Introductory Essay

Quantum Operators and An Overview of Quantum Mechanics*

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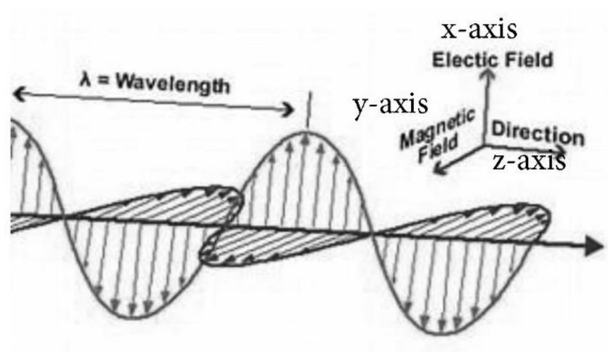
The Quantum Operators are capable of demonstrating actual quantum phenomena. In most cases, quantum behavior can only be experienced in the atomic domain, which involves complicated equipment to magnify the micro behavior sufficiently for us to perceive it. However, there is one characteristic of light (called "polarization") that manifests itself in both the atomic and the macro domain. The Quantum Operators take full advantage of this characteristic behavior to allow us to experience quantum phenomena directly.



Polarization of light is the direct result of the fact that light acts as a "transverse wave." This means that light acts as if it is vibrating perpendicular to its direction of motion, much as you can shake the end of a rope up and down or from right to left, to make vertical or horizontal vibrations travel along the rope. See diagram, which shows the electric field vibrating along the x-axis, in the vertical plane. The magnetic field, which is less important, is vibrating in the horizontal plane.

Polarization of light was first discovered by Etienne Malus in 1808. However, it was only of academic interest because there was no inexpensive, generally available way to produce and control the two independent types of polarized light.

Edwin Land invents polarizers. This all changed in 1928: Edwin H. Land, a Harvard freshman, dropped out after his first semester to invent and develop inexpensive plastic polarizing filters which transmitted one type of polarized light and absorbed the other. Land received his first patent in 1929, and he proceeded to develop an amazing cornucopia of imaginative and successful products, including the well-known Polaroid Camera, which incorporated the printing process to produce an instant photograph. He founded a major corporation, The Polaroid Corporation, and his polarizing sunglasses were adopted enthusiastically, especially by drivers and pilots, to reduce glare.

An interesting bit of scientific history that began with polarized light: In 1908, at the age of 26, Louis Pasteur started out with a brilliant series of experiments on the rotation of the plane of polarization by a solution of tartaric acid. Pasteur pushed on to discover that many organic molecules existed in two different structures that were exact mirror images. He then proved the astounding conclusion that all of life was based on only the left-handed form of the molecules. This was the beginning of the science of stereochemistry.

Quantum Operators and Quantum Phenomena. Our Quantum Operators take advantage of Land's plastic filters to make it possible to experience quantum phenomena directly, to develop the concepts and basic math needed to understand what is happening, and to predict the results correctly. The Quantum Operators are based on the polarization of light, which is a two-state system. This means that the state of polarization of a particular source or beam of light can be completely characterized by specifying the relative amounts of two independent base states of polarization, for example, the vertical and horizontal states, or the states at 45 and 135 degrees.

Analogy with electron spin. The electron spin system is another, analogous two-state system which is critically important for understanding atomic systems in general since the electron is a key component in all atoms. In many cases, such as in the silver atom, the spin of the outermost electron is the only contributor to the magnetic moment of the atom as a whole because all the other electron spins are matched up in pairs that cancel with one spin up and one down. As a result, the knowledge we gain about how the two-state system of light polarization works will be immediately useful in understanding the critically important electron spin system.

Why do we call them Quantum Operators? What do they operate on? And what do they produce? The Operators operate on **wave functions,** which are a type of mathematical function, symbolized by the Greek letter Psi (Ψ), which are assumed to exist in connection with any real-world system at the atomic level. and which contain all the information about the characteristics and state of that system. To get any information about the real-world system, that is, to make a measurement of something, one must employ an **Operator**, which is another mathematical function specifically designed to operate on the wave function so as to extract a value for whatever particular observable you want to know. The Quantum Operators we have developed are indeed operators: whenever light enters one of our Operators, it measures the state of polarization of the light.

We will be using the notation originally developed by Paul A. M. Dirac in his classic book, *The Principles of Quantum Mechanics (1930*). Quantum Mechanics is the comprehensive theory, developed by Dirac and many other physicists, to understand how things worked at the atomic level.

To use the Quantum Operators effectively in teaching, one must understand the assumptions, as well as the overall approach and structure of Quantum Mechanics, as distinct from the classical physics of Newton and Maxwell. To that end, we provide below an overview of the assumptions, approach and structure of Quantum Mechanics.

Overview of Quantum Mechanics. The theory of Quantum Mechanics was, initially, modeled on Newton's classical mechanics, as extended to cover electromagnetic phenomena by J. C. Maxwell. However, it turned out to be impossible to apply the key assumptions of classical mechanics to quantum phenomena: in particular, quantum systems were so small that measuring their key characteristics inevitably involved some disturbance of the system itself.

Conflict with Classical Mechanics. This interaction between the system and the observer immediately invalidated a fundamental assumption of classical mechanics: the assumption of determinism: based on a sufficiently accurate knowledge of the initial conditions of a given system (for example, the Solar System), classical mechanics could predict the evolution of the system and the values of its key attributes to any desired level of precision and accuracy. The apparent potential of classical mechanics for deterministic results had a strong influence on scientists' attitudes and on the way they thought about their research.

To many scientists who had been educated within the tradition of classical science and had spent their careers under the same framework, the idea that the measurement process inevitably introduced uncertainty came as a severe shock, and there was substantial resistance to the idea, including most significantly from Einstein. He explained his reservations in a 1926 letter to Max Born: "Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot but does not really bring us any closer to the secret of the 'old one'. I, at any rate, am convinced that *He* is not playing at dice."

In spite of Einstein's attitude, most scientists came to accept QM and to understand the practical implications of the probabilistic results of QM and the limitations of the deterministic model of classical science.

The Quantum – the smallest possible particle. Physicists have always tried to understand the world by separating out its constituents, studying their properties and how they interact. Particle physics represents this process carried to its ultimate conclusion with the use of more and more powerful accelerators to generate a huge "zoo" of short-lived "fundamental" particles. At the same time, physicists also came up with the idea of the Quantum, which refers to the end of the process of finding constituents. A Quantum is the ultimate constituent: the particle which doesn't have any constituents: it is the smallest possible particle.

The Photon – a Light Quantum. For many years the wave theory of light provided a comprehensive understanding of all aspects of the behavior of light, including optics, interference, and diffraction. However, the Photoelectric Effect, discovered in 1887 by Heinrich Hertz, was an exception; it seemed impossible to reconcile with an explanation based on light waves. And then in 1905, Einstein came up with a simple powerful explanation of the Photoelectric Effect based on light "quanta" called photons. This was one of the key developments that led to Quantum Mechanics. In fact, in spite of all the work of the particle physicists, no one has ever succeeded in breaking up the photon. A fraction of a photon has never been observed; it is always either a whole photon or nothing! The photon is indeed the ultimate quantum; it is the smallest possible particle of light.

The Challenge of Quantum Mechanics. Successfully developing a theory that can deal with quanta such as the photon (and the electron which has so far proved to be indivisible just like the photon) and provide a prediction of future behavior that somehow incorporates a probabilistic approach, seems difficult to say the least, if not impossible on its face. The successful development of just such a theory in Quantum Mechanics (QM) must be recognized as an extraordinary achievement that required the combined creativity and ingenuity of many of the best physicists in the world over the period from about 1900 to 1940 and beyond.

Wave Theory. The starting point for the successful development of QM turned out to be the phenomena and the well-developed theory of waves and vibrating systems, including musical instruments, rather than the model of point particles exerting forces on each other and evolving in accordance with Newton's Laws of Motion. The key model turned out to be a standing wave pattern, such as might be observed on a drumhead after it has been struck or on a violin string after it has been plucked, or in almost any musical instrument. The key characteristics of this model were the **wavelength** (the distance from one wave to the corresponding point on the next wave) and the **frequency** (the number of vibrations in every second).

The Bohr Atom. As QM developed, the analogy with waves and oscillating systems became stronger and stronger. For example, Neils Bohr developed a revolutionary model for the atom in which the key assumption was that the electron acted like a wave and that the system was stable, in what he called a "stationary state," only when the electron wavelength fitted exactly into the space around the nucleus.

Bohr's model for the atom was spectacularly successful in predicting and explaining the types of light emitted and absorbed by Hydrogen (the simplest atom). The Bohr model created as many questions as answers, but it represented obvious advantages over the Newtonian model of the electron as a miniature planet in a defined orbit around the sun, which simply couldn't escape the inevitable collapse required by Maxwell's theory of electromagnetism. In fact, the Bohr Atom led directly or indirectly to most of the rest of QM.

Quantum Mechanics today has evolved into an interlocking system of concepts that provides a robust way of thinking about phenomena at the atomic level. Every system is assumed to be described in detail by a **wave function** that is not directly observable. Measurements of the characteristics of the system, such as its position, velocity, energy, and other key variables cannot be done directly, as in classical physics, by simply using a ruler, a stopwatch, and/or a balance. All measurements must be carried out by means of an **operator**, which is a mathematical function that is specially designed to extract the desired information from the wave function. The evolution of the wave function is governed by a differential equation (the famous Schrödinger Equation). Based on the initial conditions, plus whatever is known about the interactions with the outside world, the Schrödinger Equation can produce a prediction for the wave function at a given point in the future, and the appropriate operators can then determine whatever one wants to know about the system at that time.

Classical wave functions. Since well before the development of QM, physicists had studied vibrating systems, including musical instruments, in great detail. Classical wave functions for such vibrating systems were defined such that the function itself (usually labeled with the Greek Capital Psi, Ψ) represented the physical object that was vibrating: for example, the violin string, or the pressure of the air in the organ pipe.

Quantum wave functions. As QM developed and as physicists struggled to adapt the existing classical wave functions to the new demands of the quantum world, they generally continued to use Ψ as the name of the wave function. However, quantum systems did not have any specific physical objects that were doing the vibrating. So, there was no physical object to connect with the Ψ of the wave function and scientists couldn't really explain to what it was that the Ψ was referring.

What about probability? Probability is the relative odds (usually in terms of a percentage) that a particular measurement will yield each of a series of possible values. How does the probability get incorporated into the process described above? After much questioning about what the quantum wave function was referring to, Max Born identified a new quantity, the **probability amplitude** as the proper target for the quantum wave function, with the Ψ label. Born pointed out that the probability for a particular result could be derived from the probability amplitude by simply squaring the probability amplitude, or if the probability amplitude was a complex number, multiplying it by its complex conjugate.

Born was awarded the 1954 Nobel Prize for his work on the "statistical interpretation of the wave function." In his Nobel Lecture, Born said: "I believe that ideas such as absolute certitude, absolute exactness, final truth, etc. are figments of the imagination which should not be admissible in any field of science. On the other hand, any assertion of probability is either right or wrong from the standpoint of the theory on which it is based. This loosening of thinking (*Lockerung des Denkens*) seems to me to be the greatest blessing which modern science has given to us. For the belief in a single truth and in being the possessor thereof is the root cause of all evil in the world."

Extracting values with Operators. Now, once we have used the Schrödinger Equation to find out how the quantum wave function evolves in time, we need to extract the values of the variables at the end of the period of interest. We use the appropriate operators to extract the values that we want to know, along with the related probability amplitudes. In the final step, we must square the probability amplitudes in order to find the actual probability of each of the various expected values.

The Photon and Polarization. How does the photon fit into a polarized light beam? We think of a light beam as composed of many light quanta. A light beam that is 100% polarized vertically can be considered to be composed of light quanta all of which are also vertically polarized. Now what about a light beam that is 50/50: 50% vertically polarized and 50% horizontally polarized? Are half of the photons vertically polarized and half horizontally polarized? Or are all the photons polarized 50% vertical and 50% horizontal. What about a very weak light beam, so weak that only one photon is in the apparatus at once. What is the polarization status of that photon? Exploration of these questions will take us to the topic of superposition – the shape-shifting behavior of photons and electrons that we will explore with the Quantum Operators and which defies common sense but opens the door to so much more.

*Based upon earlier work by Costas Papaliolios, as described by Jean-Francois Gauvin in "Playing with Quantum Toys: Julian Schwinger's Measurement Algebra and the Material Culture of Quantum Mechanics Pedagogy at Harvard in the 1960s," Phys. Perspect. 20 (2018) 8-42.

