1. **What you need to know about complex (im­aginary) numbers**

* Complex or imaginary numbers are all based on the square root of -1, that is, on (-1)1/2.
* This is not a “real” or a rational number and can not be represented on the number line nor as a ratio of real numbers.
* The universally-recognized symbol for (-1)1/2 is “i”.
* Thus assuming i = (-1)1/2 we can calculate,
  + i2= (-1)1/2x (-1)1/2= -1
  + l3 = -i
  + i4= +1
* In general, complex numbers can usually be written with real and imaginary parts separated, like this: a + bi.
* The complex conjugate of any complex number is the same number except with the sign of i reversed (+i becomes -i, and -i becomes +i). Complex conjugate of Z is usually designated as Z\*.
* Whenever we write z2, we actually mean z•z\*, thus (a+bi)(a-bi) = a2 + b2 .

1. **What you need to remember about traditional 3-D vectors in classical physics.**

* 3 dimensions, corresponding to the 3 dimensions of physical space.
* Basis is any 3 mutually perpendicular vectors, such as vx, vy and vz.
* Vectors have both magnitude (length) and direction.
* Length of vectors = sqroot(v2 ) = [vx2+ vy2 + vz2]1/2
* All components and coefficients are real numbers (complex numbers not allowed)
* Dot product = 0 implies that vectors are perpendicular
* Vectors usually represent measuerable quantities directly: position, speed, momentum, ….and similar quantities.

1. **What you need to know about vector spaces.**
   * Can have any number of dimensions,
   * Complex numbers are allowed and are common in components and coefficients.
   * Only the direction of the vectors has meaning. Two vectors with different lengths but the same direction are the same vector and represent the same thing.
   * There are two different types of vectors: Bras (notated like this <a|}and Kets (notated like this: |b>).
   * Most systems are represented as kets.
   * Whenever necessary, kets can be converted into their corresponding bras by simply taking the complex conjugate of all coefficients and components.
   * To multiply two vectors, use the “inner product” <a|b>, in which a Bra and a Ket come together to form a “bra-ket.” This is the key to most calculations.
   * <a|a>, the inner product of a vector with itself is usually equal to 1. All kets and bras are usually maintained in what is called “normalized form” so that <a|a> = 1. For example, if a particular light beam has equal amounts of vertical and horizontal polarization, that is, each photon in it has an equal probability of 50% of being horizontally or vertically polarized, it is represented as 1/(2)1/2 (|1> + |0>), so that the dot product with itself is 1:

[1/(2)1/2 (<1\*} + <0\*|)]• [1/(2)1/2 (|1> + |0>)]

= [1/(2)1/2 ][ 1/(2)1/2 ] [|<1\*|1> + <1\*|0\*> + <0\*|0> + <0\*|0>)

= (1/2)[1 + 0 + 0 + 1] = (1/2)(2) = 1.

* + Measurements on systems are represented in this form |a><b}
  + For example, to find out if system |a> is a vertically polarized photon, we evaluate this expression:
    - |1><1| |a>.
    - Regrouping, we get |1> (<1|a>).
    - The quantity in the parentheses is an inner product; it represents the probability that the photon represented as system |a> is in the state represented by <1|.
    - Alternatively, we can actually carry out the physical measurement with the quantum operators.
    - All systems can be analyzed into the same number of components as the number of dimensions in the underlying vector space. In direct analogy with traditional 3-D vectors, but with all numbers now potentially complex. The components in most cases are a set of “basis vectors” which are all “perpendicular” to each other with their inner products equal to zero.