{ joules} = 3.3 \text{ Cal}$$

(b) final pressure:

$$P = \frac{280M(T + 273)}{V} = \frac{2.8 \times 10^2 \times 1 \times 3.0 \times 10^2}{0.70} \\ \approx 1.2 \times 10^5 \text{ newtons/m}^2$$

The pressure increased by 20% from 1×10^5 newtons per square meter to 1.2×10^5 newtons per square meter. This is a fairly small change.

Specific heat of gases. If you try to define the specific heat of a gas as the energy transfer required to change the temperature of 1 kilogram of gas by 1 degree Celsius, you encounter an interesting and important difficulty. The specific heat appears to depend on the method of measurement! The "specific heat" of air confined in a rigid cylinder, for example, is 0.18 Calorie per degree Celsius per kilogram, while the "specific heat" of air in a cylinder with a freely moving piston (where the gas expands as it is heated) is 0.25 Calorie per degree Celsius per kilogram (Fig. 16.15). In other words, 0.07 Calorie more energy is required to raise the temperature of 1 kilogram of air in the cylinder where it can expand than to raise the temperature of 1 kilogram of air in the rigid cylinder.

You can understand this experimental result if you refer to Eq. 16.7, where the volume change of a gas was related to energy transfer in the form of work. Thus, the gas in the cylinder with the sliding piston interacts with the surrounding air, which is pushed out of the way by the piston (Fig. 16.15b). The energy transferred in the form of work from

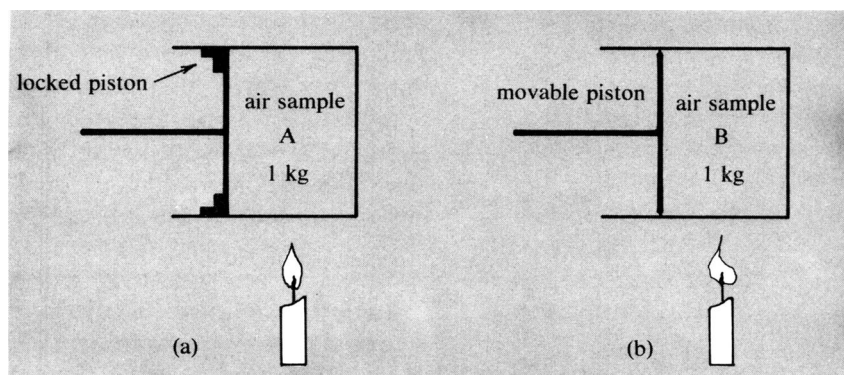


Figure 16.15 The specific heat of air.

(a) The temperature of 1 kilogram of air is raised 1 degree Celsius with the piston locked in place; 0.18 Calorie of heat is required from the candle.

(b) The temperature of 1 kilogram of air is raised 1 degree Celsius with the piston free to move; 0.25 Calorie of heat is required from the candle.

Where did the extra energy go? Answer: See top left margin on next page!

Equation 16.8 (Thermal energy of a gas)

$$\begin{aligned} \text{thermal energy (Cal)} &= E \\ \text{specific heat} & \\ (\text{Cal/deg C/kg}) &= C \\ \text{temperature} & \\ (\text{deg C}) &= T \\ \text{mass of gas (kg)} &= M \end{aligned}$$

$$E = CMT$$

(gases must be kept at constant volume)

FORMAL DEFINITION.
Isothermal processes are processes (usually expansion or contraction of a gas) that take place at one, constant temperature. In order to keep the temperature constant, other quantities (pressure, volume,...) must usually be allowed to change in carefully controlled ways.

the confined air to the surrounding air can be calculated from Eq. 16.7 and the ideal gas model for air (Eq. 16.4). It is indeed equal to 0.07 Calorie per kilogram, in agreement with the data (Example 16.5).

The air in the rigid cylinder (constant volume) does not gain or lose energy due to interactions other than that with the heat source. All the heat absorbed by this gas sample is stored in it in the form of thermal energy. The constant volume procedure, therefore, leads to the specific heat as it was defined in Section 10.2. The mathematical model for the thermal energy of a gas is very similar to the model for the thermal energy of solids and liquids (Eq. 16.8). An application of this model is described in Example 16.6. Specific heats of several gases are listed in Table 16.4. They do not vary appreciably as the gas pressure is changed.

Compression and expansion at a fixed temperature (isothermal).

Look again at the experiment for measuring the heat capacity (Fig. 16.15). The two air samples were in equal states at the beginning of the

TABLE 16.4 SPECIFIC HEATS OF GASES (AT CONSTANT VOLUME)

Gas	Temperature (deg Celsius)	Pressure (atm)	Specific heat (Cal/deg/kg)
air	-100 to +1000	0 to 10	0.18
oxygen	-100 to +1000	0 to 10	0.16
nitrogen	-100 to +1000	0 to 10	0.18
helium	-100 to +1000	0 to 10	0.75
water	+120 to + 500	0 to 10	0.37
carbon dioxide	-50 to + 50	0 to 10	0.14
hydrogen	0 to +1000	0 to 10	2.5

In Figure 16.15, the additional energy (0.07 Calories) required to raise the temperature of the air inside the cylinder at constant pressure was required because the piston moved outward, thus doing work and transferring energy to the surrounding air. When the air is heated at constant volume, the piston is locked and thus does no work.

experiment (equal masses, equal temperatures, equal pressures, and equal volumes) and therefore had equal energies. During the experiment, the energy stored in each sample increased by 0.18 Calorie while its temperature was being raised. Hence the two samples still had equal energies at the conclusion of the experiment, even though they were then in different states (equal masses and equal new temperatures, different pressures and different volumes. Fig. 16.16). It follows from

EXAMPLE 16.5. (a) Calculate the volume change of air due to a one-degree rise in temperature from 22° to 23° Celsius.

$$V = \frac{280M(T + 273)}{P}$$

At 22°:

$$V = \frac{280M \times 295}{P}$$

At 23°:

$$V = \frac{280M \times 296}{P}$$

volume change:

$$\Delta V = \frac{280M}{P}$$

(b) Calculate the work done by 1 kilogram of air while it expands as in (a).

$$W = P\Delta V = P \times \frac{280M}{P} = 280 \times 1 = 280 \text{ joules} = 0.07 \text{ Cal}$$

Note that the result depends only on the temperature change in (a) and not on the pressure of the gas.

EXAMPLE 16.6. Find the thermal energy of one “roomful” of air (4 meters by 5 meters by 3 meters) at 22° Celsius and 1 atmosphere.

Data:

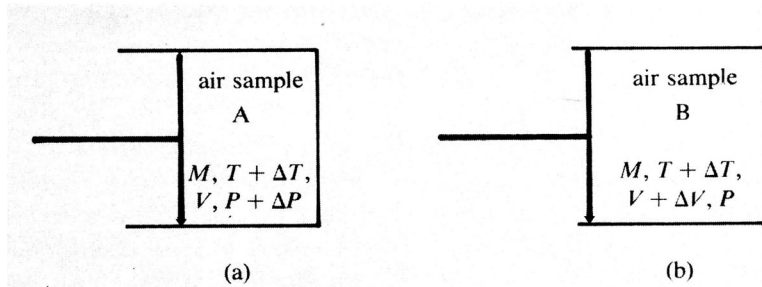
$$C = 0.18 \text{ Cal/deg/kg}; V = 60 \text{ m}^3; P = 10^5 \text{ newtons/m}^2$$

Solution: Find the mass of air from Fig. 16.2 and the energy from Eq. 16.8.

$$M = 1.2 \text{ kg/m}^3 \times 60 \text{ m}^3 = 72 \text{ kg}$$

$$\begin{aligned} E &= CMT = 0.18 \text{ Cal/deg/kg} \times 72 \text{ kg} \times 22 \text{ deg} \\ &= 280 \text{ Cal} \end{aligned}$$

Figure 16.16 (to right) Two air samples that were in equal initial states (same values of M , T , V , P) and were heated (Fig. 16.15). (a) Sample A was heated at constant volume. The temperature, the pressure, and the energy increased. (b) Sample B was heated at constant pressure. The temperature, the volume, and the energy increased.



the law of energy conservation that the isothermal expansion of sample A to bring it to the same volume and pressure as sample B requires no change of the energy stored in the sample. The same is true for the isothermal compression of sample B to bring it to the same volume and pressure as sample A.

Interaction with a heat reservoir. If this conclusion seems abstract and unreal to you, remember this: to maintain the air samples at one fixed temperature during the isothermal expansion or compression, you have to place them in contact with an environment that has been adjusted to be at the same temperature as the samples (Fig. 16.17). The environment must also be maintained at that temperature, for example by means of an external thermostat and by actively supplying and removing heat. Such an environment is called a *heat reservoir*. By supplying

Arrows to represent energy transfer in Figs. 16-17 to 16-23.

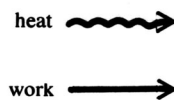
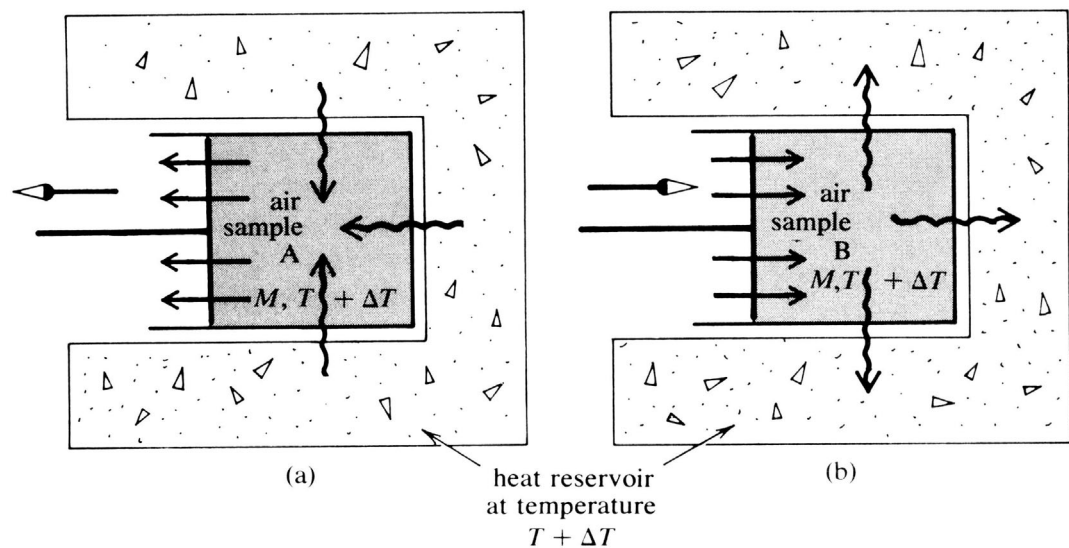


Figure 16.17 Isothermal expansion and compression of gas samples.

(a) Air sample A acts as a passive coupling element while it expands at the temperature $T + \Delta T$. Heat input equals work output.

(b) Air sample B acts as a passive coupling element while it is being compressed at the temperature $T + \Delta T$. Work input equals heat output.



heat when the air samples cool slightly and by absorbing heat when the air samples become too warm, the heat reservoir keeps them at their original temperature. The interaction with the heat reservoir ensures heat supply or heat loss that just compensates for the work done by or on the air sample during the isothermal process.

An air sample being expanded or compressed under isothermal conditions is therefore quite different from a spring: it does not store elastic energy, even though it springs back after it is compressed. Instead of storing energy, the air sample acts only as a passive coupling element between the system acting on the piston and the heat reservoir.

Identity of "elastic" energy and thermal energy of a gas. How does a gas sample store "elastic" energy? To find the answer, you must imagine an expansion or compression process during which the gas is insulated and does not exchange heat with a heat reservoir. Then all the work done on the gas sample while it is being compressed becomes energy that is stored in the gas. Since the insulated gas cannot give off heat, its temperature rises so that its thermal energy increases by an amount equal to the work. Hence, you may conclude that the gas stored the work done to compress it in the form of thermal energy.

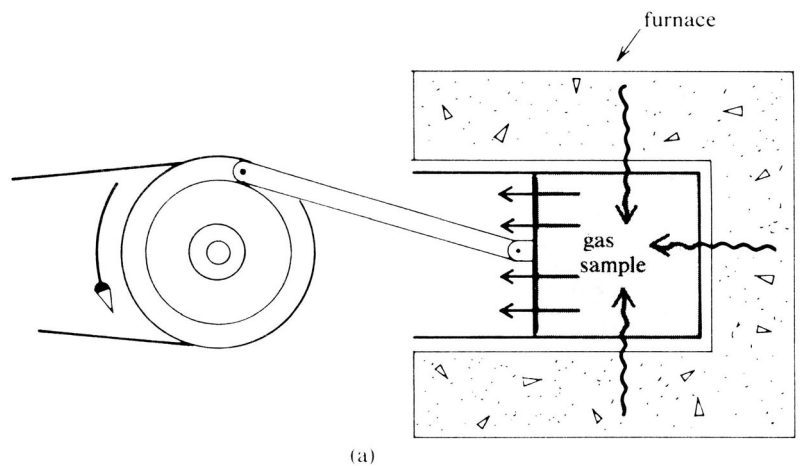
The reverse happens when an insulated gas sample is allowed to expand. Now the gas does work and its energy must decrease. Since it cannot absorb heat from the heat reservoir it becomes colder, so that its thermal energy decreases by an amount equal to the work. Again you may conclude that the thermal energy of a gas sample may be transferred in the form of work. In this respect also, a gas is quite different from matter in the solid or liquid phases, whose thermal energy is always transferred in the form of heat.

16.4 Heat engines and refrigerators

In the discussion of energy storage by gases in the previous section, we explained that a gas confined in a cylinder is a coupling element between a heat source and a piston. Consider now a gas confined at a constant pressure. If the heat reservoir is hotter than the gas, the gas pressure increases slightly so that the gas expands and converts its heat input to work output (Fig. 16.17a). If the reservoir is colder than the gas, the gas pressure decreases slightly, and work input is converted to heat output (Fig. 16.17b). This is the key property that makes gases so useful in heat engines.

We will now describe a working model for a cyclic heat engine. The model will illustrate the essential features of real heat engines, but will not include a correct treatment of many side effects, such as friction between the moving parts, heat losses to the surroundings, and so on.

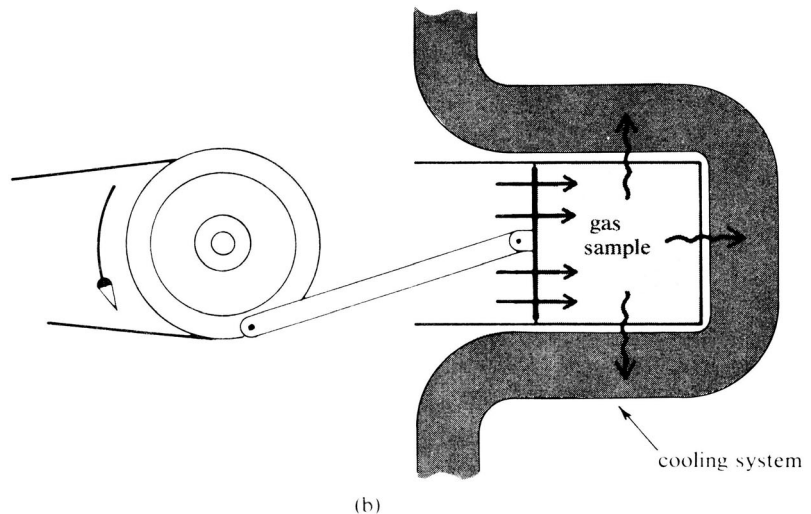
Two basic requirements that a heat engine must satisfy are that it deliver work in proportion to the heat input and that it be able to operate over a long period of time. These requirements are met by the cyclic operation of a piston in a gas-filled cylinder. The piston moves in a repeating cycle out of and into the cylinder, as the gas is alternately



(a)

Figure 16.18 Diagram of the working model for a cyclic heat engine showing two stages in the cycle,

(a) Expansion part of the cycle. The gas is heated by the furnace,



(b)

(b) Compression part of the cycle. The gas loses heat to the cooling system.

heated by being placed in a furnace and cooled by being placed in contact with a cooling system (Fig. 16.18). The piston is connected to the machinery by a shaft that converts the oscillating motion to a rotary motion. The working model for a heat engine therefore must include four major parts: a gas, a furnace, a cooling system, and a mechanical linkage (usually a piston) on which the gas can do work (Fig. 16.19).

The operating cycle. The principle of cyclic operation assures that the engine can operate for a long period, but it appears to suffer from one serious drawback. It seems that the work done during the expansion part of the cycle (Fig. 16.18a) may have to be returned when the motion of the piston is reversed during the compression part of the cycle (Fig. 16.18b). The net result would be some energy transfer from furnace to cooling system, but zero work output.

Fortunately, the properties of gases make it possible to eliminate this drawback. A gas sample at low temperature exerts a smaller pressure

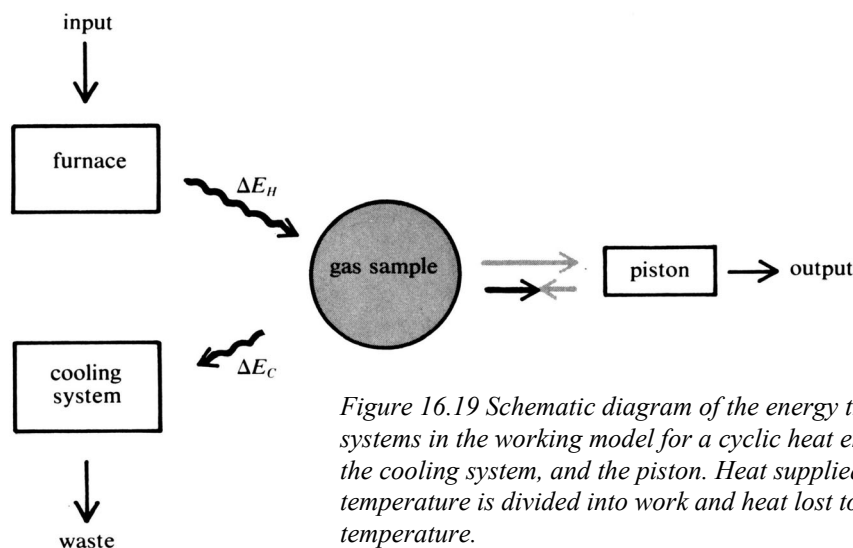


Figure 16.19 Schematic diagram of the energy transfer among the four subsystems in the working model for a cyclic heat engine: the gas, the furnace, the cooling system, and the piston. Heat supplied by the furnace at a high temperature is divided into work and heat lost to the cooling system at a low temperature.

than the same gas sample at high temperature. The work done is proportional to the pressure (Eq. 16.7). It is customary, therefore, to operate the heat engine in such a way that the gas is cold during compression and hot during expansion. Even though some work has to be returned from the machinery to compress the cold gas while it loses heat to the cooling system, this is less than the work that was done by the hot gas during the expansion part of the cycle. There is a net transfer of energy to the machinery.

The energy transfer in the operation of the working model is indicated in Fig. 16.19. Three subsystems of the engine interact by means of the gas, which is a fourth subsystem serving as coupling element among the other three. Combustion of a fuel provides energy input to the furnace. Thermal energy is transferred from the furnace, which heats the gas during the expansion stage of the cycle, while the gas is doing work on the piston. Thermal energy is transferred to the cooling system, which reduces the gas temperature during the compression stage of the cycle, while work is being done on the gas by the piston. The work returned to the gas during the compression stage is less than the work done by the gas during the expansion stage because the cold gas has a lower pressure than the hot gas. The thermal energy reaching the cooling system, however, cannot be used to heat the gas during the expansion stage, because the cooling system is at a much lower temperature than the furnace. This quantity of energy, therefore, has lost its practical usefulness in the heat engine and is indicated as "waste" in Fig 16.19. The waste of one part of the energy is the price paid for the successful conversion of another part of the energy from heat to work.

Equation 16.9

heat from furnace	ΔE_H
heat to cooling system	ΔE_C
net work	W

$$\Delta E_H = W + \Delta E_C \quad (a)$$

$$\text{efficiency} = \frac{W}{\Delta E_H} \quad (b)$$

Efficiency. The *efficiency* of a heat engine is the ratio of the work output to the thermal energy input. Even though energy is conserved, the efficiency is less than 100% because the energy transferred to the cooling system is not part of the useful work output (Eq. 16.9).

Applications. Practical heat engines, such as the steam engine, internal combustion engine, and gas turbine, are all based on the same principles

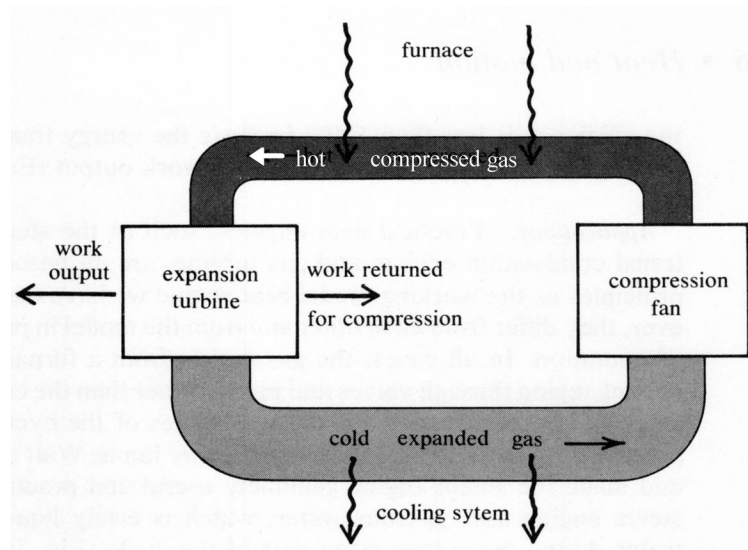
as the working model heat engine we have described (Fig 16.18). However, a heat engine built along the lines of our model inevitably wastes a substantial amount of energy because the entire cylinder must be heated and cooled for the two stages of the cycle. This was in fact the way that the first useful steam engine operated: the Newcomen engine was invented in about 1716 to pump water out of an English coal mine. It was a noisy, dirty, inefficient monster the size of a building, but it could pump water faster and at less cost than horses or humans could, and many coal mines acquired Newcomen engines during the 1700s. As you would expect, efficiency was not critical because of the abundance of fuel at a coal mine. The last working Newcomen engine was removed from active service in 1834.

James Watt's innovations. James Watt (1736-1819) addressed the problem of trying to make the steam engine more efficient and smaller in size. Watt eliminated the need to alternatively cool and heat the cylinder. He invented an engine in which, immediately after the expanding steam had pushed out the piston, a valve opened. This allowed the steam into a separate "condenser" cylinder, where it was further cooled and condensed to liquid water. This innovation meant that the main cylinder could be kept hot and the condenser kept cool at all times. This resulted in a substantial increase in the amount of work produced for every ton of coal burned. This breakthrough, supplemented by Watt's many other inventions, was the key to the vast proliferation and improvement of the steam engine.

Practical heat engines. Modern heat engines (for example, the internal combustion engine and the steam and gas turbine) differ from each other and from our ideal model in practical details of operation. In all cases, as in Watt's engine, the heating and cooling processes are separated in some way. The working fluid is heated to convert it to a gas and increase the pressure, then allowed to expand and do work, and, finally, condensed (or compressed) back to a liquid. Thus, the steam engine employs water that is first converted to steam in the boiler and then liquified during the compression part of the cycle. On the other hand, the internal combustion engine (automobile engine) uses a mixture of fuel and air as the gas in the cylinder, and the combustion takes place in the cylinder itself. This arrangement does away with the need for a separate furnace. The mixture of fuel and air is drawn into the cylinder, the fuel burns and expands, and the hot, expanded gas leaves as exhaust at a lower (atmospheric) pressure during each complete engine cycle.

In a steam turbine, high-pressure steam expands against a set of turbine blades arranged around a shaft and force the shaft to rotate. Finally, in a gas turbine there are two sets of turbine blades replacing the cylinder and piston. One set of blades acts like a fan and compresses the cold gas; the other set of blades is propelled by the pressure of the hot, exploding gas pushing against the blades. Because turbines rotate, they operate very smoothly and do not share the vibration caused by the back-and-forth motion of the piston in the steam and combustion engines. The cyclic

Figure 16.21 Diagram of a cyclic heat engine using turbines rather than pistons for expansion and compression of the gas. The work done on expansion is partly used for compression and partly becomes useful output.



flow of the gas in a turbine engine with a furnace is schematically shown in Fig. 16.21.

Refrigeration cycles. Refrigerators and air conditioners operate on the same principles as heat engines, but the action is reversed. A schematic diagram of a practical refrigerator is shown in Fig. 16.22. The coupling element is, for the sake of efficiency, not a gas but a liquid with a very low boiling temperature (about 0° Celsius), which can vaporize to form a gas. The liquid vaporizes in the evaporator at a low pressure and absorbs the heat of vaporization from its surroundings (the interior of the refrigerator), which are thereby cooled. The gas flows to

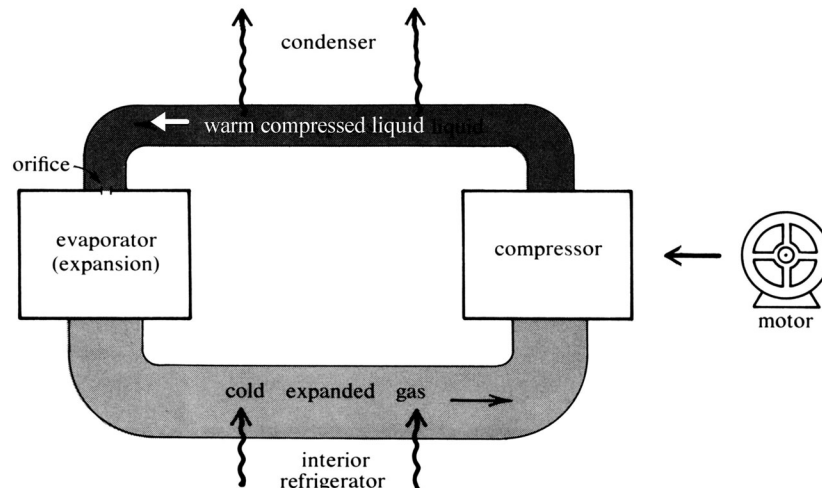


Figure 16.22 Fluid cycle in a refrigerator. Energy is transferred from the cool refrigerator interior to the warm condenser outside.

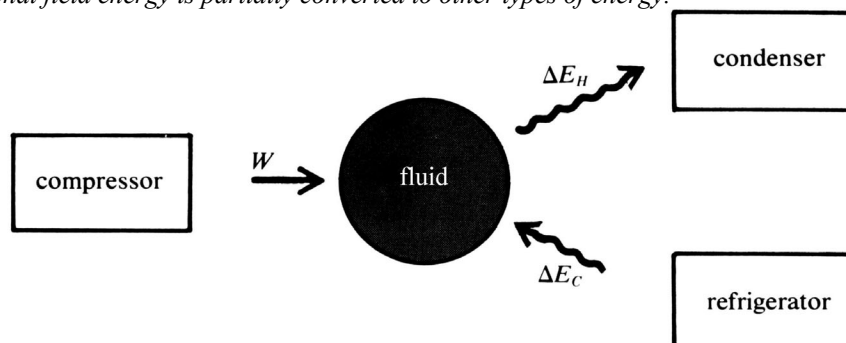
a compressor (built somewhat like a turbine) in another region of the refrigerator, where it is liquefied. The heat of vaporization, which must be removed here, is transferred in the condenser to cooling water or to air circulating around the condenser. The cooled liquid now returns to the evaporator where it expands and vaporizes once more. The net effect of the cycle is to transfer thermal energy from the environment of the place where the liquid vaporizes to the environment of the condenser where the gas liquefies. Again, three subsystems interact by means of a fluid that acts as coupling element: the inside of the refrigerator at a low temperature, the condenser at a higher temperature, and the compressor motor, which does work on the fluid (Fig. 16.23). Note that the refrigeration engine does not operate on the heat input from the refrigerator interior, but must be driven by work input to the compressor.

16.5 The kinetic theory of gases

Historical background. Newton and his contemporaries speculated about the possible micro-domain models for gases. In one model proposed by Newton, gases were composed of particles subject to a repulsive interaction-at-a-distance. This interaction spread the particles apart so they filled the entire volume of their containers and exerted a pressure on the container walls. In a second model, gases were composed of compressible particles, like fluffs of wool, touching each other. In a third model, the particles did not touch at all times, but they were in violent agitation, whirled throughout the available space within a turbulent but "subtle" fluid.

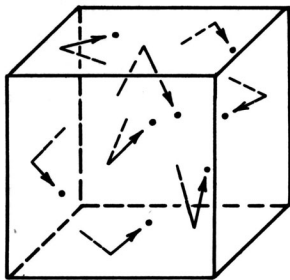
All of these particle models have some useful features, but all of them suffer shortcomings, and none of them can explain the mathematical ideal gas model of Eq. 16.4 and Table 16.3. Since the data on which the ideal gas model is based were not discovered until the end of the eighteenth century, the physicists prior to that date did not have the same reasons for eliminating their models as you have now.

Figure 16.23 Diagram of energy transfer among the four subsystems in a refrigeration cycle. Work supplied by the compressor "lifts" heat "uphill" from the cool refrigerator to the warm condenser. The fluid at higher temperature has more energy, and thus energy must be added (via work from the compressor) to raise the temperature. A heat engine (Fig. 16.19) represents the opposite process, in which heat "falls" from high to low temperature, and the engine converts some of it to external work. Thus you can think of a heat engine as similar to a waterwheel in an old-fashioned mill, where water "falls" in a controlled way, thus turning the wheel and doing useful work. The water's gravitational field energy is partially converted to other types of energy.



By the middle of the nineteenth century, practical knowledge and theoretical interpretations had progressed to the point where the decisive next step could be taken. The observation by Rumford that energy of motion could produce heat through friction and the measurement of the work done when a gas was heated at constant pressure led Julius Robert Mayer (1814-1878) to propose that energy is indestructible, but convertible from one type to another. James Joule thereupon constructed a micro-domain particle model for gases in which thermal energy of the gas was kinetic energy of the particles. Most significantly, Joule was able to show that his model permitted a calculation of the speed of the particles, led to Boyle's law, and gave new meaning to the concept of "absolute zero." In fact, other scientists had described such calculations during the preceding century, but their work had been ignored because it went against the accepted caloric theory (see Section 10.5).

According to Joule's model, a gas is composed of small, rapidly moving particles that do not interact with one another except for occasional collisions. Thermal energy of the gas is kinetic energy of the particles. The collisions between two particles are perfectly elastic; that is, the combined kinetic energy of the two colliding particles is the same before and after the collision. No energy is transferred to other types during a collision because the model provides for no other types of energy storage. The particles also experience elastic collisions with the container walls and exert a force on it by virtue of their impact. Each gas is composed of characteristic particles different in mass from the particles of other gases. This model is the basis for the kinetic theory of gases.



Qualitative properties of the particle model. This model explains qualitatively many phenomena:

Why do gases expand to fill the container they occupy? Because the particles travel until they collide with a container wall,

Why gases have a low density and are compressible? Because there is a great deal of empty space between the particles.

Why does gas pressure gives rise to forces on the walls? Because the particles striking and bouncing off the wall suffer changes of momentum.

Why does reducing the volume increase the gas pressure on the walls? Because the particles are crowded together so that they strike a particular segment of wall area more frequently.

Why do different gases diffuse rapidly through one another and mix completely? Because the particles' motion carries them into the empty spaces among the other particles.

The energy stored in the model gas is kinetic energy of the particles. Since the particles do not interact with one another, kinetic energy is the only type of energy storage possessed by the system of particles making up the gas. A crucial feature of the model, therefore, is the *particle velocity*, which determines both the momentum and the kinetic energy.

Derivation of Boyle's law. In the particle model, the pressure exerted by the gas is caused by the impacts of the particles with the container

Equation 14.1b

force	\mathbf{F}_{av}
mass	M
momentum change	$\Delta \mathcal{M}$
time interval	Δt

$$\mathbf{F}_{\text{av}} = \frac{\Delta \mathcal{M}}{\Delta t}$$

wall. As a particle strikes the wall and recoils, its momentum changes. The force exerted by the particles on the wall is equal in magnitude, and opposite in direction, to the force exerted by the wall on the particles. The net force acting on the steady stream of particles striking the wall is, according to Newton's second law, equal to the change of momentum of the particle stream divided by the time interval during which the particle stream interacted with the wall (Eq. 14.1b).

To avoid the mathematical difficulties of this calculation, we will make a still simpler particle model for gases, even though this model is in contradiction with readily observed properties of real gases. This approach, which was also used by Joule, illustrates that it is frequently more productive to use an inadequate model whose consequences can be evaluated rather than to use an adequate model that is too complicated to be exploited. Our model has the following properties.

1. The gas is in a cubical container.
2. The particles do not collide with one another at all.
3. All particles move with a speed v . Their total mass is M .
4. One sixth of the particles move at right angles toward each face of the cubical container (Fig. 16.24).

Equation 16.10

$$PV = \frac{1}{3} Mv^2 = \frac{2}{3} (\text{KE})$$

The consequences of this model for the pressure-volume relation are stated in Eq. 16.10. The force due to the particle impacts is proportional to the momentum of the particles (mass times speed) and to the frequency of the impacts (again the speed), whose combined effect gives the speed to the second power. The result is that the product of pressure times volume equals two thirds of the kinetic energy of the particles. It is therefore possible to determine the speed of the particles from the ideal gas model (Table 16.3), which was based on pressure-volume data measured by ordinary techniques (Example 16.7). The results are in excellent agreement with directly measured particle speeds.

Equation 16.11

$$\begin{aligned} \text{KE} &= 1.5PV & (a) \\ &= 420M(T + 273) & (b) \end{aligned}$$

Interpretation of temperature. By comparing Eq. 16.10 with the ideal gas model for air (Eq. 16.4), you can see that the total kinetic energy of the air particles is directly related to the temperature (Eq. 16.11). In fact, it is possible to show that the factor $(T + 273)$ is directly proportional to the average kinetic energy of one gas particle, with the same

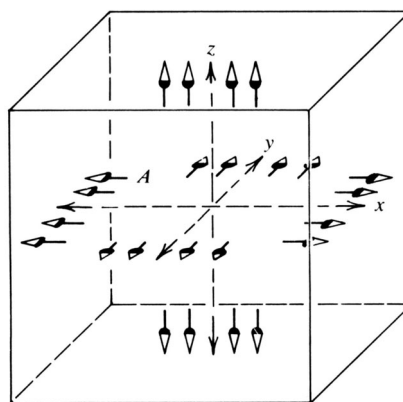


Figure 16.24 The particles move only parallel to the cube edges, and perpendicularly to the cube faces. One sixth of the particles move toward each cube face. The velocity of particle A is $(-v, 0, 0)$.

constant of proportionality for all gases. The actual value of the constant of proportionality requires knowledge of the mass of one gas particle, which was only determined at the beginning of the twentieth century.

This, then, is the micro-domain interpretation of temperature: average kinetic energy of a gas particle. The model thereby explains what happens at a temperature of -273° Celsius: the average kinetic energy and therefore the speed of the particles is zero, so that the gas exerts zero pressure and has no "spring" at this temperature.

Thermal energy. The second principal prediction of the model concerns the specific heat of gases. If the thermal energy is equal to the kinetic energy of all the particles, as was claimed, then the mathematical model relating kinetic energy to temperature should be similar to the model for thermal energy. The result shows good agreement for helium, except that the two formulas clearly refer to different thermal reference states (Example 16.8). In the particle model, the reference state is -273° Celsius (absolute zero) where the stored energy is zero Calories. The thermal energy determined according to the operational definition (Chapter 9) is measured from the arbitrary reference state at 0° Celsius. The specific heats in both models, however, are equal to 0.75 Calorie per degree per kilogram.

For gases other than helium, the agreement is not as good as it is for helium (Example 16.9). The specific heat derived from their thermal energy is greater than the specific heat derived from their kinetic energy. This discovery led to improvements in the model and ultimately to good agreement with the data.

EXAMPLE 16.7. Find the speed of oxygen particles at 0° Celsius.

Solution: Combine Eq. 16.10 with the ideal gas model for oxygen in Table 16.3.

$$\begin{aligned} PV &= \frac{1}{3} Mv^2 = 260M(T+273) \\ v &= \sqrt{\frac{3 \times 260(T+273)}{M}} = \sqrt{(3 \times 2.6 \times 10^2 \times 2.7 \times 10^2)} \\ &= \sqrt{21 \times 10^4} \approx 4.6 \times 10^2 = 460 \text{ m/sec} \end{aligned}$$

This rather high speed, almost 1000 miles/hr, may seem unreasonable, but actual measurements have confirmed it! How does it compare with the speed of sound? Would gas particles moving this fast also be able to transmit sound? This result depends only on the temperature, not on the pressure of the oxygen, its volume or the mass of one particle.

EXAMPLE 16.8. Find the combined kinetic energy of all helium particles in 1 kilogram of helium gas at the temperature T and compare it with the thermal energy.

Solution:

From Eq. 16.10 and Table 16.3,

$$\text{KE} = 3100M(T + 273) \text{ joules} = 0.75M(T + 273) \text{ Cal}$$

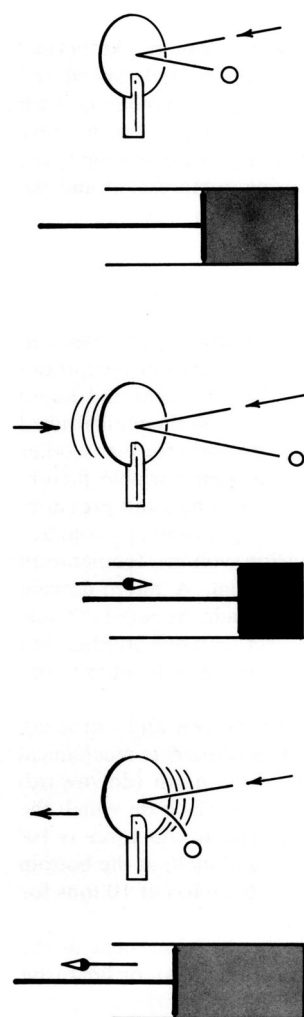
From Eq. 16.8 and Table 16.4,

$$E = CMT = 0.75MT \text{ Cal}$$

Comment. Rewrite the kinetic energy formula as follows:

$$KE = (205M + 0.75MT) \text{ Cal}$$

The first term is the combined kinetic energy at 0° Celsius; the second term is the change in kinetic energy when the helium temperature is raised or lowered from 0° Celsius to the temperature T. The second term is equal to the thermal energy of a system as defined in Section 10.2.



EXAMPLE 16.9. Find the combined kinetic energy of all air particles in 1 kilogram of air at the temperature T and compare it with the thermal energy.

Solution:

From Eq. 16.11b,

$$KE = 420M(T + 273) \text{ joules} = 0.10M(T + 273) \text{ Cal}$$

From Eq. 16.8 and Table 16.4,

$$\text{Thermal Energy} = E = CMT = 0.18MT \text{ Cal}$$

Comment. Rewrite the kinetic energy formula as follows:

$$KE = (27M + 0.10MT) \text{ Cal}$$

The second term here is less than the thermal energy. The first term is greater than the thermal energy so long as $0.18T$ is less than 27, or T is less than $(27/0.18) = 150^\circ\text{C}$.

Application. With the theory of this section, you can understand qualitatively the particle mechanism whereby the gas temperature increases while a gas is being compressed. Consider the gas in a cylinder with a movable piston. The cylinder and the piston are thermally insulated and are at the same temperature as the gas. The particles strike all the walls and recoil with the same speed with which they impinged on the walls. Now, while the gas is being compressed, the piston moves forward and the particles recoil with an increased speed, just as a ping-pong ball recoils with an increased speed when it is struck by an advancing paddle. The increased speed implies an increased kinetic energy and, therefore, an increased temperature. While the gas is being compressed, therefore, the temperature rises. After the piston stops moving, incident and recoil speeds are once more equal and the temperature remains at its elevated value.

While the piston is being pulled out, the particles striking the piston recoil with a reduced speed, just as a ping-pong ball recoils with reduced speed when it strikes a receding paddle. The decreased speed implies a decreased temperature. After the piston stops moving, the temperature again remains at its reduced value. Please note that this

TABLE 16.5 MANY-INTERACTING-PARTICLES (MIP) MODEL FOR GASES

<i>Micro domain</i>	<i>Macro domain</i>
<i>many particles</i>	<i>gas</i>
<i>total mass of particles</i>	<i>mass of gas</i>
<i>particle impacts with the walls</i>	<i>pressure</i>
<i>volume of container</i>	<i>volume of container</i>
<i>kinetic energy per particle</i>	<i>temperature</i>
<i>particle speed</i>	-
<i>particle collisions</i>	<i>heat flow</i>

Equation 16.4 (Ideal gas model for air)

$$\begin{aligned}
 \text{pressure (newtons/m}^2\text{)} &= P \\
 \text{volume (m}^3\text{)} &= V \\
 \text{mass (kg)} &= M \\
 \text{temperature (deg C)} &= T
 \end{aligned}$$

$$\begin{aligned}
 PV &= 280 M (T + 273) \\
 &\text{(for air)}
 \end{aligned}$$

model includes no collisions at all between the gas particles. At thermal equilibrium, the gas temperature is related to the particle speed and not to the frequency of particle collisions with one another or with the wall. The collisions operate only to *transfer* energy, not to store it. Thus, the collisions of particles with the moving piston are important only to bring about the temperature rise during compression and the temperature drop during expansion.

Summary

The gas phase of matter differs from the solid and liquid phases in several significant ways. Gases have lower density, greater compressibility, and show greater thermal expansion than do solid and liquid materials. These properties are described by the mathematical ideal gas model for air (Eq. 16.4), which has the same form for many other gases as well. The ideal gas model relates the four variable factors that describe a gas sample: mass, temperature, volume, and pressure. The gas density (mass-volume ratio), the compressibility (volume-pressure relationship, Boyle's Law), and the thermal expansion (volume-temperature relationship, Charles' Law) may all be inferred from this model. A micro-domain particle model for gases can explain several of their properties (Table 16.5). The particles in the model do not interact with one another, but move at a high speed and collide (interact) with the walls of the container that confines the gas.

The most common gas is air (a mixture of oxygen and nitrogen), which surrounds the planet earth. The air is in approximate mechanical equilibrium subject to the force of gravity of the earth (downward) and the force of support exerted by the earth's surface on which the air rests (upward). The air pressure that gives rise to this force is 10^5 newtons per square meter (14.7 pounds per square inch) at the bottom of the atmosphere. This pressure, which corresponds to a mass of 10 tons for the column of air resting on 1 square meter, is known as one atmosphere, and it can support a column of mercury approximately 0.76 m (30 inches) in height.

Gas samples may exchange energy with their environment in the form of either work (the gas is compressed or expands), or heat (the

Equation 16.7

$$\begin{aligned} \text{work (joules)} &= W \\ \text{pressure (newtons /m}^2\text{)} &= P \\ \text{volume change (m}^3\text{)} &= \Delta V \end{aligned}$$

$$W = P\Delta V$$

Equation 16.8

$$\begin{aligned} \text{thermal energy (Cal)} &= E \\ \text{specific heat} &= C \\ \text{(Cal/deg C/kg)} &= C \\ \text{temperature (deg C)} &= T \\ \text{mass of gas (kg)} &= M \end{aligned}$$

$$E = CMT$$

gas is colder or warmer than its environment), or both at the same time. The work done by the gas is equal to the gas pressure times the volume change (Eq. 16.7). No matter how the energy transfer is accomplished (heat or work), a gas stores energy *only* in the form of thermal energy (Eq. 16.8). Cyclic heat engines and refrigerators are mechanical devices that use a gas as a coupling element to accomplish desired energy transfer. In a heat engine, thermal energy from a furnace at high temperature is transferred partly to mechanical energy of machinery and partly to thermal energy of the cooling system at low temperature. A refrigerator transfers thermal energy from the interior at low temperature to the exterior at a higher temperature, but some mechanical energy must be supplied by the motor that operates the refrigerator.

List of new terms

pressure	heat reservoir
atmosphere (unit of pressure measurement)	heat engine
barometer	efficiency
compressibility	refrigeration cycle
ideal gas model	

List of symbols

F	force	Δs_F	displacement component along force
P	pressure	T	temperature
A	area	C	specific heat
V	volume	M	mass
ΔV	volume change	$\Delta \mathcal{M}$	momentum change
W	work	Δv	velocity change
		v	speed

Problems

1. Carry out library research to trace the development of heat engines from the time of James Watt to the present.
2. Describe the impact on western civilization of the invention and development of heat engines.
3. Obtain a plastic syringe and explore the properties of the air in the syringe (Fig. 16.3).
4. Tire pressure gauges measure the excess of the pressure in the tire over the pressure of the atmosphere outside the tire.
 - (a) What is the actual air pressure in an automobile tire?
 - (b) The gauge shown in Fig. 16.8 measures the actual pressure. Modify the diagram so that the gauge measures the excess pressure.

5. Either the theory of atmospheric pressure or the theory that "nature abhors a vacuum" (suction) can explain many everyday phenomena.
 - (a) Select three or more everyday phenomena that illustrate these theories and explain them from both points of view.
 - (b) Explain the concept of suction by both theories.
 - (c) Which of these theories is closer to your own common-sense approach? Explain briefly.
6. Formulate one or two operational definitions of gas pressure.
7. The ocean floor must support both the ocean water and the atmospheric air. The pressure on the ocean floor and other objects under water is therefore greater than atmospheric pressure.
 - (a) Find the pressure at a depth of 30 meters (100 feet) under the surface of the water.
 - (b) Find the force acting on 1 square centimeter of submarine hull 30 meters under water.
 - (c) Find the force acting on 1 square centimeter of ocean bottom under 10^4 meters of water. (The bottom of the deepest ocean trench, the Marianas Trench near the Phillipines, is almost 11,000 m., or about 7 miles, below sea level. In comparison, the peak of the highest mountain, Everest, is only about 8800 m above sea level.)
8. Observe by approximately how many meters your altitude above sea level must change suddenly to create an uncomfortable feeling in your ears. Estimate the force on your eardrum that causes your discomfort, treating the eardrum as a circle 0.006 meter (1/4 inch) in diameter. (Hint: Use the data on air density, Fig. 16.2. Find the sudden change of the mass of air in a column "resting" on your eardrum.)
9. Estimate the average atmospheric pressure in Denver, which is 1600 meters above sea level.
10. Use your mathematical model for atmospheric pressure (from Problem 9) to estimate the pressure at the peak of Mount Everest and compare it with the value stated at the end of Section 16.2. Comment on any discrepancies between the two.
11. Give a critical discussion of the analogy between a gas and an elastic system.
12. Compare Boyle's law with Hooke's law and point out qualitative similarities and differences. Mention extreme examples of deformations in your discussion and compare the laws' limitations.

13. The "molecular weight" is a useful concept in chemistry and for the analysis of gases. State the ideal gas model in a form in which the mass of gas is expressed in "molecular weight units." The molecular weights of the five gases in Table 16.3 are: carbon dioxide (44); helium (4); hydrogen (2); nitrogen (28); oxygen (32).
14. Use the results of Problem 13 to estimate the "molecular weight" of air. Interpret the result in light of the knowledge that air is a mixture of oxygen and nitrogen. (The calculation has low accuracy because the numerals in Eq. 16.4 and Table 16.3 have been rounded off to two digits.)
15. (a) Use the ideal gas model to estimate the mass of 1 cubic meter of air at the peak of Mount Everest, which is about 8800 m above sea level.
(b) What are the implications of this result for the operation of internal combustion engines or of human beings at the top of Mount Everest?
16. Discuss the concept of "elastic" energy of a gas.
17. Work is done on a gas while it is being compressed at a fixed temperature. What system acts as energy receiver in this process?
18. Use the data in Table 16.4 and the ideal gas model (Table 16.3) to answer the following questions.
(a) How much heat must be supplied to 1 kilogram of hydrogen gas when it is warmed by 1 degree Celsius at constant pressure?
(b) Do the same for helium gas.
19. Estimate the increase of thermal energy stored in 1 cubic mile of air when it is heated from 50° Fahrenheit to 100° Fahrenheit. (In metric units: 4 cubic kilometers, from 10° Celsius to 38° Celsius.)
20. (a) Apply the result of Problem 19 to a 1-mile-thick air layer over the Central Valley of California (approximate dimensions: 400 miles long, 120 miles wide) to find the air's thermal energy increase on a hot summer day.
(b) Solar energy reaches the earth at the rate of approximately 1/3 Calorie per square meter per second. This numerical value is called the solar constant. How long would it take for solar energy to raise the temperature of the air over the Central Valley from 50° Fahrenheit to 100° Fahrenheit if all the solar energy is transferred to the air layer 1 mile thick? Interpret the result of your estimate.
21. Devise an experiment to measure the solar constant approximately (see Problem 20b).

22. Do you think the air being heated by the sun (Problems 19 and 20) is heated at constant pressure, at constant volume, or under still other conditions? Explain your answer.
23. When 1 kilogram of water is heated from 10° to 30° Celsius, it expands by 4×10^{-6} cubic meters. Compare the work done by water when it is heated from 10° to 30° Celsius with the work done by air when it is heated by the same amount.
24. Describe some energy sources used by man for the performance of work prior to the invention of the steam engine (other than manual and animal labor).
25. Select two cyclic heat engines from your everyday experience and identify the "furnace," the cooling system, and what happens to the "output" in each one (refer to Fig. 16.19).
26. Describe what happens to the waste heat in the following situations: automobile engine; electric power plant in your region; air conditioner.
27. Waste heat is sometimes said to give rise to "thermal pollution" in the environment of a power plant or factory. Describe some possible undesirable effects of "thermal pollution."
28. Comment on some shortcomings of the three gas models proposed in Newton's time (first paragraph, Section 16.5).
29. Discuss the three gas models described in the first paragraph of Section 16.5 with respect to their explanation of the thermal and elastic energies of gases. (Note: the conclusion of Section 16.3 about the identity of thermal and elastic energies dates to the nineteenth century.)
30. In what respect are the qualitative properties of Joule's particle model for gases superior to the models proposed in Newton's time?
31. (a) Find the speed of air "particles," hydrogen "particles," helium "particles," and carbon dioxide "particles" at room temperature.
(b) Compare the particle speeds you have calculated with the speed of sound in these gases as listed in Table 7.1.
(c) Comment on the results of the comparison in (b) by relating the particle model for gases to the wave theory of sound.
32. (a) Calculate the total kinetic energy of the "particles" in 1 kilogram of helium at room temperature.
(b) How does this energy compare with the thermal energy of 1 kilogram of helium gas, if you use absolute zero as the reference temperature for thermal energy?

- (c) How does the particle model explain the fact that the thermal energy of helium gas at room temperature does not depend on the pressure or the volume of the gas?
33. Solve Problem 32 for nitrogen gas instead of helium.
34. (a) Calculate the specific heats predicted by Joule's model for hydrogen, nitrogen, and carbon dioxide.
(b) Compare the specific heats in (a) with those listed in Table 16.4 and point out which gas has the largest and which the smallest percentage discrepancy.
35. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

Bibliography

- M. Born, *The Restless Universe*, Dover Publications, New York, 1951.
- S. G. Brush, *Kinetic Theory, Volume I: The Nature of Gases and of Heat*, Pergamon, New York, 1965.
- P. W. Bridgman, *The Nature of Thermodynamics*, Harper and Row, New York, 1961.
- D. Cassidy, G. Holton, J. Rutherford, *Understanding Physics*, Springer Verlag, 2002.
- T. G. Cowling, *Molecules in Motion*, Hutchinson, London, 1950.
- H. M. Leicester and H. S. Klickstein, *A Source Book in Chemistry*, McGraw-Hill, New York, 1952.
- J. F. Sandfort, *Heat Engines*, Doubleday, Garden City, New York, 1962.
- Articles from *Scientific American*.** Some or all of these, plus many others, can be obtained on the Internet at <http://www.sciamarchive.org/>.
- L. Bryant, "The Origin of the Automobile Engine" (March 1967).
- W. Ehrenberg, "Maxwell's Demon" (November 1967).
- J. F. Hogerton, "The Arrival of Nuclear Power" (February 1968).
- M. B. Hall, "Robert Boyle" (August 1967).
- G. A. Hoffman, "The Electric Automobile" (October 1966).
- J. W. L. Kohler, "The Stirling Refrigeration Cycle" (April 1965).