Chapter 5: Models for Light and Sound

The nature of light and sound has intrigued mankind over the centuries. Light and sound are connected with sight and hearing and are therefore vital sources of information about our environment as well as essential for survival. The control of sound has led to spoken language and music. The use of light has led to written language and the visual arts. Hearing and seeing, sound and light, enable us to communicate with one another and to derive pleasure from the natural world as well as from music and art.

5.1 Properties of light and sound

Primary sources. We tend to associate light and sound because both are important for sense perception through interaction-at-a-distance. In this way sight and hearing are different from touch, taste, and smell: these three latter senses depend on physical contact of the sense organ and the material (solid, liquid, or gaseous) to be sensed. Both light and sound originate in so-called primary sources: a candle flame, a lightning flash, and the sun emit light; a bowed or plucked violin string, a thunderclap, and a croaking bullfrog produce sound. Both light and sound interact with detector systems: light with the human eye, photographic film, or a video camera; sound with the human ear or a microphone. Both light and sound, therefore, function as intermediaries in interaction-at-a-distance.

Information and energy. Sound and light transmit not only information but also significant amounts of energy. The energy the earth receives from the sun maintains the earth's temperate climate and makes possible the photosynthesis of food material by green plants. The sound blast from dynamite explosions (or Joshua's trumpets at Jericho) breaks windows and may even topple buildings. Even though sound and light always transmit some energy, the amounts involved in seeing and hearing are very small. Therefore it is worthwhile to distinguish between situations in which the transmitted information is more important (such as in sense perception) and those in which the energy transferred is of greater interest (such as in photosynthesis, sunlamps, and dynamite blasts).

Reflection. In describing the similarity of sound and light, we mentioned the existence of primary sources for each. Even though you recognize these primary sources, you are also aware of the reflection of sound and light by all the objects in your environment. As a result of this reflection process, you receive light and sound from all directions and thus appear to be bathed in sound and illumination. Reflecting objects may be called "secondary sources" because they do serve as sources, but their action depends on that of the primary sources.

In other words, the primary sources of light and sound lose energy, but the reflecting objects do not. Rather, the reflecting objects often absorb some of the energy that strikes them and reflect only a part of it. The difference in level of illumination in a room with light-colored walls and in one with dark-colored walls is due to the poor reflection, and greater absorption, of light by the dark walls. A similar effect with respect to sound is achieved by adding rugs and drapes to a bare room. The rugs and drapes are poor sound reflectors (and better absorbers) compared to the bare walls, which reflect most of the sound energy that strikes them.

Sound sources. Sound is generated by vibrating or suddenly moving objects, such as a plucked violin string, a bursting balloon, or a jet engine in operation. Sound travels in air and is blown aside by the wind, but it also travels along a stretched string, through water, and along steel rails. Sound does not propagate in the absence of matter. You can recreate a speaking tube, like the one used by a ship's captain to communicate with the engine room, with the help of an air-filled garden hose. You can also put your ear against a table and listen to the sound produced when you scrape the table surface with your fingernail. Sound is transmitted well by almost all materials, but it is not transmitted well from one material to a very dissimilar material, as from a tuning fork to air.

Can you find evidence that most of the sound is transmitted by the air column inside the hose and not by the walls of the hose?

Loudness and pitch. A sound signal has certain recognizable properties, such as its loudness and, in the case of musical notes, its pitch. The loudness of a sound is related to the energy being transferred from the source to the air and then to the ear. You can reduce the loudness of a sound by weakening the primary source, increasing the distance from the source, or placing obstacles near the source. The pitch of a note is not affected by any of these procedures, but it can be altered by a process called "tuning" of the primary source. A violinist, for instance, tunes his instrument by tightening or loosening the strings. A complex sound, which is a mixture of notes, can be changed by nearby reflecting objects because such objects are usually more efficient reflectors of low notes than of high ones. The sound therefore becomes muffled. You are probably familiar with the effect of a long pipe on the sound of the human voice as well as the fact that the sound of a speaker system depends as much on the room surroundings as on the speakers themselves.

Light sources. Light is usually generated by extremely hot objects, such as a candle flame, an incandescent light bulb filament, or the sun, but it may also come from a fluorescent tube or a firefly. Light travels through air, transparent liquids (like water), and transparent solid materials (like glass), but it also travels through interstellar space, where there is no appreciable amount of material present (vacuum). Most solid objects are not transparent and do not transmit light. They may be white or light-colored and thereby act as efficient reflectors. Photographers have to be especially alert to the color of objects surrounding their photographic subject, because these objects act as "secondary light sources" and influence the exposure conditions. Snow or beach sand may result in overexposure, and dark foliage in underexposure, unless precautions are taken.

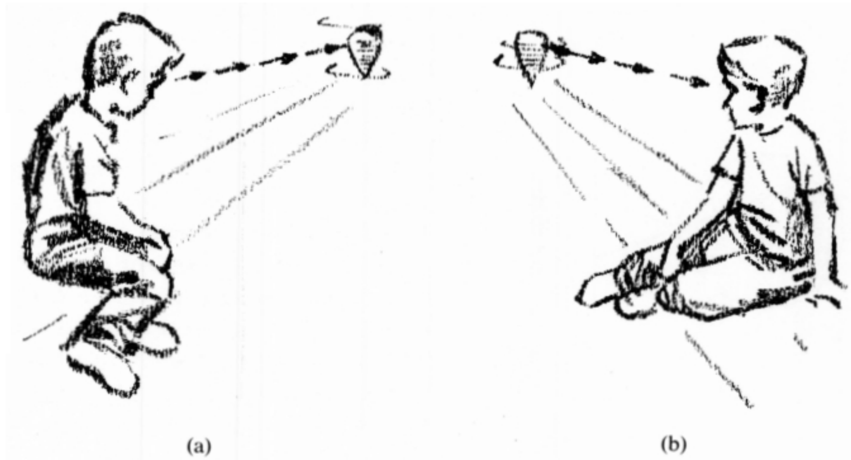


Figure 5.1 Two models for vision.
(a) Child's model that eyes reach out.
(b) Scientific model that light reflected by the object reaches the eye.

Properties of a light signal are intensity and color. As with sound, the intensity is related to the energy being transmitted. You can decrease the perceived intensity of light by weakening the source, going to a greater distance from it, or interposing obstacles. The color depends on the nature of the light of the primary light source, and it can be modified by interposing color filters (green, red, or yellow transparent plastic sheets), or by the presence of strongly colored reflectors whose hue influences the general illumination.

Images. For purposes of sense perception, the most important difference between light and sound is that your eyes give you an incredibly detailed picture (image) of the position, shape, color, and so on, of the objects that reflect light (secondary light sources) in your surroundings, whereas your ears mainly give you information about the primary sound sources. Related to this circumstance is the fact that you possess a primary sound source (your vocal cords) to enable you to communicate with others, but your body is only a reflector and not a source of light. Therefore, your body can be seen only if there is an external source of light and is invisible in the dark. Even when you do control a primary light source, such as automobile headlights, you use it to illuminate and enable you to see *other* objects that reflect light. In other words, you rarely look directly at a primary light source, and you rarely pay attention to secondary sound sources.

Misconceptions about vision. This difference between sound and light, which results in your attending mainly to primary sound sources and secondary light sources, gives rise to a curious misconception among many children. They believe that their eyes (or something from them) "reaches out" to the objects they see, much as their hands reach out to touch objects they wish to feel (Fig. 5.1). In fact, infants must both see and touch objects to develop their visual perception and hand-eye coordination. All of us at one time or other have probably had the feeling that our eyes "reach out." The control we have over selecting the objects we look at and the great detail we can perceive when we look closely, plus the apparent similarity with the way we use our hands, seem to create a strong, but false, impression that the eye is an "active" organ rather than a passive receiving system like the ear. Can you cite any specific evidence that shows that the eye simply receives and senses light, rather than "reaching out"? How might you go about convincing a child who claims his eyes "reach out"?

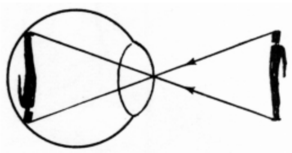


Image on the retina. The sharp visual images are created by the composite effect of many light signals striking the retina of the eye (as shown to the left). Light signals can create an image of a reflecting object because adjacent parts of the object register on adjacent portions of the retina. The retinal image is therefore usually a reliable indicator of the shape of the object. An exception to this rule occurs when the light on its way to your eye passes through a rain-covered automobile windshield or the hot exhaust of a jet engine; then the object appears blurry and you do not register a sharp image.

Ray model for light. Our ability to have visual images, the obstruction of images by objects intervening between object and eye, the silhouette-like shadows cast by objects in bright sunlight, and the projection of film pictures on a screen, all suggest that light usually travels in straight lines. In the ray model, light signals are made up of light rays that travel in straight lines and obey simple geometrical rules as they propagate (move) from source to detector. In the next two sections we will describe details of the ray model and how it may help in the analysis or construction of optical instruments using lenses (eyeglasses, projectors, cameras, microscopes, and telescopes). The ray model, which explains the formation of images and shadows very directly, does not directly explain the existence of a background level of illumination on a cloudy day, when the sun is concealed and there are no sharp shadows. This problem, as well as the problem of the color of light, can nevertheless be solved within the framework of the ray model.

Wave model for sound. It is quite clear that a ray model is not adequate for describing sound because obstacles placed between the source and receiver do not block a sound signal in the same way as they block a light signal. Sound does not appear to travel in straight lines. Instead, it appears to diffuse through space around obstacles much like water waves, which radiate out from a dropped pebble, pass on both sides of an obstacle, and then close again behind it. Sound (and water waves) rarely give rise to a distinct "shadow," that is, a region behind an obstacle from which the effect of the source is completely blocked. This analogy has given rise to the wave model for sound. We will introduce the concept of wave motion in Chapter 6 and apply it to sound in Chapter 7. There you will see how the pitch of a note and the construction of musical instruments are explained by a wave model.

Combination of two sound or light signals. So far we have been describing generally well-known properties of sound and light. We will now describe what happens when light or sound from two sources is combined. The resultant effects of such combinations are surprising

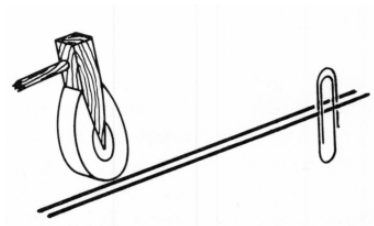


Figure 5.2 Low notes on the piano have two strings. Change the tuning of one string by clipping a paper clip on it. Then hit the key and listen for beats. To compare the sound of two strings with the sound of one, stop the vibration of one string with your finger.

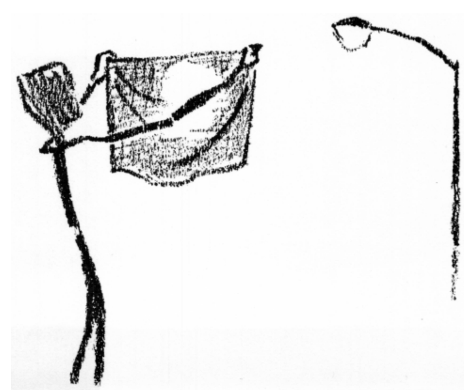
and the implications derived from them, though not widely known, are decisive in formulating working models to explain the phenomena.

Beats. The first phenomenon is the beats that may be heard from a poorly tuned piano. When a single note with two or three strings is struck, there is a rhythmic pulsation of the sound level (so-called beats) even though the tone does not change (Fig. 5.2). To produce beats, it is necessary to have two primary sound sources with pitches that are almost the same. The closer the pitches of the two notes, the slower the beats. The detection of beats is of great importance to the piano tuner, who adjusts the tension of the strings until no beats are heard.

The scientific significance of beats is that there are instants when the combined sound produced by the two sources together (strange as it may seem) is *weaker* than the sound produced by either one alone. This is a very surprising observation, since you would ordinarily expect two similar sources to produce about twice the sound intensity of one alone. You are forced to conclude that the two sources somehow act "in opposition" to one another at the time when the combined sound intensity is very low. To be acceptable, the wave model for sound must therefore make possible this "opposition" as well as the expected reinforcing of two sound sources.

Interference. The second phenomenon involves light. To observe it, look through a handkerchief, a fabric umbrella, or window curtains at a bright, small source of light such as a distant street lamp at night (Fig. 5.3). You will see the light source broken into a regular array of

Figure 5.3 Look through a handkerchief at a bright but distant light. Identify the interference pattern



bright spots. The array of spots will be much larger than the source seen directly, and cannot be explained as arising because the light source is seen through the holes in the fabric. The array of spots may even have touches of color. It is called an *interference pattern*.

An interference pattern is visible whenever one small primary light source is viewed through an obstacle, such as a piece of fabric that has many holes or slits in a regular pattern. Each of the slits functions as a separate "source" and the light passing through each slit "interferes" with the light from all the other slits to produce the observed effect. If you compare the interference patterns caused by coarse and fine fabrics, you will find that the fine fabric creates a larger-scale pattern than the coarse, exactly the opposite from what would happen if you saw the source through the holes in the fabric. With very coarse fabrics (1-millimeter spacing or more) the pattern is too closely spaced to be seen. You can also see rainbow-colored interference effects when you look along a compact disc (CD) or record and let the grooves catch the light, or when you look at thin layers of material in a bright light, such as soap bubbles, mother-of-pearl, or an oil film on water, all of which appear iridescent in sunlight.

5.2 The ray model for light

Many common conditions create the appearance that light rays are visible objects. A powerful searchlight at night is the source of a beam that stabs the sky. A crack in a shutter or curtain admits a shaft of light into a darkened room. Gaps between clouds on a hazy day or trees in mist allow the sun to create streaks of illumination (Fig. 5.4). Actually, light is registered by your eyes only if it strikes the retina. What you see in these examples, therefore, are the massed dust particles or water particles, which reflect the light striking them to your eyes. The form of a beam, shaft, or streak is created by the pattern of the incident light, which is restricted to a slender region of space. The most impressive property of this region of space is that it is straight. Hence the concept of a light ray that travels in straight lines.

Figure 5.4
Streaks of sunlight at dawn.

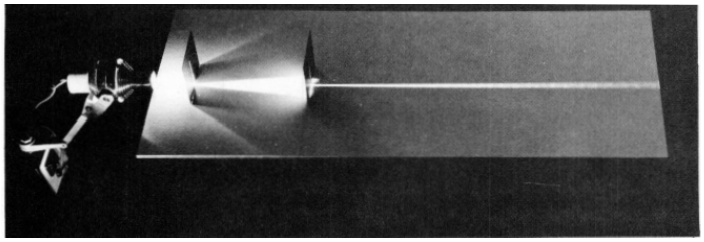


Figure 5.5 A beam and a pencil of light are formed by the small holes in the two screens illuminated from the left.

Figure 5.6 A laser producing a pencil of light.

The phrases "light beam," "pencil of light," and "shaft of light" refer to light traveling in a particular direction, usually directly from a primary source. The distinction among these phrases is a subjective one, with "light beam" referring to the widest lit area and "pencil of light" referring to the narrowest. Their meaning is to be compared with that of "illumination," which refers to light traversing a region of space from all directions, usually coming from reflecting surfaces, so that there are no dark shadows and all objects in the space are visible.

In the ray model, a light beam is represented as a bundle of infinitesimally thin light rays. The width and shape of the bundle determines the area and shape of the beam. The propagation of each ray determines the behavior of the entire beam. In other words, a ray of light is to a beam just as a particle in the MIP model is to a piece of matter (Section 4.5). The function of the ray model is to explain the observations made on light in terms of the assumed properties of light rays. The ray model does not attempt to explain the assumed properties of rays, which are justified (or undermined) by the successes (or failures) of the model.

Properties of light beams. As is true of all models, the observations to be explained by the ray model are built into its assumptions in a simplified and/or generalized form. We therefore begin with the observation of light beams, which the model will have to explain.

Light beams are usually produced by letting light from a powerful source impinge on a screen in which there is a small hole (Fig. 5.5). The light passing through the hole forms the beam. A modern device, the laser (Fig. 5.6), is a primary source that produces a pencil of light.

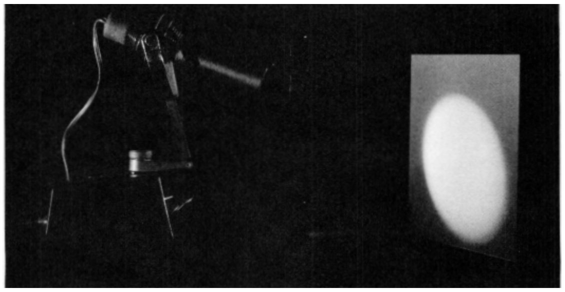


Figure 5.7 The light source and the light spot on the screen can be seen, but not the light beam between them.

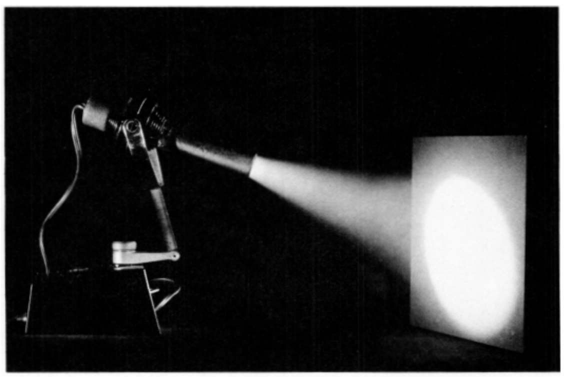


Figure 5.8 Fine smoke particles in the path of the light beam reflect the light and make the beam visible.

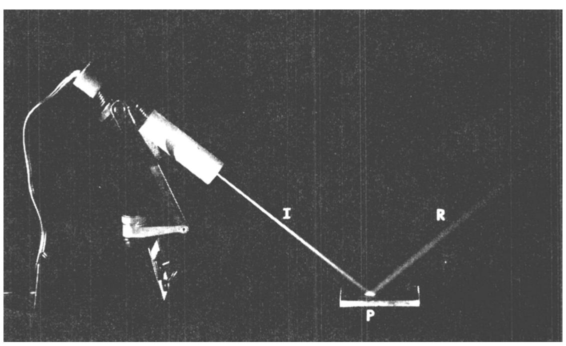


Figure 5.9 A beam of light is reflected from a polished metal mirror (P). Note the incident beam (I) and the reflected beam (R). Why is the mirror mostly dark? Why is the reflected beam dimmer than the incident beam?

If you try this experiment with a flashlight, you wouldn't expect to be able to see the paths of the two light beams as in this photograph. Why not? How might you make the beams visible?

A beam of light cannot be seen from the side, because our eye detects light only when the light strikes our retina (Fig. 5.7). To make the beam "visible," chalk, smoke, or dust particles must be introduced into the region of space (in air or in water) traversed by the beam. The illuminated particles act as secondary light sources and can be seen by reflection, while the un-illuminated particles outside the beam remain dark and unseen (Fig. 5.8). Another technique for tracing a light beam's path is to let it strike a white sheet at a glancing angle (Fig. 5.5).

Three properties of light can be easily observed with beams rendered visible by one of these procedures. 1) A light beam travels in straight lines. 2) A light beam is *reflected* by a polished surface or mirror (Fig. 5.9), and 3) A light beam is partially reflected and partially *refracted*, when it

TABLE 5.1 INDEX OF REFRACTION FOR TRANSPARENT MATERIALS

Material	*Index of refraction*
air	*1.00*
water	*1.33*
glass	*1.5*
diamond	*2.42*
ethyl alcohol	*1.36*

crosses the boundary between two transparent materials such as air and glass or air and water (Fig. 5.10). Simple mathematical models have been found to fit the observations on the relationships of the angles of reflection, refraction, and incidence (Fig. 5.11, Eqs. 5.1 and 5.2). The symbol "n" in Eq. 5.2 stands for the index of refraction, a number that is determined experimentally for each transparent material (Table 5.1 and Example 5.1).

Equation 5.1
(Law of ref<u>le</u>ction)

*angle of ref**le**ction* $= \theta_R$
angle of incidence $= \theta_i$
(Figure 5.11, below, shows θ_R *and* θ_i*.)*

$$\theta_R = \theta_i$$

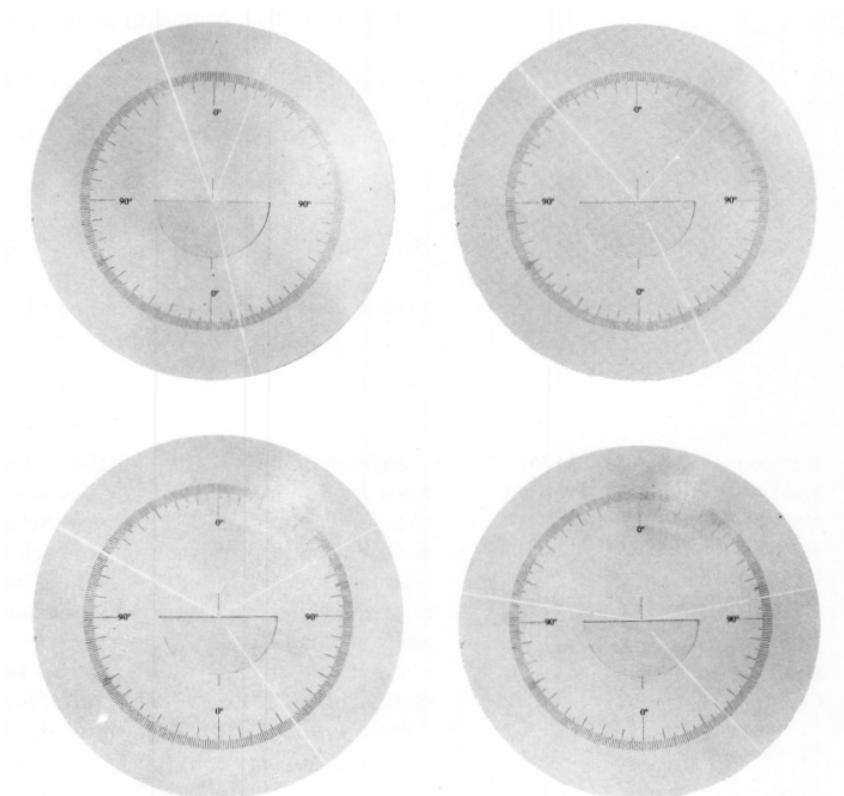
Equation 5.2
(Snell's law of refraction)

*angle of ref**ra**ction* $= \theta_r$
angle of incidence $= \theta_i$
(Figure 5.11, below, shows θ_r *and* θ_i*.)*
*index of ref**ra**ction in material of refraction* $= n_r$
*index of ref**ra**ction in material of incidence* $= n_i$

$$n_r \text{ sine } \theta_r = n_i \text{ sine } \theta_i$$

See Appendix for definition (Eq. A.6) and values (Table A.7) of sine θ.

Figure 5.10 A beam of light enters a semicircular slab of glass at various angles. Note the incident beam, reflected beam, and refracted beam.



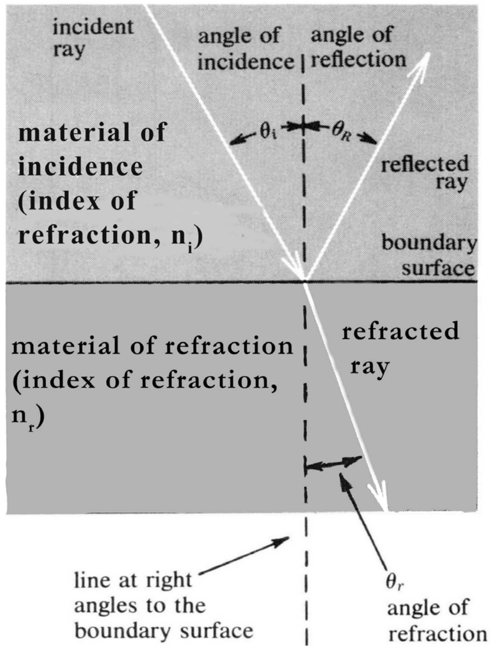


Figure 5.11 The angles of incidence (θ_i), *reflection* (θ_R), *and refraction* (θ_r) *are defined in relation to the line at right angles to the boundary surface between two media.*

Willebrord Snell (1591-1626). Snell's law is an example of an empirical law that summarizes data but does not rest on a theoretical foundation.

Claudius Ptolemy (approx. 140 A. D.) is much better known for his work in astronomy (see Chapter 15).

Equation 5-3 (Ptolemy's Law of Refraction)

$n_r\theta_r = n_i\theta_i$

(less adequate than Snell's Law of Refraction)

EXAMPLE 5-1. Applications of Snell's law.

(a) Light incident on glass: $\theta_i = 40°$, $n_i = 1.00$, $n_r = 1.50$

$$\text{sine } \theta_r = \frac{n_i}{n_r} \text{ sine } \theta_i = \frac{1.00}{1.50} \text{ sine } 40° = 0.67 \times 0.64 = 0.43$$
$$\theta_r = 25°$$

(b) Light incident on diamond: $\theta_i = 60°$, $n_i = 1.00$, $n_r = 2.42$

$$\text{sine } \theta_r = \frac{n_i}{n_r} \text{ sine } \theta_i = \frac{1.00}{2.42} \text{ sine } 60° = 0.41 \times 0.87 = 0.36$$
$$\theta_r = 21°$$

Refraction. Even though the index of refraction of each material must be found, Eq. *5.2* is a useful mathematical model because it uses only one empirical datum (the index of refraction) and yet predicts an angle of refraction for each angle of incidence. This mathematical model is called *Snell's law,* and was formulated by Snell early in the seventeenth century on the basis of experiments conducted with air, water, and glass.

Long before Snell, Ptolemy had tabulated and proposed a mathematical model (Eq. 5.3) for refraction, but the Arabian investigator Alhazen (965-1038) pointed out the inadequacy of Ptolemy's model. The two models are represented graphically in Fig. 5-12, where their similarity for small angles can be recognized. Johannes Kepler (1571-1630), better known for

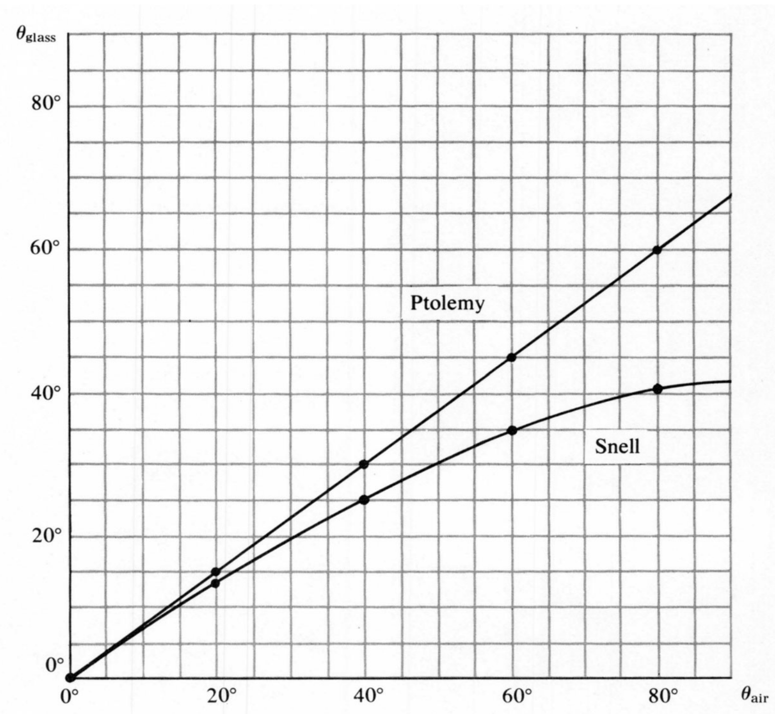


Figure 5.12 Refraction of light passing between air and glass (n_{glass} = 1.5). Ptolemy's model (Eq. 5.3) is compared with Snell's model (Eq. 5.2). For experimental results, see Fig. 5.10. The graph applies for light passing from air into glass or from glass into air. Note that θ_{glass} is always less than θ_{air}.

"I procured me a triangular glass prism, to try therewith the celebrated phenomena of colours... having darkened my chamber, and made a small hole in my window-shuts, to let in a convenient quantity of the sun's light, I placed my prism at its entrance... It was ... a very pleasing divertissement, to view the vivid and intense colours ... I [thereafter] with admiration beheld that all the colours of the prism being made to converge ... reproduced light, entirely and perfectly white, and not at all sensibly differing from the direct light of the sun...."
Isaac Newton
Philosophical Transactions, *1672*

his planetary models for the solar system (see Section 15.2), made measurements of refraction, but was not able to construct a mathematical model better than Ptolemy's (Eq. 5.3). Long after Snell's time, refraction was a key to the acceptance of new models that replaced the ray model for light (Section 7.2).

White and colored light. Isaac Newton achieved a breakthrough in the understanding of light. He found (as others had before) that a glass prism refracted a pencil of light in such a way that a rainbow-colored streak appeared (Fig. 5.13). Newton theorised that the white light was composed of a mixture of various colors. He tested this idea by using a second prism to refract the colored streak back toward its original direction of propagation (Fig. 5.14). The original white light was restored, confirming his theory! When Newton used a screen to isolate only one color in his streak to impinge on the second prism, he found that further refraction of this one color did not alter the color of the light (Fig. 5.15). These findings led Newton to elaborate on the ray model generally accepted in his time to include an explanation of color.

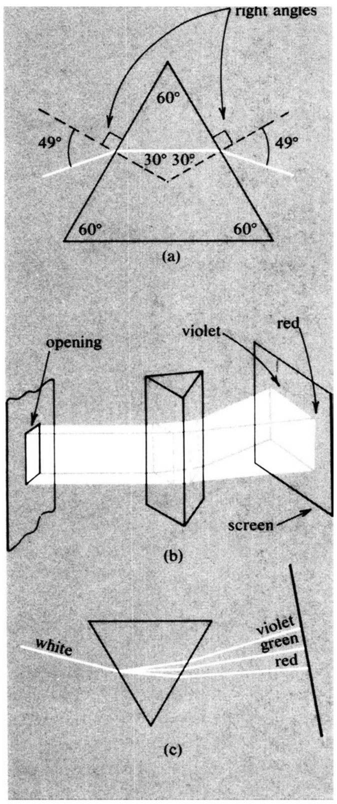


Figure 5.13 (to left) Refraction of light by a glass prism (index of refraction, n = 1.5).
(a) Refraction of a ray of light according to Snell's Law.
(b) and (c) Refraction of a beam of white light. Violet light is bent (or refracted) more than red light.

Figure 5.14 Refraction of a pencil of white light in opposite directions, by two prisms. The colors recombine to make white light. This experiment was first carried out and reported by Isaac Newton.

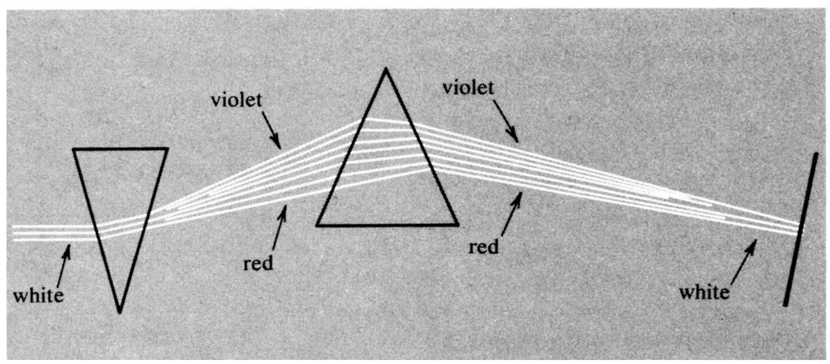
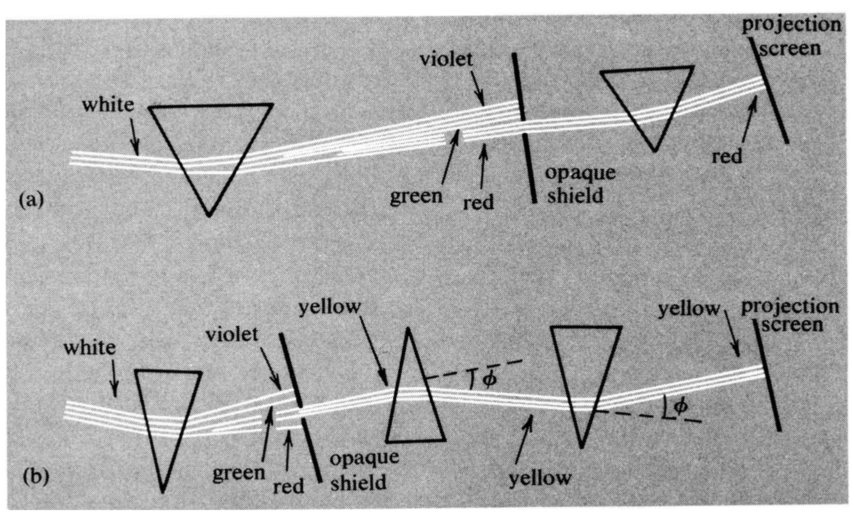


Figure 5.15 Refraction of light of a single color does not change the color. The color is selected from the spectrum by an opening at the appropriate place in the opaque shield.
(a) Red light selected for passage through second prism.
(b) Yellow light selected for passage through two further prisms.



Assumptions of the ray model. Isaac Newton gathered the assumptions of the ray model in his *Opticks, or a Treatise on Reflections, Refractions, Inflections , and Colours of Light*. In his version of the ray model, the elemental light rays were colored (monochromatic = single color), and their color was an intrinsic, unchangeable property. Visual impressions of color were produced by monochromatic rays or by combinations of monochromatic rays, as we will explain below.

The assumptions of the ray model as adapted for this text are summarized in Fig. 5.16 to the right. It is clear that the assumptions were selected so as to be consistent with the observation on light beams described in the previous paragraphs. The task of the model is to explain, in addition, more complicated phenomena, such as the formation of images, the concept of illumination, why not all surfaces act as mirrors, and how colors are produced in mixtures of paint pigments. In all these situations, light is not a single bundle of rays, but a combination of diverging, converging, and crossing rays of various colors and directions of propagation. Assumptions 6 and 7 enable us to apply the model in situations where many rays, following different paths, combine to form an *image* (Section 5.1 and Figure 5.17). Their essence is that only rays that begin by diverging from one point on an object can later converge to form an image of the object point where they started.

Color. The ray model provides three techniques for the production of colored light. One of these is by selective transmission through a color filter. The color filter absorbs rays of some colors and transmits rays of other colors. The colored beam obtained in this way may happen to be monochromatic in Newton's sense, or it may be composed of several monochromatic rays. The second technique is selective reflection by an opaque object. The surface of the object absorbs rays of some color and reflects rays of other colors. The third technique is refraction, through a glass prism or other suitably

Figure 5.16 Assumptions of the ray model for light.

Assumption 1. Each point in a primary light source emits rays which diverge in all directions from that point.

Assumption 2. Light rays in a uniform medium travel in straight lines.

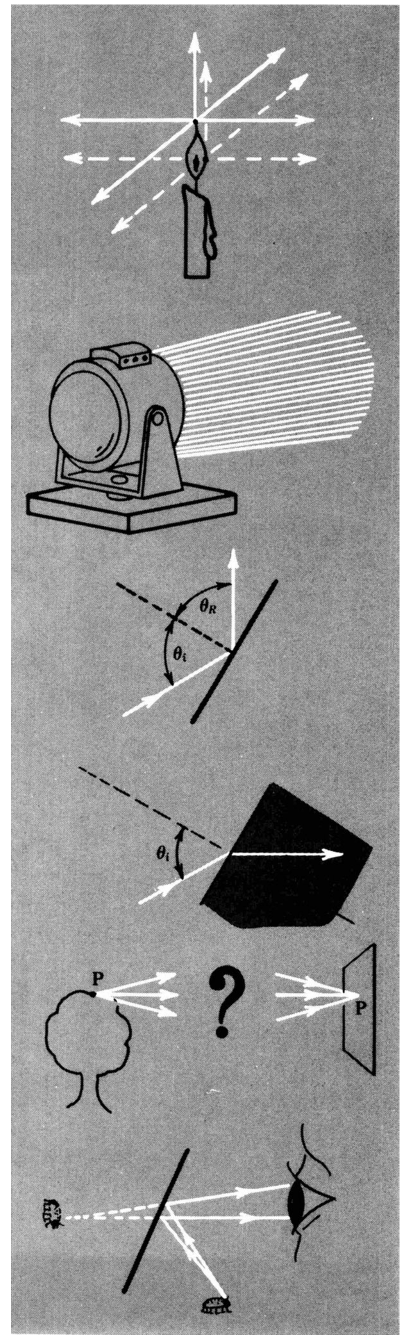
Assumption 3. Each point in an object may transmit, absorb, or reflect the rays incident upon it.

Assumption 4. Light rays are reflected, with the angle of incidence equal to the angle of reflection, $\theta_i = \theta_R$.

Assumption 5. Light rays are refracted, with the angles of incidence and refraction of monochromatic rays related by the formula $n_r \sin \theta_r = n_i \sin \theta_i$. *The index of refraction may differ for different colored rays (dispersion).*

Assumption 6. Whenever rays that diverge from one point in an object meet again at another point on a white screen as the result of reflection and refraction, they make an image of the point in the object.

Assumption 7. An object seen directly, by reflection, or by refraction, appears to be in that place from which the rays diverge as they fall into the observer's eye.



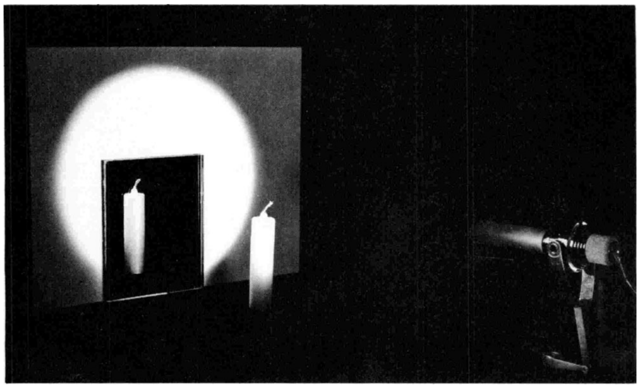


*Figure 5.17 (to left) Specular reflection from a mirror. At first glance this photo may seem unremarkable. However, notice that the mirror is dark and the screen behind it is bright. In addition, both the candle and its image are bright. Finally, notice the light source at the right side: why don't you see it in the mirror? If you can make a ray diagram that shows clearly what is going on here, you understand the ray model very well. (This is **not** in any way a trick photograph; no hidden lenses, mirrors or other apparatus were used.)*

shaped object, such as described earlier (Fig. 5.13). For example, the colors of the rainbow are produced by refraction of light in rainwater droplets. Only refraction is sure to give rise to monochromatic light. Selective transmission and reflection may or may not, depending on the materials in the filter or reflector.

Addition of colors. The ray model can explain the modification of colored light by addition or subtraction of colors. Color addition occurs when several colored beams illuminate the same screen. Where they overlap, the screen acts as a reflecting surface of all the incident colors. This is the process by which Newton obtained white light from the colored streak (Fig. 5.14); after refraction through the second prism, the various colored rays were brought to overlap on the screen.

Subtraction of colors. Color subtraction occurs when filters are inserted into the path of a beam of light, or pigments are mixed in paint.

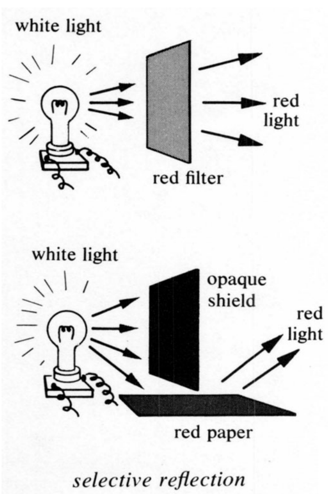
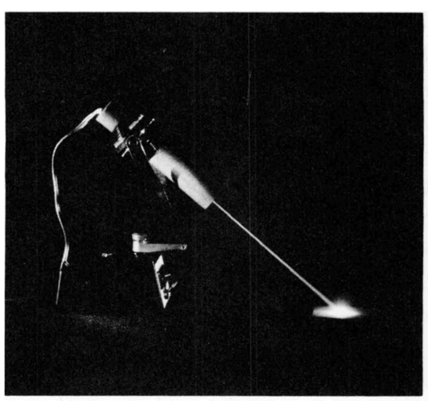
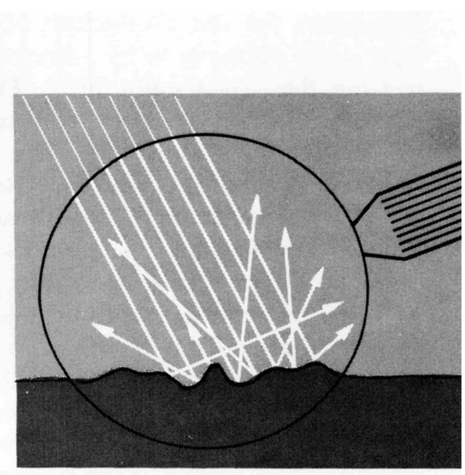


Figure 5.18 Diffuse reflection.
(a) A pencil of light strikes a dull white surface. Note the diffuse reflected light near surface.
(b) Working model for a dull surface that shows diffuse reflection of light.



(a)



(b)

The light absorption in the filter removes some colors while others are transmitted. The insertion of successive filters results in the subtraction of more and more colors, until none (that is, no light at all) may be left. Inferences about the absorption by color filters are indirect, because the observer sees only the transmitted light and not what is absorbed (or reflected). To find out what a filter absorbs, you have to illuminate it with monochromatic light and determine whether or not any light at all is transmitted. All the monochromatic light that is not transmitted is absorbed or reflected.

Experiment A. Observations of light absorption by single-color filters

red filter absorbs blue, green, and yellow

yellow filter absorbs blue

green filter absorbs red

blue filter absorbs red and yellow

Experiment B. Observations of light transmission by combinations of single-color filters

red and yellow filters: red light transmitted

red and green filters: no light transmitted

yellow and green filters: yellow and green light transmitted

yellow and blue filters: green light transmitted

On the basis of the ray model and the information in Experiment A above, can you predict the results summarized in Experiment B?

Interference colors. There is a fourth technique that produces colored light: interference. This was mentioned briefly at the end of Section 5.1. It is an observation that cannot be explained by the ray model, since the experiment uses no refraction or selective absorption. Curiously enough, Newton carried out investigations of the color of thin films (air between glass plates, soap bubbles) and tried to adapt the ray model by including multiple reflections and refractions back and forth between the two surfaces of the film. This attempt, however, while suggestive, was not successful and the observations remained unexplained for more than 100 years (Section 7.2).

Models for reflecting surfaces. We have already explained how the color of surfaces arises from selective reflection and absorption of monochromatic light rays. Still to be discussed is the difference between mirrors on the one hand and dull surfaces that reflect light but do not reflect images on the other hand.

Mirror surfaces. A mirror is a surface that maintains the divergence or convergence of rays incident upon it. When you look into a mirror, therefore, you do not see the mirror itself. Rather, you see the light rays reflected from the mirror to your eye, or, more precisely, you see the apparent *sources* of those reflected light rays. (Fig. 5.17). A mirror has a smoothly polished surface to ensure the orderly reflection of all incident rays. This is an example of *specular reflection*. It is well known that fingerprints on a mirror interfere with the specular reflection.

Dull surfaces. A model for reflecting surfaces that are not mirrors must be designed to destroy the divergence or convergence of rays incident upon it. Accordingly, the model surface is highly irregular, with minute irregularities. As light rays strike various points of the surface, they are absorbed or reflected according to Assumption 6. Since the surface is irregular, however, the reflected rays diverge in all directions from a small area on the surface and do not propagate in a direction simply related to the placement of the primary source (Fig. 5.18). This phenomenon is called *diffuse reflection*. According to Assumption 7, an observer will see the surface from which the rays diverge after reflection.

Diffuse reflection, selective reflection and the straight-line propagation of light from primary source to reflecting surface and to the eye account for most of the everyday properties of light and vision. The phenomenon

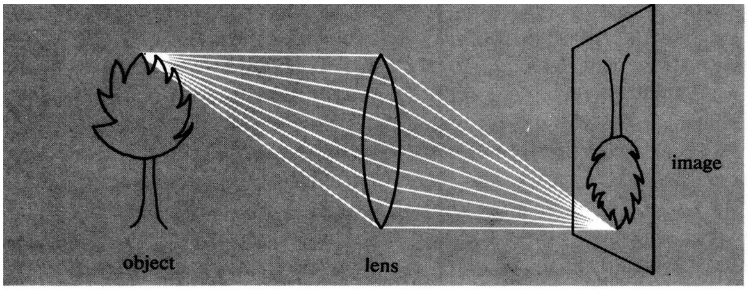


Figure 5.19.
The convex glass lens in air produces an image.

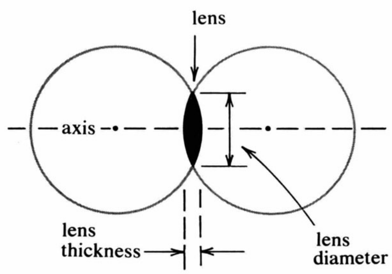


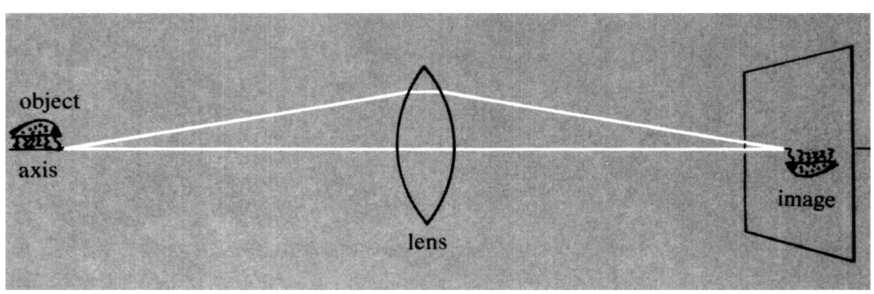
Figure 5.20 A thin spherical lens has the shape of the overlapping region of two slightly interpenetrating spheres.

of refraction enters mainly through the widespread use of lenses in eyeglasses and optical instruments. The next section contains an introduction to the theory of lenses, as derived from the ray model for light.

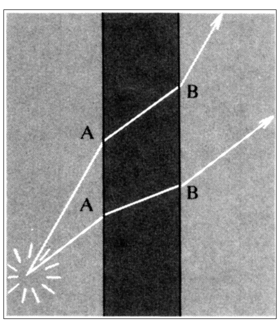
5.3 Application of the ray model to lenses

Binoculars, telescopes, cameras, and projectors are so common today that it is difficult to believe that none of them existed 400 years ago (and most of them not even 150 years ago). Even eyeglasses were invented only in the fourteenth century, and their use spread very slowly. Until about the time of Galileo the world must have been a blur to

Figure 5.21 Two rays that diverge from an object are refracted by the lens to converge at one point of the image. One ray passes along the axis.



In contrast to a lens, a sheet of glass with parallel faces (see diagram below) refracts light rays in such a way that their convergence or divergence is not altered. Snell's law applied at the two points labeled B leads to refraction which exactly compensates the refraction at the two points labeled A. Each ray emerges in a direction parallel to its incident direction (see Problem 17).



The concept of focal ***point*** *applies to all lenses, not only thin ones. The focal* ***length****, however, is definable only for* <u>*thin*</u> *lenses, which do not extend far from the lens center on either side.*

quite a few people, considering how many are now wearing glasses. The principles of the ray model occupied scientists in a great flurry of discovery during the sixteenth and seventeenth centuries and led to the development of optical instruments.

The converging lens. A convex glass lens in air has the property of taking rays that diverge from one point on a source object and refracting them so they converge to a point on the other side of the lens (Fig. 5.19). This point is called an *image* of the source point for the following reason. Once the rays have passed through the image point, they diverge again, just as though they had been emitted by a source at the image point. To an observer, therefore, the rays diverging from the image point appear to come from an object located at the image point.

The vast majority of lenses are spherical; that is, their two surfaces are sections of two spheres (Fig. 5.20). The axis of the lens is the line that joins the centers of those spheres. We will restrict our discussion to lenses that are thin compared to their diameter.

Image formation. Consider two rays emitted from a small object on the lens axis, one traveling along the axis of the lens, the other at an angle to the axis, striking the lens at an off axis point (Fig. 5.21). The ray on the axis goes through the lens undeviated (Snell's law, Fig. 5.12, Eq. 5.2) because it strikes the glass at right angles to its surface. As the oblique ray strikes the first surface and enters the glass, it is refracted toward the right angle direction (Fig. 5.11). Upon emerging from the second surface into air, it is refracted again. Both times refraction bends the ray toward the axis, similar to refraction by a prism (Fig. 5.13). Because the two rays in Fig. 5.21 are *converging* to one point after passing through the lens even though they were diverging before, the lens is called a *converging lens*. The lens has focused the two rays to produce an image. Other rays from the same point of the object are also refracted to the same image point.

The basic problem to be solved in the operation of a converging lens is to find the position and size of the image, given the position and size of the source object and some properties of the lens. The problem is solved in the ray model by taking the rays diverging from each point on the source object and finding where they converge after passing through the lens.

Focal length. It turns out that the shape and index of refraction of the lens, which clearly must influence the image position, can be used to derive a property of a thin spherical lens called its *focal length*. The focal length is the distance from the center of the lens to the point called the *focal point* where all rays parallel to the axis converge (Fig. 5.22(a)). In addition, as shown in fig. 5.22 (b), rays diverging from the focal point are refracted by the lens so that they emerge parallel to the axis.

The lens formula. The lens problem is solved by the geometrical construction illustrated in Fig. 5.23. This construction is based on a thought experiment in which a source object to the left of the lens produces an image to the right of the lens. In the thought experiment, you consider

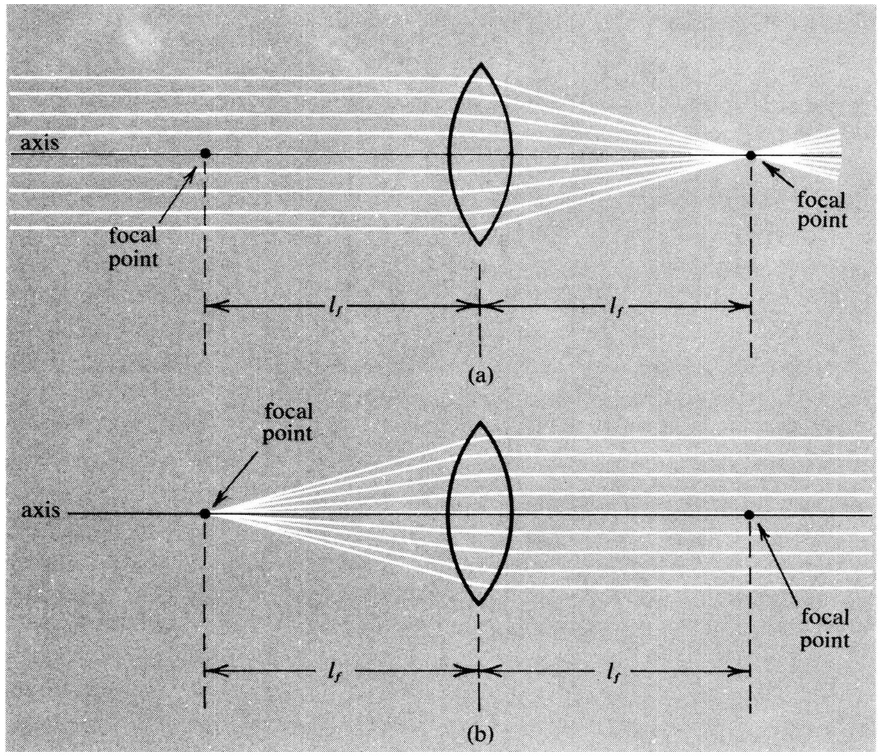


Figure 5.22 Focal length (l_f) of a convergent lens.
(a) Parallel rays entering a convergent lens converge at the focal point.
(b) Rays leaving the focal point are refracted so they emerge parallel to the axis.

Equation 5.4

object distance = l_o
image distance = l_i
focal length = l_f

$$\frac{1}{l_o} + \frac{1}{l_i} = \frac{1}{l_f}$$

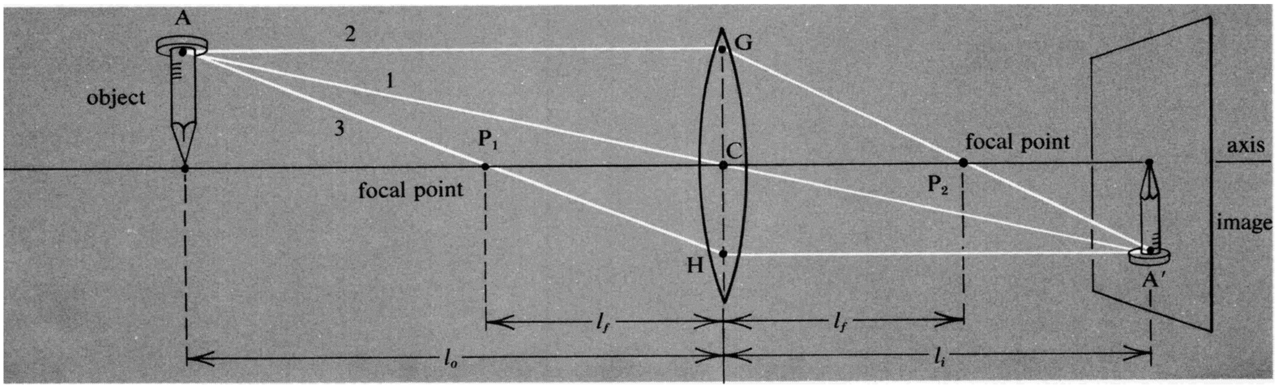
Equation 5.5

image size = s_i
object size = s_o

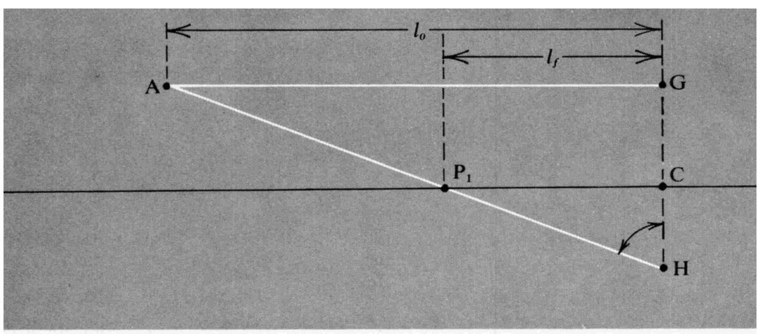
$$\frac{s_i}{s_o} = \frac{l_i}{l_o}$$

three rays that diverge from the top of the source object and whose refraction by the lens is easy to predict. You trace those rays to find where they converge after passing through the lens. The point where they converge forms the "top" of the image. The result is a mathematical model called the lens formula relating the object distance, image distance, and focal length (Eq. 5.4). In words, the reciprocal of the object distance plus the reciprocal of the image distance equals the reciprocal of the focal length. The formula applies only to thin, spherical lenses because the concepts of focal length, image distance, and object distance have no consistent meaning otherwise.

Since Eq. 5.4 does not make reference to the distance between the axis and the source point at the top of the object, the lens formula predicts that light diverging from all other points of the object will converge to form the image of the object as indicated in Fig. 5.24. The size of the image can therefore be found from the same thought experiment, which is illustrated in a simplified form in Fig. 5.25. The result is a mathematical model relating object and image sizes to object and image distances (Eq. 5.5). The ratio of image size to object size is equal to the ratio of image distance to object distance. The properties of converging lenses are illustrated in Examples 5.2 and 5.3.

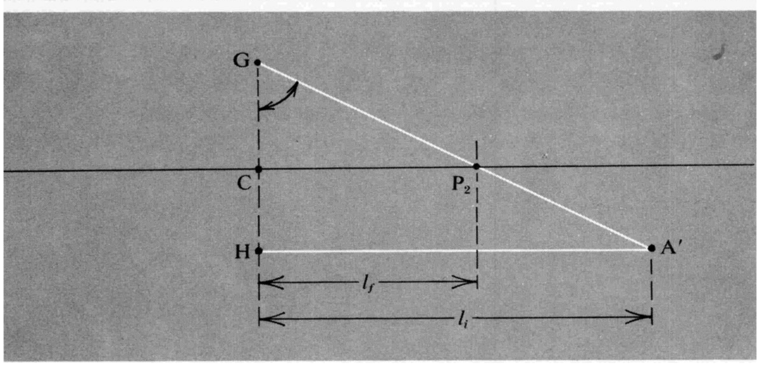


(a) A nail at a distance l_o from the converging lens produces an image at the distance l_i from the lens. To find the position of the image, three rays diverging from the top of the nail A are traced to their point of convergence A'. Ray 1 passes through the center of the lens C. It is not deflected because it traverses two glass-air surfaces that are parallel. Ray 2 propagates parallel to the lens axis and is refracted by the lens G to pass through the focal point P_2. Ray 3 propagates through focal point P_1 and is refracted by the lens at H to emerge parallel to the axis.



(b) The two right triangles P_1CH and AGH are similar, because they have a common acute angle at H. Corresponding sides P_1C (length l_f) and AG (length l_o) have the same ratio as sides CH and GH. Therefore,

$$\frac{l_f}{l_o} = \frac{\overline{\text{length of line CH}}}{\overline{\text{length of line GH}}} \quad (1)$$



(c) The two right triangles P_2CG and A'HG are similar, because they have a common acute angle at G. Corresponding sides P_2C (length I_f) and A'H (length I;) have the same ratio as sides CG and GH.

$$\frac{l_f}{l_o} = \frac{\overline{\text{length of line CG}}}{\overline{\text{length of line GH}}} \quad (2)$$

Figure 5.23 Solution of the thin lens problem and derivation of Equation 5.4. The construction in parts (a), (b), and (c) leads to Eqs. (1) and (2). By adding Eqs. (1) and (2) and noting that segments CH and CG add up to GH, you obtain Eq. (3) below:

$$\frac{l_f}{l_o} + \frac{l_f}{l_i} = \frac{\overline{CH}}{\overline{GH}} + \frac{\overline{CG}}{\overline{GH}} = \frac{\overline{CH} + \overline{CG}}{\overline{GH}} = \frac{\overline{GH}}{\overline{GH}} = 1 \qquad (3)$$

When both sides of Eq. (3) are divided by l_f , the result is Eq. 5.4.

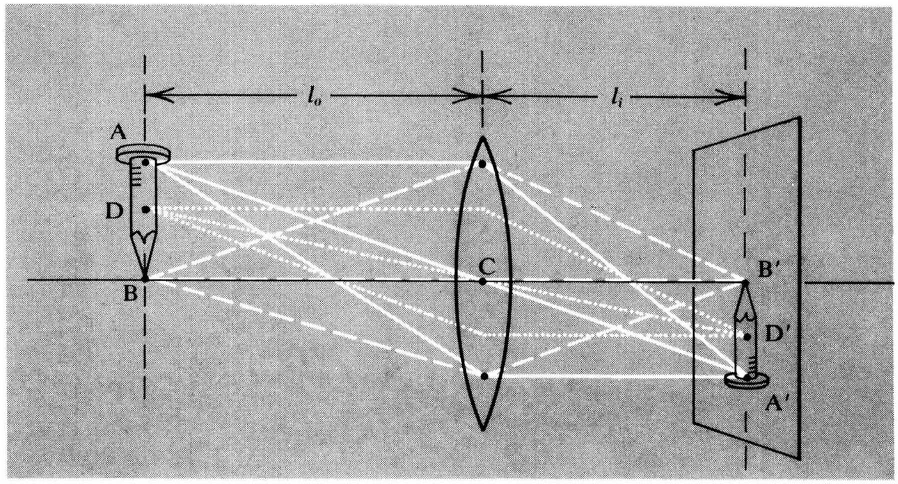


Figure 5·24 Rays diverging from A, B, and D converge at A′, B′, and D′, respectively, to form the image.

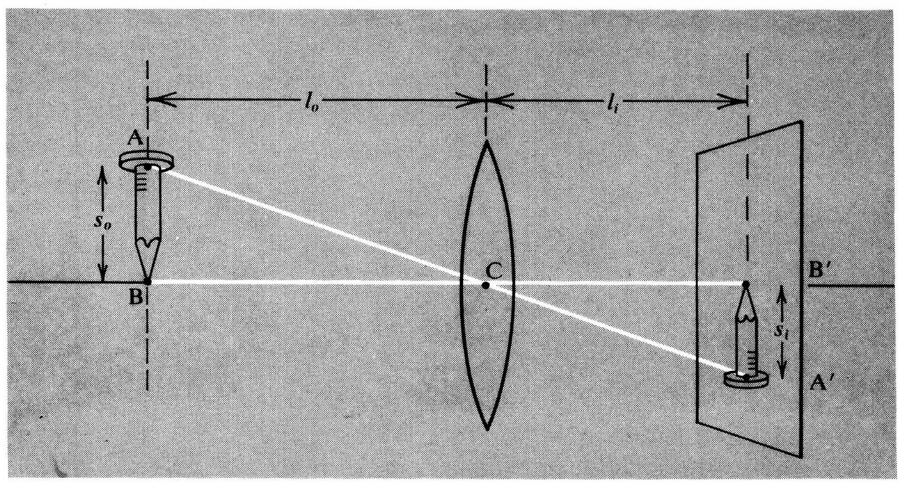
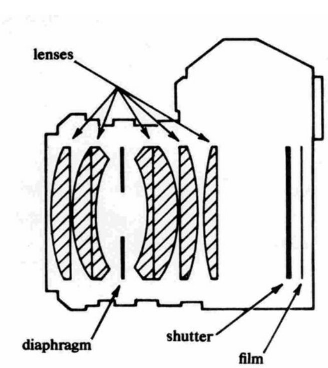


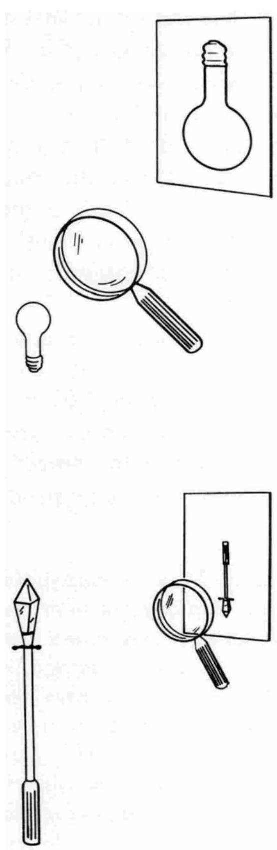
Figure 5·25 The ratio of image size to object size is related to image and object distances because triangles A′B′C and ABC are similar.



Typical 35 mm camera.

Applications. Based on this abbreviated discussion of a lens, we can describe several optical instruments.

Cameras. Let us consider a camera. A camera consists of a lens system of fixed focal length (usually about 50 millimeters) with some movement possible for focusing the image on the film. Since most objects are thousands of millimeters distant from the lens, Eq. 5.4 predicts that the image will be very close to the focal point, and will vary only slightly in position even though the object distance varies from 5 meters to 100 meters (see also Example 5.2). Movement of the lens of only a few millimeters in or out is needed to focus the image on the film unless the subject of the photograph is very close to the lens. The very fact that a camera lens has to move at all (as opposed to the more common "fixed focus" lenses) tells you something of the quality of the lens. The image of a standard 35 mm camera lens is so sharp that a 1-millimeter movement of the lens, placing the image 1 millimeter away from the film, results in very significant deterioration of the image.



EXAMPLE 5.2. A lens with focal length of 0.10 meter is used to focus the image of a light bulb on a piece of paper. The lens is 0.15 meter from the light bulb. Where do you have to hold the paper? How big is the image compared to the bulb itself?

$$\text{Solution}: l_f = 0.10\,m,\ l_o = 0.15\,m, l_i = ?,\ \frac{s_i}{s_o} = ?$$

$$Equation\,5.4: \ \frac{1}{l_o} + \frac{1}{l_i} = \frac{1}{l_f},$$

$$or\ \frac{1}{l_i} = \frac{1}{l_f} - \frac{1}{l_o} = \frac{1}{0.10\,m} - \frac{1}{0.15\,m} = 10.0 - 6.7 = 3.3$$

$$thus: l_i = 0.30\,m$$

$$Equation\,5.5: \ \frac{s_i}{s_o} = \frac{l_i}{l_o} = \frac{0.30\,m}{0.15\,m} = 2.0$$

The paper must be held 0.30 meter from the lens. The image is two times as large as the bulb.

EXAMPLE 5.3. The same lens is used to focus the image of a street lamp 30 meters away on a piece of paper. Where do you have to hold the paper? How big is the image compared to the street light?

$$\text{Solution}: l_f = 0.10\,m,\ l_o = 30\,m, l_i = ?,\ \frac{s_i}{s_o} = ?$$

$$Equation\,5.4: \ \frac{1}{l_o} + \frac{1}{l_i} = \frac{1}{l_f},$$

$$or\ \frac{1}{l_i} = \frac{1}{l_f} - \frac{1}{l_o} = \frac{1}{0.10\,m} - \frac{1}{30\,m} = 10.0 - 0.033 \approx 10$$

$$thus: l_i = 0.10\,m$$

$$Equation\,5.5: \ \frac{s_i}{s_o} = \frac{l_i}{l_o} = \frac{0.10\,m}{30\,m} = \frac{1}{300}$$

The paper must be held at the focal point 0.10 meter from the lens. The image is 1/300 as large as the street lamp itself.

If the diameter of the camera lens is large, more rays from every source point on the object pass through the lens. That is, a large lens collects and focuses on the film more of the light energy emitted by the object than does a small one. For poorly lit objects, the size of the lens can therefore be of great significance to the photographer. He may want to gather all the light from the object he possibly can to interact with the film. Lens size is given by the f-number, which is equal to the focal length divided by the diameter of the lens. Thus high f-numbers describe small lenses and low f-numbers describe large lenses. To make a lens of large diameter with good focusing properties is very difficult, thus accounting for the high cost of lenses with low f-number.

A lens has a focal length of 50 mm and a diameter of 25 mm. Its f-number is f = 50 mm/25 mm = 2. Opening or closing the camera iris increases or decreases the effective f-number of the lens, since only the non-covered part is used.

A telephoto lens enlarges the image of distant objects. It does so by having a longer focal length than an ordinary camera lens. The reason the focal length is so significant is that the image distance of a very distant object is almost equal to the focal length (Fig. 5.22). A longer focal length therefore leads to a larger image distance, and this in turn leads to a larger image size according to Eq. 5.5. Telephoto lenses are physically large and expensive. Powerful ones have a focal length of 200 millimeters or more, and therefore enlarge the image fourfold or more over the size obtainable with a regular lens (focal length about 50 mm.). Wide-angle lenses function in the opposite way. They are of short focal length, say 20 millimeters, and are used to make the image of all objects smaller so as to include a broader panorama on the film.

Magnifying instruments. The magnifying glass, in its usual application, does not project an image by converging the rays to a focus, but transmits divergent rays, which appear to come from a larger object (Fig. 5.26). Refracting telescopes and binoculars are a little more complicated, consisting of two lenses. Since the object is enormously distant with respect to the focal length, the image produced by the first

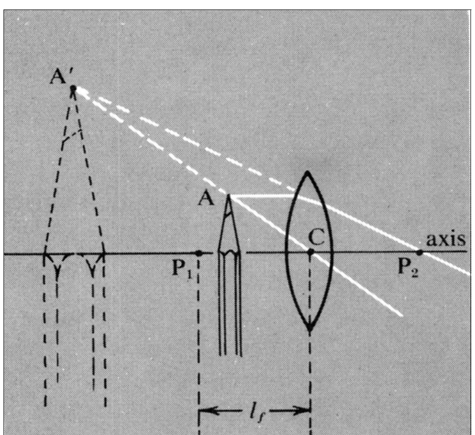


Figure 5.26 A magnifying glass (lens) is placed so near the object (A) to be examined that the latter is closer than the focal point P_1. The two rays traced from the point A are refracted by the lens and appear to diverge from the point A' behind the object. The mathematical model in Eqs. 5.4 and 5.5 can still be used. However, these equations will yield negative numbers for the image distance l_i and for the image size s_i. This means that the image is on the opposite side of the lens compared to the way they were defined in Fig. 5.23a. The distance from the eye to point A' has to be large enough so the eye can focus the incoming divergent rays on the retina.

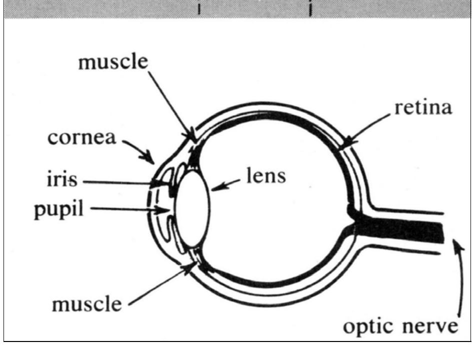


Figure 5.27 Cross section of the eye.

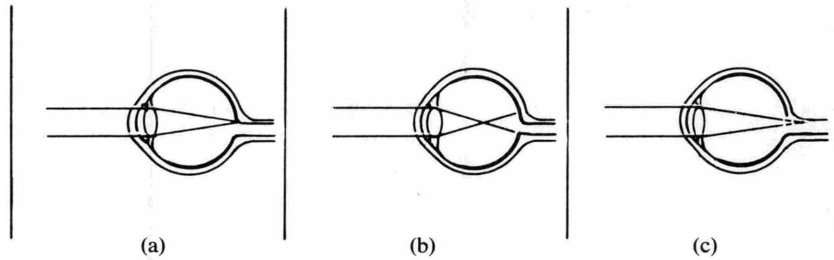


Figure 5·28 Focusing of images on the eye's retina. (a) normal eye. (b) nearsighted eye. (c) farsighted eye.

lens is just slightly beyond its focal point. This image then acts as "object" just inside the focal point of the second lens, which functions like a magnifying glass.

The diameter of the outer (or objective) lens determines the light-gathering ability of a telescope or a pair of binoculars. This is very importance for astronomers who look at distant, faint stars. Binoculars are usually characterized by two dimensions, the magnifying power and the diameter of the outer (objective) lenses. Thus, a 7 x 35 rating means the image is magnified 7 times and the lens has a diameter of 35-millimeters. A larger magnification (such as 10) makes it easier to view small, distant objects; however, a larger magnification also means that the image jumps around more. For hand-held binoculars, most people find that a magnification of 7 or 8 is most practical. For greater magnifications, a tripod must be used.

The human eye. Human eyes function somewhat like a camera with the retina taking the place of the film (Fig. 5.27). The eye's lens focuses the image on the retina. To accommodate objects at various distances, muscles in the eye modify the shape of the lens so as to change its focal length. This process differs from that in a camera, where the distance from lens to film can be adjusted but the focal length cannot.

Eyeglasses have become an indispensable optical instrument. For various reasons, the rays from an object are often brought to focus in front of or behind the retina, in spite of the changes of shape of which the eye's own lens is capable (Fig. 5.28). If the rays are focused in front of the retina, the rays are too strongly converged by the eye's lens and the convergence must be reduced by what is called a negative correcting lens in front of the eye. For the other case, when the image falls behind the retina, the rays are not converged enough and a converging lens must be put in front of the eye.

An ophthalmologist expresses the strength of the necessary correction in the unit *diopters* by taking the reciprocal of the focal length (in meters) of the necessary lens. For example, a 0.33-meter focal length lens gives a correction of 3 diopters. The thickest glasses made give a correction of about 15 diopters. Eyeglasses can also correct for other defects in your vision. These defects are due to the asymmetric shape of your cornea, lens, or retina. To correct for such defects, the lens is not ground to a simple spherical shape. Often the inside is ground to a cylindrical contour while the outside surface may be spherical.

Summary

Light and sound are intermediaries in the interaction-at-a-distance of light or sound sources (candle, loudspeaker) and human sense organs or other detectors (photographic film, microphone). Light and sound transmit information and energy. The very detailed images transmitted by light have led to the development of the ray model. The lack of images and the analogy with water waves have led to the wave model for sound, which will be described further in Chapter 7. Assumptions of the ray model for light provide for emission, propagation, absorption, reflection, and refraction of monochromatic light rays, and the formation of images by converging and diverging rays. These assumptions may be used to explain color, the eye's ability to see objects that reflect light, and the function of mirrors, lenses, cameras, magnifiers, and telescopes.

List of new terms

primary source	wave model	diffuse reflection
reflecting surface	beats	lens
loudness	refraction	converging lens
pitch (of sound)	interference pattern	focal point
light intensity	Snell's law	focal length
color	monochromatic	f-number
image	selective transmission	negative lens
retina	selective reflection	diopter
ray model	specular reflection	

List of symbols

θ_i	angle of incidence	l_f	focal length of a lens
θ_R	angle of reflection	l_o	length (distance) of object from lens
θ_r	angle of refraction	l_i	length (distance) of image from lens
n	index of refraction	s_o	size of object
		s_i	size of image

Problems

1. List similarities and differences of light and sound.
2. Obtain an empty bottle and make a sound by blowing across the top. Do you have control over the intensity and/or the pitch of the sound? Explain.
3. Whistle through your lips. Describe how you control the intensity and the pitch of the whistle.
4. Experiment at talking to yourself through a tube, placing one end at your mouth and the other end over one ear. Use as many different materials and lengths of tubing as you can find (garden hose, rubber tubing, plastic tubing, and so on). Describe the effect of the material and length of the tube on the transmitted sound.

5. Collaborate with a colleague to find whether you can produce sound "shadows."

6. Interview four or more children (ages 6-10) to investigate their concept of vision and the role of light in vision. (A useful aid is an opaque piece of flexible tubing. Bend the tube into various shapes and orientations and ask whether a flashlight at the far end would be seen through the tube in each case.)

7. Use the ray model for light to explain the concept of illumination.

8. Investigate beats produced by an untuned piano. If you can, use paper clips to change the pitch (as in Fig. 5.2), try various notes and various positions of the paper clip (end of string, center of string, two paper clips on one string, and so on). Report your findings, including the rate of beats. (Count the number of beats in 5 or 10 seconds whenever they are not too fast.)

9. Investigate the interference pattern produced when a small, strong light source (a very distant street light, for example) is viewed through a piece of woven fabric (fig 5.3). Draw and describe your observations. (Variations to try: hold the fabric at various distances from your eye; stretch the fabric; distort the fabric so the threads do not cross at right angles; compare dense and loose fabrics. Give evidence that the pattern you see is not simply a "shadow" cast by the threads and holes of the fabric. Include measurements as well as you can.

10. Look at the reflection of a bright light source at medium distance (10-50 meters) from the grooved area of a phonograph record or compact disc (CD). Describe your observations.

11. Describe conditions under which you have "seen" light rays (Fig. 5.4).

12. Inside the tube at the left of Fig. 5.8 is the primary light source from which the beam originates. Use the shape of the visible beam to make inferences about the position and size of the primary source.

13. Measure the angles of incidence and reflection of the pencil of light in Fig. 5.9. (See Fig. 5.11 for the definitions.)

14. Tabulate the angles of incidence, reflection, and refraction in the four parts of Fig. 5.10. Explain why the refracted beam is not refracted again when it emerges from the glass. Find the index of refraction of the glass piece used.

15. (a) A beam of light is generated in an underwater lamp and hits the smooth water-air surface at an angle of incidence of 30°. Make a diagram to show the incident, reflected, and refracted beams. Calculate the angles of reflection and refraction and indicate

them in the diagram. Make the diagram true to the real angles.
(b) Do the same for an underwater angle of incidence of 40°.
(c) Do the same for an underwater angle of incidence of 50°.
(Hint: You will find that part (c) of the problem does *not* have the same kind of solution as parts (a) and (b). You should use a ray diagram to interpret your calculations. Discuss your conclusions.)

16. Compare the magnitude of refraction effects you expect to observe at an air-water, air-glass, air-diamond, water-glass, water-diamond, and water-alcohol (separated by a thin plastic sheet) boundary surface. Investigate one of more of these situations experimentally.

17. A pencil of light enters a pane of glass with parallel faces. Calculate the angle at which it emerges for various angles of incidence. Relate your result to your observations of noticeable refraction (or lack thereof) by the glass in a window. (Hint: the index of refraction can be found in Table 5.1.)

18. Look at the fish or other objects in a rectangular aquarium from all side directions and the top. Describe your observations, especially when you are looking into the aquarium along a diagonal line.

19. A pencil of light is incident on a glass prism (Fig. 5.13) at an angle of 60°. Find the angle at which the pencil of light emerges from the prism. The index of refraction is in Table 5.1.

20. Stick a pencil into a glass of water. Look at the submerged part of the pencil through the water-air boundary and through the water-glass-air boundary. Report the refraction effects you observe and explain them by the ray model.

21. Experiment with pieces of colored cellophane combined with one another and held over colored pictures to explore the subtraction of colors. Report your findings and relate them to Newton's model.

22. The transparent film in a 35-millimeter slide is about 1 inch high. A student wishes to buy a projector that will give a 20-inch-high picture in his room, which is 12 feet long. What focal length projector lens should he buy?

23. Use a magnifying lens for the following experiments.
(a) Find the focal length by focusing a distant object outside your window on a piece of paper at right angles to the lens axis.
(b) Hold the lens between a lit light bulb and a piece of paper so that the image of the bulb is focused on the paper. Measure the object and image distances and check whether they satisfy Eq. 5.4 with the focal length measure in (a). Comment whether discrepancies are due to experimental error or to limits of the mathematical model. If you get very poor results, repeat the measurement with a different distance between bulb and paper.
(c) Hold the lens very close to your eye and bring a small object (pencil point, fingertip, print) into focus (Fig. 5.26). Have a friend measure the distance from lens to object. Relate this distance to the

focal length.

(d) Look at an object (about 2 feet away) through the lens held close to the object. Slowly move the lens away from the object and toward your eye. Observe and describe the changes in the image seen through the lens and explain them in terms of the lens theory described in Section 5.3.

(e) Hold the lens close to your eye and look toward a very distant object. Gradually move the lens away from your eye until it is at arm's length. Observe and describe the image you see through the lens. Explain in terms of the lens theory of Section 5.3.

(f) When you use the lens as magnifier, the image of the object is projected on your retina. No measurable image is projected on a screen. It is difficult to measure the "magnification," since object size and image size cannot be compared directly. Formulate an operational definition of the magnification of a lens used as a magnifier and apply the definition to your lens.

24. Assumption 2, Fig. 5.16, refers to a "straight line." Describe how you would define "straight line" operationally and/or formally. Discuss the application of your definition to the assumption.

25. Identify one or more explanations or discussions in this chapter that you find inadequate. Describe the general reasons for your judgment (conclusions contradict your ideas, steps in the reasoning have been omitted, words or phrases are meaningless, equations are hard to follow, . . .), and make your criticism as specific as you can.

Bibliography

E. N. daC. Andrade, *Sir Isaac Newton - His Life and Work*, Doubleday (Anchor), Garden City, New York, 1958.

Sir William Bragg, *The Universe of Light*, Dover Publications, New York, 1959.

M. Minnaert, *Light and Color in the Open Air*, Dover Publications, New York, 1954.

Isaac Newton, *Opticks*, Dover Publications, New York, 1952.

U. Haber-Schaim *et. al.,* PSSC *Physics*, *Seventh Edition,* Kendall/Hunt, Dubuque, Iowa, 1991.

Michael Rodda, *Noise and Society*, Oliver and Boyd, Edinburgh and London, 1968.

R. A. Weale, *From Sight to Light*, Oliver and Boyd, Edinburgh, London, 1968.

Article from Scientific American (see archives on the Internet at http://www.sciamarchive.org/.)

L. L. Beranek, "Noise" (December 1966).